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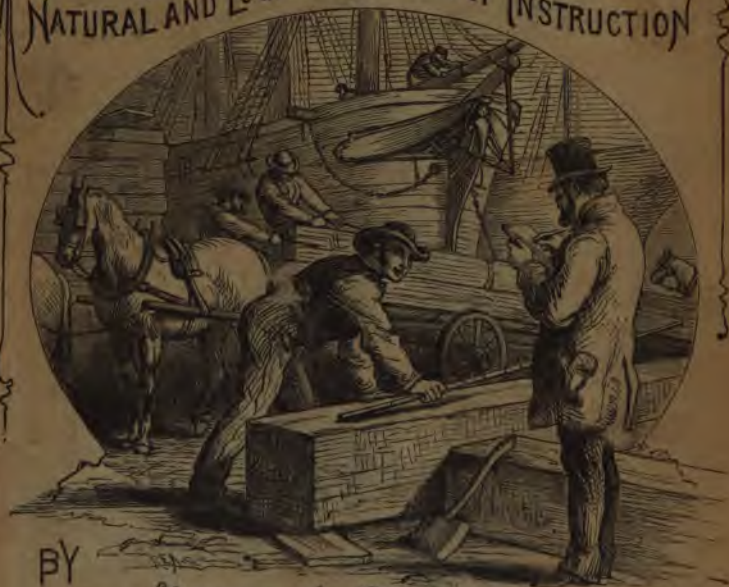
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The
COMPLETE
Arithmetic
COMBINING
ORAL AND WRITTEN
EXERCISES
IN A
NATURAL AND LOGICAL SYSTEM OF INSTRUCTION



BY

ALBERT N. RAUB A.M.

PRINCIPAL OF THE CENTRAL PENNSYLVANIA
STATE NORMAL SCHOOL

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PREFACE.

IN the author's experience as a teacher during the last twenty years, the truth that too much time is given in our schools to the subject of Arithmetic has constantly forced itself on his attention, and year by year the necessity for a shorter course in Arithmetic has increased. It is with the hope of successfully meeting this growing want that the author ventures to offer this book to the educational public.

Teachers everywhere seem to be awaking to the reality that what has been called Mental or Intellectual Arithmetic is best taught, and only properly taught, in connection with Written Arithmetic. The two are inseparable, and together constitute but one subject or branch of study.

The author believes that the term "Mental Arithmetic," as applied to the study as pursued in our schools, is a misnomer. The same mental or intellectual process is necessary however the problems be solved, whether orally or written. He, therefore, believing that the difference lies mainly in the manner of solution—that is, as to whether it be by speech or by written characters—considers the subject under the general head Arithmetic, and the processes as either *oral* or *written*.

All important processes of analysis as found in the best so-called Mental Arithmetics are assimilated in this work, but in every case the oral analysis and oral exercise are given

as preliminary to the solution of problems by the written process.

Oral Arithmetic is not ignored. The author believes in its valuable features. In no case, however, are the problems and processes carried to such an extent as to make them mere puzzles for testing the retentive power of the pupil's memory. The problems for oral solution are introductory to a fuller discussion by the written process. The written problems will be found to contain many of a more complex character than could be solved with profit orally. The author claims that by this arrangement a better knowledge of Arithmetic can be gained in a much shorter time and at much less expense than by the method of teaching *Mental Arithmetic* from one book and *Written* from another, the class usually studying and reciting on the same day topics having no connection and in no way related, except that they both belong to the science of Arithmetic.

This book, as well as its predecessor, is believed to be strictly and systematically graded, from the simplest problem for oral analysis to the most complex for written solution.

It is also practical: the problems are drawn from the various business interests of life, and are so presented as to evolve thought on the part of the pupil.

The author desires to call the special attention of teachers and others to the methods of treatment of Longitude and Time, Compound Proportion, Interest, Annual Interest, Notes, Partial Payments, Banking and Discount, Bonds, Interest on Monthly Payments, Interest on Instalments, including building association shares, Interest on Deferred Payments, Annuities, and Circulating Decimals. Many new processes are introduced also in the remaining topics discussed.

As the pupil gains growth in the proper use of language, he is called upon, in many cases, to give his own explanation of the written solutions; and as his mental strength increases still more, he is required to frame his own rules from the preceding solutions and principles. Thus he is trained to think from the beginning, and master the whole subject in such a way as will not only make him a thorough practical arithmetician, but at the same time develop in him sound mental strength.

The author hopes the progressive, practical teachers of America will find this book suited to their wants, and that by its aid the subject of Arithmetic may be thoroughly taught without the consumption of more time than necessary.

A. N. R.

CENTRAL STATE NORMAL SCHOOL, }
June 27, 1877.

SUGGESTIONS TO TEACHERS.

THE author takes the liberty of offering a few suggestions here as to the manner of using this book. He hopes that they may be of much service, especially to inexperienced teachers. A few of them are in a measure repetitions of those given in the author's *Elementary Arithmetic*.

Beginners need much practice in both reading and writing numbers. Give, therefore, a great number of examples for exercise from other sources.

The author has given a number of practical problems under each rule; it will be well for the teacher to add largely to the number.

Combination problems—that is, problems which combine the operations of several rules in their solution—will do much to evolve thought on the part of the pupil. Since the pupil cannot solve these problems by any one rule, it is necessary that he “think out” his own method of solution. It also shows him the practical application of *Arithmetic*. The teacher should add to those given by the author.

Thorough and frequent drill should be given in Addition, particularly in the addition of ledger columns. Give a thorough drill in all the fundamental rules; all others are based on these.

See that the work in *Written Arithmetic*, whether on the slate or on the blackboard, is neat and in proper order. See

also that pupils give all their solutions, analyses and explanations in grammatical language.

Give the class frequently problems selected from actual business operations and from other books. Encourage them to think for themselves and give original solutions.

Have your pupils originate problems to illustrate the principles and rules, and thus make an application of the science as they learn it.

Be sure that the pupils understand each part of their work before they pass to the next. Be thorough!

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THE
COMPLETE ARITHMETIC.

CHAPTER I.
INTEGERS.

SECTION I.
DEFINITIONS.

1. Arithmetic is the science of numbers, and the art of computing by them.

As a *science* it treats of the principles, properties and relations of numbers. As an *art* it applies this science in computation.

2. A Unit is a single thing, as *one*.

3. A Number is one or more units.

A whole number; as, 1, 5, 10, etc., is called an *Integer*.

4. A Concrete Number is one applied to a particular unit; as, three men, six horses, 173 dollars.

5. An Abstract Number is one not applied to any particular unit; as, three, six, 173, 120.

6. Similar numbers have the same unit; as, 6 *boys*, 8 *boys*, 13 *boys*.

7. Dissimilar numbers have not the same unit; as, 6 *eggs*, 8 *horses*, 13 *sheep*.

8. A Problem is a question to be solved.

9. A Principle is a general truth.

Principles are illustrated by the solution of problems, called **examples**.

10. A Rule is a statement of the method of solving a problem.

11. A Mental solution is one performed mentally, and the results held or retained in the mind.

12. A Written solution is one in which the results are written on a slate, paper or other substance.

EXERCISES.

How many units in 3? In 3 books? In ten pencils?

What is the unit of 3? Of 3 books? Of 7 apples?

Tell which are abstract and which concrete of the following numbers: 6, 7 books, 8 men, 9, 17, 16, 45 apples, 10 dollars, 4 cents, 719, 875 acres.

Are 6 boys and 8 boys similar or dissimilar numbers? Are 10 apples and 8 peaches similar or dissimilar numbers? Why?

What is the unit of 8 miles? 10 miles? 17 miles?

What is an integer?

SECTION II.

NOTATION AND NUMERATION.

13. Notation is the art of writing or expressing numbers by figures or letters.

14. Numeration is the art of reading numbers expressed by figures or letters.

Numbers may be expressed as follows:

a. By *words*; as, six, sixteen, twenty-five, etc.

b. By *letters*, called the Roman method.

c. By *figures*, called the Arabic method.

15. In practice the Arabic method is used almost exclusively, except in numbering headings, chapters, volumes, sections, etc.

16. In Roman notation seven capitals are used, I standing for one, V for five, X for ten, L for fifty, C for one hundred,

D for five hundred, and M for one thousand. Other numbers are expressed by combining or repeating these seven letters.

The value of these combinations is governed by the following principles:

1. Whenever a letter is repeated its value is repeated; thus, X equals ten; XX, twenty; XXX, thirty.

2. Whenever a letter is placed before one of greater value, the difference of the two values is the value expressed; thus, X equals ten, L equals fifty, but XL equals forty.

3. When a letter is placed after one of greater value, the sum of the two values is the value expressed; as, LX equals sixty.

4. A dash or line drawn over a letter or combination increases its value a thousand-fold; thus, $\overline{\text{LX}}$ equals sixty thousand.

ARABIC NOTATION.

17. In Arithmetic the Arabic system of notation is used. Its characters are called *figures*. They are as follows:

Printed,	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Written,	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Named,	Zero,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

By a combination of these figures any number may be represented.

18. A figure standing alone, or in the first place at the right, represents units or ones; thus, 7 in the numbers 7, 37, 417, in each case represents seven ones.

19. When a figure stands in the second place it represents tens; in the third, hundreds; in the fourth, thousands, etc. Thus, 6 in the following numbers: 60, 362, represents six tens, and in the following: 673, 5622, the 6 represents six hundreds.

One ten is called ten (10);	Six tens, sixty (60);
Two tens, twenty (20);	Seven tens, seventy (70);
Three tens, thirty (30);	Eight tens, eighty (80);
Four tens, forty (40);	Nine tens, ninety (90);
Five tens, fifty (50);	Ten tens, one hundred (100).

One ten and one is called eleven (11);
 One ten and two, twelve (12);
 One ten and three, thirteen (13);
 One ten and four, fourteen (14);
 One ten and five, fifteen (15);
 One ten and six, sixteen (16);
 One ten and seven, seventeen (17);
 One ten and eight, eighteen (18);
 One ten and nine, nineteen (19).

ORAL EXERCISE.

How many tens and units in 27? 75? 42? 83? 92? 87?
 17? 64?

How many hundreds, tens and units in 426, 793, 879, 478,
 777, 894, 625?

How many thousands, hundreds, tens and units in 6784,
 3746, 8758, 7964, 5678?

Read the following: 86, 97, 734, 568, 973, 468, 976, 472,
 4689, 9864, 8649, 6498, 8964.

WRITTEN EXERCISE.

- | | |
|--|--------------------------------|
| 1. Write 7 tens 8 units. | 6. 8 hundreds 4 tens 9 units. |
| 2. 9 tens 3 units. | 7. 5 hundreds 6 tens 3 units. |
| 3. 6 tens 7 units. | 8. 6 tens 8 units. |
| 4. 8 tens 4 units. | 9. 3 hundreds 4 tens 5 units. |
| 5. 3 hundreds 6 tens 7 units. | 10. 8 hundreds 8 tens 8 units. |
| 11. 5 thousands 6 hundreds 7 tens 8 units. | |
| 12. 3 thousands 4 hundreds 2 tens 2 units. | |
| 13. 7 hundreds 7 tens 7 units. | |
| 14. 7 thousands 9 hundreds 8 tens 1 unit. | |

PERIODS.

20. For convenience, figures are arranged in periods of three places each, the first three at the right being called the units period; the next three, the thousands period; and thus in order, going to the left, millions, billions, trillions, quadrillions, quintillions, sextillions.

TABLE.

NAMES OF PERIODS,	of Quadrillions.			of Trillions.			of Billions.			of Millions.			of Thousands.			of Units.		
	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
NUMBERS,	6	7	3,	4	8	6,	9	2	3,	4	7	8,	9	6	5,	3	0	4.

21. Figures have two values, *Simple* and *Local*.

The *Simple* value of a figure is its value when taken alone.

The *Local* value of a figure is that which arises from the place in which it is used. Thus, 5, standing alone, is 5 units; but put in the third order or place is 5 hundreds; or in the seventh order, 5 millions.

Vacant places are each filled with a cipher.

PRINCIPLES.—1. Ten units of the first order equal one of the second order; ten of the second equal one of the third; and, in general, ten of any order equal one of the next higher order.

2. The value of figures increases from right to left in a ten-fold ratio.

3. Each remove of a figure from right to left increases its value tenfold.

4. Each remove of a figure from left to right decreases its value tenfold.

EXERCISE.

Read the following:

423, 1683, 38475, 49687, 763473, 8964270, 7896843, 9134682, 70840, 9006, 8100034, 96875862, 983437869, 3434343434, 680403708, 5009, 50009, 500009, 10000001, 350490000000, 60060060, 560068000, 700701701, 120408, 102408, 10248.

Write the following :

1. Seven hundred sixty-six, 608, 870, 807.
2. 4607, 40607, 920300, 9642003.
3. 60060, 60006, 60606, 60660.
4. 100700, 100070, 107000, 100007, 107007.
5. 4506872, 350706, 3507060, 3507006.
6. 7304268721, 7804693784.
9. 9643897643847.

What place is occupied by tens? Thousands? Millions? Trillions? Hundreds of billions? Quadrillions? Sextillions? Hundreds of thousands? Tens of millions?

SECTION III.

ADDITION.

ORAL EXERCISES.

1. If I have six books and you have three, how many books have we both?
2. If Mary has 7 roses and Sarah 4, how many roses have they together?
3. A man sold a pig for 7 dollars and a sheep for 6 dollars : how much did he receive for both?
4. If you have 3 roses, 6 pinks and 7 dahlias, how many flowers have you?
5. How many are 7 roses and 4 roses? 6 boys and 3 boys? 8 books and 4 books?
6. How many are 7 and 3? 6 and 8? 4 and 9? 8 and 11?
7. Add by 2's from 1 to 25; 25 to 49.
 By 3's from 2 to 38; 38 to 72.
 By 4's from 5 to 53; to 101.
 By 5's from 7 to 67; to 127.
 By 6's from 11 to 83; to 155.
 By 7's from 9 to 93; to 177.
 By 8's from 13 to 101; to 197.
 By 9's from 10 to 118; to 226.

8. William earned 6 dollars on Monday, 4 dollars on Tuesday, 5 dollars on Wednesday, and 7 dollars on Thursday: how much did he earn in the four days?

9. A hat cost 7 dollars; a vest, 4 dollars; a coat, 12 dollars; a pair of boots, 10 dollars: how much did they all cost?

10. There are 27 boys in school, and 34 girls: how many pupils in the school?

11. Harry walked 20 miles in one day, 25 miles the next, 27 miles the next, and 30 miles the next: how far did he walk in the four days?

12. There are 12 acres in one field, 15 in another, 16 in another, 20 in another, and 18 in another: how many acres in all?

13. A merchant sold 24 dollars' worth of goods one day, 30 dollars' worth the next, 42 dollars' worth the next, and 65 dollars' worth the next: how much did he sell in the four days?

14. A farmer paid 35 dollars for one cow, 50 dollars for another, 45 dollars for another, 47 dollars for another, 60 dollars for another, 30 dollars for another, and 42 dollars for another: how much did he pay for all?

15. A horse cost 125 dollars, a carriage 150 dollars, and a set of harness 50 dollars: what did they all cost?

16. The distance from Philadelphia to Lancaster is 69 miles; from Lancaster to Harrisburg, 37 miles; and from Harrisburg to Huntingdon, 97 miles: how far from Philadelphia to Huntingdon?

17. The distance from Philadelphia to Pittsburg is 354 miles; from Pittsburg to Cincinnati, 313 miles; and from Cincinnati to Louisville, 110 miles: how far from Philadelphia to Louisville?

18. The distance from Harrisburg to Sunbury is 54 miles; from Sunbury to Williamsport, 40 miles; and from Williamsport to Elmira, 78 miles: how far from Harrisburg to Elmira?

19. A man has 17 turkeys, 27 ducks, 11 geese and 93 chickens: how many fowls has he?

20. A farmer paid 140 dollars for a horse, and 27 dollars for his keeping : for how much must he sell him to gain 23 dollars ?

21. If two men start at Pittsburg, one going east 85 miles a day, and the other west 36 miles a day, how far apart will they be at the end of one day ?

22. A farmer sold a horse for 127 dollars, a cow for 37 dollars, a wagon for 82 dollars, and a sleigh for 40 dollars : how much did he get for all ?

DEFINITIONS AND PRINCIPLES.

22. **Addition** is the process of finding the sum of two or more similar numbers.

23. The result obtained by the addition is called the *sum* or *amount*.

24. The sign of addition, +, is called *plus*, and when placed between two numbers indicates that they are to be added.

25. The following sign, =, is called the sign of equality, and when placed between two expressions it shows that they are equal. It is read *equals*, or *is equal to* ; thus, $6 + 5 + 3 = 14$, is read, 6 plus 5 plus 3 equals 14.

PRINCIPLES.—1. Only similar numbers or like orders of figures can be added.

2. The sum is of the same kind or order as the numbers added to produce it.

3. The sum is the same in whatever order the numbers are added.

WRITTEN PROBLEMS.

CASE I.

Where the Sum of no Column exceeds Nine.

(1.)	(2.)	(3.)	(4.)
Add 12 books.	24 cents.	44 apples.	42 miles.
And <u>33</u> "	<u>12</u> "	<u>33</u> "	<u>26</u> "
(5.)	(6.)	(7.)	(8.)
21 cents.	20 apples.	12 girls.	322 dollars.
33 "	33 "	30 "	143 "
<u>22</u> "	<u>44</u> "	<u>45</u> "	<u>312</u> "

(9.)	(10.)	(11.)	(12.)
312	6342	1502	453242
213	1223	2034	124314
<u>422</u>	<u>2314</u>	<u>3213</u>	<u>210313</u>

13. A man was paid 320 dollars one day, 245 dollars the second day, and 223 dollars the third day: how much was he paid in the three days?

CASE II.

To Add where the Sum of One or more Columns exceeds Nine.

Ex. 1. Add 3762, 9325, 4316, 7139.

Explanation.—For convenience in adding, write the numbers so that figures of the same order stand in the same column. For convenience also, begin to add at the right; thus, 9 units + 6 units + 5 units + 2 units are 22 units, or 2 tens and 2 units; write the two units under the column of units and add the 2 tens to the column of tens.

2 tens + 3 tens + 1 ten + 2 tens + 6 tens are 14 tens, or 1 hundred and 4 tens; write the 4 tens under the column of tens and add the 1 hundred to the column of hundreds.

1 hundred + 1 hundred + 3 hundreds + 3 hundreds + 7 hundreds are 15 hundreds, or 1 thousand and 5 hundreds; write the 5 hundreds under the column of hundreds and add the 1 thousand to the column of thousands.

1 thousand + 7 thousands + 4 thousands + 9 thousands + 3 thousands are 24 thousands, or 2 tens of thousands and 4 thousands; write the 4 thousands under the column of thousands and the 2 tens of thousands in the place of tens of thousands.

Hence, the sum of 3762, 9325, 4316, 7139 is 24542. From the principles, Section III., and the foregoing explanation, we derive the following

RULES FOR ADDITION.

1. Write the numbers so that figures of a like order shall stand in the same column.
2. Begin at the right and add each column separately.
3. When the sum of a column exceeds nine, write the right-hand figure of the result under the column just added, and add the remaining figure or figures to the next left-hand column.

PROOF.—Add the numbers in a different order ; the result is the same by both methods if the work be correct.

WRITTEN PROBLEMS.

Add—

(1.)	(2.)	(3.)	(4.)	(5.)
Dollars.	Miles.	Birds.	Pupils.	
4320	129	2743	7846	44684
6893	343	8746	9734	6763
<u>4732</u>	<u>856</u>	<u>9857</u>	<u>8697</u>	<u>891</u>

6. What is the sum of 724, 896, 347 ? *Ans.* 1967.

7. What is the sum of 842, 695, 873, 429 ?

8. What is the sum of 7104, 9325, 3462, 8973 ?

9. What is the sum of 7824, 642, 7563, 17 ?

10. Six bales of cotton weigh respectively 421, 387, 418, 391, 405 and 412 pounds : what do they all weigh ?

Ans. 2434 pounds.

11. A miller bought 5 loads of wheat, the first weighing 6224 pounds ; the second, 6420 pounds ; the third, 6583 pounds ; the fourth, 5857 pounds ; and the last, 6172 pounds : how much did the five loads weigh ? *Ans.* 31256 pounds.

12. A man had four farms ; the first contained 422 acres ; the second, 179 acres ; the third, 327 acres ; and the fourth, 197 acres : how many acres of land had he ? *Ans.* 1125 acres.

13. In a school-building there are nine rooms : in the first there are 65 pupils ; in the second, 43 ; in the third, 47 ; in the fourth, 34 ; in the fifth, 38 ; in the sixth, 54 ; in the seventh, 72 ; in the eighth, 59 ; and in the ninth, 107 : how many pupils in the building ? *Ans.* 519.

14. How many dollars are 7482 dollars, 6843 dollars, 4974 dollars, 6845 dollars ? *Ans.* 26144.

15. The deposits in a bank for a week were as follows : first day, 9437 dollars ; second, 21643 dollars ; third, 18942 dollars ; fourth, 5620 dollars ; fifth, 21947 dollars ; sixth, 12620 dollars : what were the deposits for the week ? *Ans.* 90209 dollars.

16. A farmer made in one year as follows: from sale of wheat, 232 dollars; of corn, 127 dollars; of oats, 73 dollars; of sheep, 175 dollars; of cattle, 545 dollars; of vegetables, 29 dollars; of butter, 349 dollars; of poultry, 88 dollars: how much did he make in the year? *Ans.* 1618 dollars.

17. A coal-dealer furnished me 8 loads of coal, weighing as follows: 2120 lbs., 2312 lbs., 2218 lbs., 1927 lbs., 2063 lbs., 2284 lbs., 1995 lbs., 1987 lbs.: what was the weight of all? *Ans.* 16906 lbs.

18. A lumber-dealer has 10 piles of boards, containing the following amounts respectively: 9724 ft., 8695 ft., 9653 ft., 9642 ft., 10864 ft., 9687 ft., 9586 ft., 9945 ft., 8977 ft. and 9785 ft.: how many feet of boards had he? *Ans.* 96558 ft.

Add the following:

(19.)	(20.)	(21.)	(22.)
15038	57068	12494	15838
26881	17255	56131	41919
41919	18647	80983	12173
93808	53806	38107	66768
41371	74628	81981	26081
19577	68246	49411	93808
12173	86742	13192	14971
34971	65480	98351	33249
39665	51157	10777	41919
74736	77834	98173	41371
66768	26862	11337	39665
33249	85768	24946	30350
30359	15768	19677	93808
16660	83486	77636	19577
33249	40244	31568	34736
34705	57686	63313	16669
66768	85132	63063	41371
12673	70355	72362	12173
80597	24778	64564	76768
95299	48666	89099	33249
<u>70466</u>	<u>19628</u>	<u>57165</u>	<u>86463</u>

(23.)	(24.)	(25.)	(26.)	(27.)
742	570	457	311	819
938	681	686	337	280
371	725	851	240	814
133	551	327	461	971
763	864	305	967	332
924	753	524	777	494
967	806	778	636	191
772	746	486	315	941
362	286	661	686	371
776	824	249	331	396
524	686	456	136	653
799	742	131	306	305
299	654	809	372	903
952	805	833	362	808
597	115	810	645	195
583	777	781	648	773
367	834	981	909	473
909	268	494	915	616
381	628	111	838	669
627	576	319	419	413
623	815	209	191	711
946	768	351	217	217
777	834	107	366	376
666	864	779	768	768
<u>503</u>	<u>204</u>	<u>817</u>	<u>260</u>	<u>490</u>

28. Three men enter into partnership : A puts in 6420 dollars ; B, 5130 dollars ; and C, as much as A and B together : how much did C put in, and how much A, B and C ?

Ans. C, 11550 dollars ; all, 23100 dollars.

29. A man bought a house, a farm, a store and a mill. The house cost 3500 dollars ; the farm, 7800 dollars ; the store, as much as the house and the farm ; and the mill, as much as the house and the store : what was the cost of the store and of the mill, and how much did they all cost ?

Ans. Store, 11300 dols. ; mill, 14800 dols. ; all, 37400 dols.

30. Two men start at Chicago. One travels east as follows: 1st day, 327 miles; 2d, 462 miles; 3d, 194 miles: the other travels west as follows: 1st day, 349 miles; 2d day, 621 miles; 3d, 347 miles: how far apart are they at the end of three days?
Ans. 2300 miles.

31. Rome was founded 753 years before the birth of Christ: how long since that time?

32. If the thermometer is 21 degrees below zero in January, and 105 degrees above zero in July, how much warmer is it in July than in January?
Ans. 126 degrees.

33. By railway New York is 911 miles east of Chicago, and Omaha 401 miles west: how far is it from Omaha to New York?
Ans. 1312 miles.

34. New York is east of Omaha 1312 miles, and San Francisco is 1961 miles west of Omaha: how far is it from New York to San Francisco?
Ans. 3273 miles.

35. A man dying willed his estate as follows: to his wife, 5500 dollars; to each of his four sons, 3200 dollars; to each of his three daughters, 2800 dollars; to a church, 1950 dollars; to a school, 2430 dollars: how much was the estate?
Ans. 31080 dollars.

SECTION IV.

SUBTRACTION.

ORAL EXERCISE.

1. If you have 12 turkeys, and give 7 away, how many have you left?

2. A barrel contained 18 gallons of wine; 6 gallons were sold: how much remains?

3. If you have 27 dollars, and spend 12 dollars, how much will you have remaining?

4. A horse was bought for 120 dollars, and was sold for 115 dollars: what was the loss?

5. Charles had 50 cents, and bought a book for 18 cents: how much money had he remaining?

6. A man sold a cow for 25 dollars that cost him 37 dollars: what was the loss?

7. Edwin had 75 hens, but the foxes caught 18: how many had he remaining?

8. There are 90 trees in an orchard, 27 of which are apple, and the remainder peach: how many peach trees are in the orchard?

9. A horse was bought for 110 dollars, and was sold for 128 dollars: what was the gain?

10. A man's house cost him 2850 dollars; he sold it for 2775 dollars: how much did he lose?

26. The sign of subtraction, $-$, is called and also read *minus*. When placed between two numbers it denotes that the one on the right is to be taken from the one on the left; thus, $20 - 4$ is read 20 minus 4, and means that 4 is to be subtracted from 20.

What are the values of the following expressions?

11. $15 - 5$, $12 - 3$, $18 - 7$, $16 - 4$, $27 - 9$, $18 - 5$, $90 - 12$.

12. $16 - 8$, $17 - 8$, $33 - 12$, $94 - 27$, $65 - 13$, $105 - 8$, $420 - 125$.

13. $6 + 4 - 3$, $7 + 6 - 5$, $10 + 12 - 9$, $5 + 6 - 4$, $11 + 13 - 8$.

What is the value of

14. $5 + 12 - 3$?

15. $8 + 12 - 6$?

16. $9 + 20 - 7$?

17. $18 + 12 - 9$?

18. $40 + 19 - 12$?

19. $94 + 27 - 13$?

20. $45 + 63 - 108$?

21. $33 + 26 + 9 - 30$?

22. $25 + 78 - 42 - 4$?

23. $18 + 26 + 25 - 17 + 6$?

24. $22 + 48 + 73 - 16 + 5$?

25. $84 + 28 + 15 - 70 - 10 + 6$?

26. A boy having 75 cents, earned 27 cents and spent 52 cents: how much had he then?

27. John having 50 cents, gave to Mary 15 cents, and to James 12 cents: how much had he remaining?

28. A merchant paid 40 dollars for goods, 4 dollars for freight, and sold the goods for 50 dollars: what did he gain?

29. Bought a watch for 100 dollars, a chain for 37 dollars, and sold both for 92 dollars: how much did I lose?

30. A piece of calico contained 46 yards; there were sold from it 8 yards, 12 yards, 11 yards and 13 yards: how many yards remained?

31. A roll of carpet contained 154 yards; there were sold from it 30 yards, 44 yards, 15 yards and 18 yards: how much remained?

32. A farmer had 600 bushels of corn; he sold 225 bushels and fed 127 bushels to his stock: how many bushels had he remaining?

33. A lady bought a bonnet for 12 dollars, a pair of shoes for 4 dollars, and a fan for 1 dollar; she gave the merchant a twenty-dollar bill: how much change did she get?

34. A man owed his grocer 62 dollars; he brought him 12 dollars' worth of meat, 6 dollars' worth of butter, 10 dollars' worth of lard, 20 dollars' worth of chickens, and paid the balance in cash: how much cash did he pay?

35. Bought a pair of boots for 9 dollars, a hat for 6 dollars, a coat for 12 dollars, a vest for 4 dollars, and gave the merchant a fifty-dollar bill: how much change should I get?

36. Bought potatoes for 25 dollars, cabbages for 12 dollars, beets for 7 dollars: how must I sell all to gain 5 dollars?

37. A man had 49 sheep in one field, 63 in another, and 71 in another: how many must he buy to make his number three hundred?

DEFINITIONS AND PRINCIPLES.

27. **Subtraction** is the process of finding the difference between two numbers.

28. The **Difference** is the result obtained by subtracting one number from another.

29. The **Minuend** is the number from which the other is to be taken.

30. The Subtrahend is that which is taken from the minuend.

PRINCIPLES.—1. Only similar numbers can be subtracted.

2. The sum of the subtrahend and the difference equals the minuend.

WRITTEN PROBLEMS.

CASE I.

Where the Units of the Subtrahend do not exceed those of the same Order in the Minuend.

Ex. 1. From 689 take 274.

Minuend, 689 *Explanation.*—For convenience, write the figures
Subtrahend, 274 of the subtrahend under figures of the same order in
Difference, 415 the minuend. Begin at the right to subtract. 9 units
 – 4 units = 5 units, which write in the place of units;
8 tens – 7 tens = 1 ten, which write in the place of tens; 6 hundreds –
2 hundreds = 4 hundreds, which write in the place of hundreds.

Hence the difference between the two given numbers is 4 hundreds,
1 ten and 5 units, or 415.

	(2.)	(3.)	(4.)	(5.)	(6.)
From	696	873	966	4648	878 sheep.
Subtract	<u>484</u>	<u>362</u>	<u>544</u>	<u>3237</u>	<u>125</u> sheep.

(7.)	(8.)	(9.)	(10.)
6743	96489	868964	48976
<u>5423</u>	<u>34473</u>	<u>327853</u>	<u>24515</u>

(11.)	(12.)	(13.)
98748	9878436	8986473
<u>43612</u>	<u>1346135</u>	<u>4834453</u>

14. A man bought a farm for 8768 dollars, and sold it for 6424 dollars: what was the loss?

15. Bought a house for 3426 dollars, and sold it for 4538 dollars: what was the gain?

CASE II.

Where Units of one or more Orders of the Subtrahend exceed those of the same Order or Orders in the Minuend.

Ex. 1. From 4381 subtract 2864.

Explanation.—Write the numbers as in the preceding case.
 4381 Since 4 units cannot be subtracted from 1 unit, convert 1 ten of
 2864 the 8 tens to units, thus leaving 7 tens and giving 10 units+1
 1517 unit, or 11 units; 11 units-4 units=7 units, which write in the
 place of units; 7 tens-6 tens=1 ten, which write in the place of tens;
 since 8 hundreds cannot be subtracted from 3 hundreds, convert 1 thou-
 sand of the 4 thousands to hundreds, leaving 3 thousands, and giving 10
 hundreds+3 hundreds, or 13 hundreds; 13 hundreds-8 hundreds=
 5 hundreds, which write in the place of hundreds; 3 thousands-2
 thousands=1 thousand, which write in the place of thousands. Hence
 the difference between the two given numbers is 1517.

From the explanation and the foregoing principles we derive the following

RULES FOR SUBTRACTION.

1. Write the less number under the greater, so that units of the same order shall be placed in the same column.
2. Begin at the right, and subtract each figure from the one above, writing the result beneath.
3. When a figure of the minuend is of less value than the one of the same order in the subtrahend, add ten to the figure in the minuend by converting one of the next higher order of the minuend to ten of the number from which the subtraction is to be made.

PROOF.—If the subtrahend and the difference be added, and they equal the minuend, the work is correct.

PROBLEMS.

	(2.)	(3.)	(4.)	(5.)	(6.)
From	248	963	972	785	3422
Take	<u>179</u>	<u>475</u>	<u>864</u>	<u>649</u>	<u>1478</u>
	(7.)	(8.)	(9.)	(10.)	(11.)
	4527	7846	9825	12649	7857
	<u>3872</u>	<u>4938</u>	<u>4763</u>	<u>8672</u>	<u>6859</u>

SUBTRACTION.

(12.)	(13.)	(14.)	(15.)
46835	94368	78639	86493
<u>37842</u>	<u>87272</u>	<u>46894</u>	<u>7642</u>

What is the value of

- | | |
|---------------------|---------------------|
| 16. 9000 - 909 ? | 19. 90605 - 40820 ? |
| 17. 9001 - 1009 ? | 20. 20000 - 2 ? |
| 18. 100001 - 3003 ? | 21. 40608 - 263 ? |

PRACTICAL PROBLEMS.

- Washington was born in 1732 and died in 1799: how old was he? *Ans.* 67 yrs.
- Franklin was born in 1706: how long ago was Franklin born?
- Columbus discovered America in 1492: how long before Washington's birth was America discovered? *Ans.* 240 yrs.
- How long since the discovery of America by Columbus?
- How long was it from the discovery of America to the Declaration of Independence in 1776? *Ans.* 284 yrs.
- Paid 9645 dollars for a farm, and sold it at 9780 dollars: what was the gain? *Ans.* 135 dols.
- At an election A received 225670 votes, and B, 212517 votes: what was A's majority? *Ans.* 13153.
- A town which ten years ago had a population of 3745, now has a population of 6996: what is the gain? *Ans.* 3251.
- A man bought a farm for 6720 dollars, a house for 3740 dollars, a store for 10160 dollars, and sold all for 25000 dollars: what was his gain? *Ans.* 4380 dols.
- A man's salary is 900 dollars, his income from other sources 1205 dollars; he spends 1425 dollars: how much does he save? *Ans.* 680 dols.
- A has 2500 dollars; he spends 1200 dollars for family expenses, 325 dollars for rent, and 220 dollars for travel: how much does he save? *Ans.* 755 dols.
- Bought land for 12500 dollars, sold half for 7334 dol-

lars, and the remainder for 8627 dollars: what was the gain?
Ans. 3461 dols.

13. A drover had 640 sheep; he sold 225 to one man, 217 to another, and bought enough to make his number 500: how many did he buy?
Ans. 302.

14. A grocer having 720 pounds of sugar, bought 617 pounds, and sold to one man 327 pounds, and to another 445 pounds: how many pounds had he remaining? *Ans.* 565.

SECTION V.

MULTIPLICATION.

ORAL EXERCISE.

1. What cost 8 pigs at 5 dollars each?

SOLUTION.—If 1 pig cost 5 dollars, 8 pigs will cost 8 times 5 dollars, or 40 dollars.

2. What cost 12 colts at 40 dollars apiece?

3. What will 11 books cost at 20 cents each?

4. A fly has 6 legs: how many legs have 15 flies?

5. A pigeon flies 35 miles in an hour: how far can it fly in 12 hours?

6. What cost 10 pairs of chickens at 50 cents a pair?

7. What cost 12 pounds of butter at 25 cents a pound?

8. What cost 16 bushels of potatoes at 75 cents a bushel?

9. A man earns 33 dollars a month: how much does he earn in 12 months?

10. When flour is worth 9 dollars a barrel, what are 14 barrels worth?

11. A man bought 20 acres of land at 160 dollars an acre: how much did the land cost?

12. Sold 6 pounds of butter at 22 cents a pound, and 9 dozen of eggs at 18 cents a dozen: what did I receive for both?

13. Bought 11 pounds of beef at 15 cents a pound, and 4 pounds of veal at 12 cents a pound: how much did both cost me?

14. Two men start at the same point and travel in opposite directions, one at the rate of 6 miles an hour, and the other at the rate of 9 miles an hour: how far apart are they in 12 hours?

15. I hire three men—one at 5 dollars a day, one at 4 dollars a day, and one at 3 dollars a day: how much do I pay them in a week?

16. Bought 9 pencils at 5 cents each, 3 copy-books at 15 cents each, and 12 pens at 2 cents each: how much did they all cost?

17. A drover bought 25 sheep at 4 dollars apiece, 9 cows at 30 dollars apiece: what did all cost?

18. What cost 5 pounds of sugar at 15 cents a pound, 4 pounds of rice at 12 cents a pound, 5 pounds of soap at 9 cents a pound?

19. Bought 3 pairs of shoes at 3 dollars a pair, 2 pairs of boots at 9 dollars a pair, and 2 hats at 4 dollars each, and gave the merchant a fifty-dollar bill: how much change did I get?

20. If I give 4 cows at 35 dollars each, and receive in return 20 pigs at 6 dollars each, how much is due me?

21. Sold 12 chickens at 35 cents apiece, and took in trade 10 caps at 40 cents apiece: how much is due me?

22. A has 24 apples; B has 3 times as many as A, plus 12; C has as many as A and B both: how many have they all?

23. A man earns 50 dollars a week, and spends 15 dollars a week: how much does he save in 12 weeks?

24. If 9 men dig a ditch in 15 days, how long will it take one man to dig it?

25. A man bought 25 cows at 30 dollars each; he sold 10 at 40 dollars each, 7 at 35 dollars each, and 8 at 25 dollars each: how much did he gain?

MULTIPLICATION TABLE.

1	0	1	2	3	4	5	6	7	8	9	10	11	12
	1	1	1	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	1	2	3	4	5	6	7	8	9	10	11	12
	2	2	2	2	2	2	2	2	2	2	2	2	2
	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	1	2	3	4	5	6	7	8	9	10	11	12
	3	3	3	3	3	3	3	3	3	3	3	3	3
	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	1	2	3	4	5	6	7	8	9	10	11	12
	4	4	4	4	4	4	4	4	4	4	4	4	4
	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	1	2	3	4	5	6	7	8	9	10	11	12
	5	5	5	5	5	5	5	5	5	5	5	5	5
	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	1	2	3	4	5	6	7	8	9	10	11	12
	6	6	6	6	6	6	6	6	6	6	6	6	6
	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	1	2	3	4	5	6	7	8	9	10	11	12
	7	7	7	7	7	7	7	7	7	7	7	7	7
	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	1	2	3	4	5	6	7	8	9	10	11	12
	8	8	8	8	8	8	8	8	8	8	8	8	8
	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	1	2	3	4	5	6	7	8	9	10	11	12
	9	9	9	9	9	9	9	9	9	9	9	9	9
	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	1	2	3	4	5	6	7	8	9	10	11	12
	10	10	10	10	10	10	10	10	10	10	10	10	10
	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	1	2	3	4	5	6	7	8	9	10	11	12
	11	11	11	11	11	11	11	11	11	11	11	11	11
	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	1	2	3	4	5	6	7	8	9	10	11	12
	12	12	12	12	12	12	12	12	12	12	12	12	12
	0	12	24	36	48	60	72	84	96	108	120	132	144

DEFINITIONS AND PRINCIPLES.

31. Multiplication is the process of taking one of two numbers as often as there are units in the other.

32. The number to be multiplied or repeated is called the *Multiplicand*.

33. The number denoting how often the multiplicand is taken is called the *Multiplier*.

34. The result obtained by the process of multiplying is called the *Product*.

35. The sign, $\times$, of multiplication is read *multiplied by*, or *times*; thus, 16×4 is read 16 *multiplied by* 4, or 16 *times* 4.

PRINCIPLES.—1. The multiplier is always an abstract number.

2. The product and the multiplicand are similar numbers.

3. When finding the product of two numbers, either, regarded abstractly, may be used as the multiplier.

WRITTEN PROBLEMS.

CASE I.

When the Multiplier consists of a single Order of Units.

Ex. 1. Multiply 6845 by 5.

Multiplicand, 6845	<i>Explanation.</i> —For convenience, write the multi-
Multiplier, <u>5</u>	plier under the multiplicand, and begin at the right
Product, 34225	to multiply.

5 times 5 units = 25 units, or 2 tens and 5 units; write the 5 units in the place of units and add the 2 tens to the product of tens. 5×4 tens = 20 tens; 20 tens + 2 tens = 22 tens, or 2 hundreds and 2 tens; write the 2 tens in the tens' place, and add the 2 hundreds to the product of hundreds. 5×8 hundreds = 40 hundreds; 40 hundreds + 2 hundreds = 42 hundreds, or 4 thousands and 2 hundreds; write the 2 hundreds in the hundreds' place, and add the 4 thousands to the product of thousands. 5×6 thousands = 30 thousands; 30 thousands + 4 thousands = 34 thousands, or 3 tens of thousands and 4 thousands; write the 4 thousands in the thousands' place, and the 3 tens of thousands in the tens of thousands' place. Hence, 6845 multiplied by 5 equals 34225.

Abbreviated Explanation.—5 times 5 are 25; write the 5, and reserve the 2 to add to the next column; 5 times 4 are 20, plus 2 are 22; write

the 2 tens and reserve the 2 hundreds; 5 times 8 are 40, plus 2 are 42; write the 2 and reserve the 4; 5 times 6 are 30, plus 4 are 34, which write.

	(2.)	(3.)	(4.)	(5.)
Mult.	743 men	6487 dollars	78642	468253
by	<u>7</u>	<u>6</u>	<u>8</u>	<u>4</u>

6. What cost 9 horses at 187 dollars apiece?

7. A man earns 5763 dollars in one year: how much at the same rate will he earn in 7 years?

8. There are 9765 feet in each of 9 piles of lumber: how much in all the piles?

9. If light moves 192500 miles in a second, how many miles will it move in 7 seconds?

10. If a bank earns 16743 dollars in a year, how much will it earn in 8 years at the same rate?

CASE II.

When the Multiplier consists of Two or more Orders of Units.

Ex. 1. Multiply 467 by 45.

SOLUTION. *Explanation.*—45 times 467 equals 4 tens times 467 and 5 units times 467. By solution in Case I, $5 \times 467 = 2335$.
 467 5 units times 467. By solution in Case I, $5 \times 467 = 2335$.
45 4 times 467 = 1868, and 4 tens times 467 = 1868 tens, or 1
 2335 ten thousand, 8 thousands, 6 hundreds and 8 tens; writing
 1868 these numbers in their proper places and adding, we have
 21015 21015. Hence, 467 multiplied by 45 equals 21015.

From the preceding solutions and principles we derive the

RULES FOR MULTIPLICATION.

1. Write the multiplier under the multiplicand, placing units of the same order in the same column.

2. Multiply the multiplicand by each figure of the multiplier, and write the first or right-hand figure of each product directly under the figure or term of the multiplier which produced it.

3. Add the partial products; their sum is the entire product.

2. Multiply 324 by 63.

3. Multiply 427 by 75.

4. Multiply 8763 by 27.
5. Multiply 9456 by 34.
6. Multiply 763 by 246.
7. Multiply 849 by 587.
8. Multiply 7844 by 689.
9. Multiply 8542 by 875.
10. Multiply 87642 by 35; by 305; by 8005.
11. Multiply 49600 by 544; by 5406.

CASE III.

When the Multiplier is a number of Tens, Hundreds, etc.

Ex. 1. Multiply 78 by 30.

78 *Explanation.*—It will be noticed that the plan is to place the
30 first right-hand significant figure in the units' column, and multi-
2340 ply as in the preceding cases, writing the cipher or ciphers at the
right of the product.

2. Multiply 84 by 70; 96 by 80; 875 by 60; 927 by 40; 3054 by 90; 648 by 500; 847 by 9000; 6484 by 5000; 27600 by 5400; 18015 by 54000000.
3. What cost 900 sheep at 6 dollars each?
4. What cost 600 acres of land at 175 dollars an acre?

PRACTICAL PROBLEMS.

1. There are 1440 minutes in a day: how many minutes in 30 days? How many in 365 days?
2. If one town-lot costs 650 dollars, how much will 25 similar lots cost? *Ans.* 16250 dollars.
3. If there are 640 acres in a square mile, how many acres in 36 square miles?
4. What cost 2750 bushels of corn at 95 cents a bushel?
5. There are 5280 feet in a mile: how many feet in 1312 miles, the distance from New York to Omaha.
6. If a train of cars run 28 miles an hour, how far will it run in 144 hours?
7. There are 63 gallons in a hogshead: how many gallons in 2345 hogsheads?

8. A man spent 945 dollars in a year: how much would he spend at the same rate in 27 years?

9. Bought 27 pieces of calico, each containing 43 yards, at 9 cents a yard: what did they cost me?

10. Bought a farm of 160 acres at 125 dollars an acre, and another of 123 acres at 162 dollars an acre: what did both cost?
Ans. 39926 dollars.

11. A drover bought 120 sheep at 6 dollars each, 27 cows at 47 dollars apiece, 65 pigs at 7 dollars apiece, and 24 horses at 134 dollars each: how much did all cost?

Ans. 5660 dollars.

12. A merchant bought 154 yards of carpet at 73 cents a yard, and sold it at 80 cents a yard: what was the gain?

Ans. 1078 cents.

13. Exchanged 75 bushels of corn at 95 cents a bushel, and 47 bushels of oats at 52 cents a bushel, for 53 yards of cloth at 90 cents a yard, and 125 yards of carpet at 87 cents a yard: how much cash must I pay?

Ans. 6076 cents.

14. Bought two farms for 7245 dollars each; paid 175 dollars for fencing and 227 for repairs; sold them for 16000 dollars: what was the gain?

Ans. 1108 dollars.

15. Bought 12 cows at 27 dollars each, 9 horses at 95 dollars each, and 250 sheep at 3 dollars each; sold all for 2000 dollars: what was the gain?

Ans. 71 dollars.

16. A man travels from Chicago to New York, 911 miles. He travels 10 hours a day at the rate of 6 miles an hour; how far will he be from New York at the end of 15 days?

Ans. 11 miles.

17. Two vessels are 4500 miles apart, and travel toward each other—one at the rate of 91 miles a day, and the other at the rate of 85 miles a day: how far apart are they at the end of 24 days?

Ans. 276 miles.

18. Two ships start from the same port, and travel in opposite directions—one at the rate of 75 miles a day, the other at the rate of 85 miles a day: how far apart will they be at the end of 15 days?

Ans. 2400 miles.

19. Two vessels start from New York for Liverpool; one sails at the rate of 127 miles a day, the other at the rate of 204 miles a day: how far apart will they be at the end of 9 days?

Ans. 693 miles.

ABBREVIATIONS.

CASE I.

When the multiplier is 10, 100, 1000, etc., annex to the multiplicand as many ciphers as there are ciphers in the multiplier. Thus, $725 \times 1000 = 725000$.

1. Multiply 427 by 100; by 10000.
2. Multiply 4280 by 1000; by 10.
3. Multiply 4684000 by 100; by 1000.

CASE II.

When the multiplier is a little less than 10, 100, 1000, etc., multiply by 10, 100, 1000, etc., and subtract as many times the multiplicand as the multiplier is less than 10, 100, 1000, etc., as the case may be. Thus, 425 multiplied by 97, is solved as follows:

$$\begin{array}{r} 100 - 97 = 3 \\ 425 \times 100 = 42500 \\ 425 \times 3 = 1275 \\ \text{Dif.} \quad = \overline{41225} \end{array}$$

- | | |
|--------------------------|---------------------------|
| 1. Multiply 5642 by 99. | 4. Multiply 8746 by 95. |
| 2. Multiply 7844 by 198. | 6. Multiply 87567 by 998. |
| 3. Multiply 864 by 97. | |

SECTION VI.

DIVISION.

ORAL EXERCISE.

1. At 5 dollars each, how many hats can you buy for 25 dollars?

SOLUTION.—If one hat costs 5 dollars, for 25 dollars you can buy as many hats as 5 is contained times in 25, or 5 hats. Therefore, at 5 dollars each, for 25 dollars you can buy 5 hats.

2. For 21 cents how many pens can be bought at 3 cents each?
3. There are 7 days in a week: how many weeks in 77 days?
4. How many kegs containing 9 gallons each can be filled from a hogshead of 63 gallons?
5. How many oranges at 5 cents each can be bought for 60 cents?
6. If 6 barrels of flour cost 54 dollars, how much will 1 barrel cost?
7. There are 96 trees in an orchard, and 12 trees in each row: how many rows are there?
8. If a horse travel 6 miles an hour, how long will it take to travel 48 miles?
9. If 12 dozen of eggs cost 300 cents, what will 1 dozen cost?
10. How many pairs of shoes at 7 dollars a pair can be bought for 84 dollars?
11. If 6 pounds of meat cost 72 cents, what will 8 pounds cost?

SOLUTION.—If 6 pounds of meat cost 72 cents, 1 pound will cost 12 cents, and 8 pounds will cost 8 times 12 cents, or 96 cents.

12. If 6 pairs of boots cost 54 dollars, what will 10 pairs cost?
13. If 7 yards of muslin cost 84 cents, what will 9 yards cost?
14. Mary bought 10 yards of silk at the rate of 4 yards for 12 dollars: what was the cost of 1 yard?
15. If 5 horses cost 650 dollars, how much will 12 horses cost?
16. A man traded 5 sheep at 9 dollars each, and 2 cows at 30 dollars each, for pigs at 5 dollars each: how many pigs did he get?
17. If 16 bushels of wheat make 4 barrels of flour, how many bushels will be required to make 9 barrels?

18. If 16 bushels of wheat make 4 barrels of flour, how many barrels of flour will 84 bushels of wheat make?

19. Five men bought a horse for 160 dollars; they hire him out at 4 dollars a day for 25 days, and sell him for 120 dollars: what will each one gain?

20. A man had 100 dollars; he bought a suit of clothes for 25 dollars, a hat for 6 dollars and a pair of shoes for 4 dollars: how many books at 5 dollars apiece can he get for the remainder of his money?

21. A grocer bought 15 barrels of flour for 100 dollars; he sold it so as to gain 20 dollars: how much did he receive per barrel?

22. Five times my age + 4 times my age + 6 times my age - 5 times my age, equals 130 years: how old am I?

23. A farmer buys 40 pigs at 7 dollars each; he desires to keep 5, and sell the remainder so that those he keeps shall cost him nothing: how much does he get for each pig sold?

DEFINITIONS AND PRINCIPLES.

36. **Division** is the process of finding how many times one number or quantity is contained in another.

37. The number to be divided is called the *Dividend*.

38. The number which is contained in the other is called the *Divisor*.

39. The result of the division is called the *Quotient*.

40. When the divisor is not contained an exact number of times, the number left undivided is called the *Remainder*.

41. The sign of division, $\div$, when placed between two numbers, indicates that the one on the left is to be divided by the one on the right, and is read *divided by*.

42. Division is also sometimes denoted by writing the dividend above the divisor, with a short horizontal line between; thus, $\frac{36}{9}$, or thirty-six divided by nine.

43. There are two methods of Division, called *Long Division* and *Short Division*.

44. In Short Division the process is performed mentally, and the partial quotients only are written.

45. In Long Division each process of the solution is written.

PRINCIPLES.—1. The dividend and the divisor are always similar numbers.

2. The quotient is an abstract number.

3. The remainder and the dividend are similar numbers.

4. The dividend is equal to the divisor multiplied by the quotient, plus the remainder.

5. Division is the reverse of multiplication.

6. Division may be regarded as a concise method of subtraction.

SHORT DIVISION.

WRITTEN PROBLEMS.

Ex. 1. Divide 8656 by 4.

SOLUTION. *Explanation.*—For convenience, write the divisor to the

4)8656 left of the dividend, and divide the highest order of units first. 4 is contained in 8 thousands 2 thousands times, which write in the thousands' place; 4 is contained in 6 hundreds

1 hundred times, with a remainder of 2 hundreds; write the 1 in the hundreds' place, and convert the remainder into tens, making 25 tens; 4 is contained in 25 tens 6 tens times, with a remainder of 1 ten; write the 4 in the tens' place, and convert the 1 ten into units, making 16 units; 4 is contained into 16 units 4 units times, which write in the units' place.

Abbreviated Explanation.—4 is contained in 8 twice; in 6 once, with 2 remaining; in 25, 6 times, with 1 remaining; in 16, 4 times.

Hence 4 is contained in 8656, 2164 times.

2. Divide 644 by 4; 786 by 3.

3. Divide 968 by 4; 875 by 5.

4. Divide 8750 by 5; 80705 by 5.

5. Divide 72360 by 8; 91600 by 8.

6. Divide 6687 by 9.

7. Divide 81396 by 9.

8. Divide 13704 by 12.

9. Divide 37356 by 11.

10. If 1 hat cost 8 dollars, how many hats can you buy for 9136 dollars?

11. If 1 sheep cost 7 dollars, how many can you buy for 1953 dollars?

LONG DIVISION.

Ex. 1. Divide 9675 by 26.

SOLUTION. *Explanation.*—26 is not contained in 9 thousands any thousands times; hence we convert the 9 thousands to 26)9675(372 hundreds, making with 6 hundreds 96 hundreds.

78	
187	
182	
55	
52	
3	

26 is contained in 96 hundreds 3 hundreds times; 3 hundreds times 26 are 78 hundreds, which subtracted from 96 hundreds leaves 18 hundreds, or 180 tens, making with 7 tens 187 tens.

26 is contained in 187 tens 7 tens times; 7 tens times 26 are 182 tens, which, taken from 187 tens, leave 5 tens or 50 units, making with 5 units 55 units.

26 is contained in 55 units 2 units times; 2 units times 26 are 52 units, which, taken from 55 units, leave a remainder of 3 units.

Hence, 26 is contained in 9675, 372 times, with a remainder of 3.

From the foregoing we derive the following

RULES FOR DIVISION.

1. For convenience, draw curves to the right and the left of the dividend, placing the divisor on the left.

2. Divide the least number of left-hand figures that will contain the divisor, and place the quotient at the right of the dividend.

3. Multiply the divisor by the quotient, write the product under the partial dividend used, and subtract; to the remainder annex the next figure of the dividend. Proceed in a like manner until the whole dividend has been used.

4. If any partial dividend will not contain the divisor, place a cipher in the quotient, annex the next figure of the dividend and proceed as before.

PROOF.—Multiply the quotient by the divisor and add the remainder. If this result equals the dividend, the work is correct.

2. Divide 2709 by 7.
3. Divide 5024 by 8.
4. Divide 18880 by 5.
5. Divide 8112 by 12.
6. Divide 19344 by 13.
7. Divide 9840 by 15.
8. Divide 27744 by 17.
9. Divide 102144 by 19.
10. Divide 81098 by 23.
11. Divide 1332936 by 27. *Ans.* 49368.
12. Divide 477576 by 134. *Ans.* 3564.
13. Divide 1678 by 127. *Ans.* 13. *Rem.* 27.
14. Divide 208035 by 345. *Ans.* 603.
15. Divide 2121 by 101. *Ans.* 21.
16. Divide 1485267 by 321. *Ans.* 4627.
17. Divide 290625 by 465. *Ans.* 625.
18. Divide 1279026 by 63. *Ans.* 20302.
19. Divide 252144 by 612. *Ans.* 412.
20. Divide 1122586 by 983. *Ans.* 1142.
21. Divide 4192226 by 1021. *Ans.* 4106.
22. Divide 4964916 by 1276. *Ans.* 3891.
23. Divide 65240856 by 4404. *Ans.* 14814.
24. Divide 18286268 by 9134. *Ans.* 2002.
25. Divide 187242 by 413. *Ans.* 453. *Rem.* 153.

NOTE.—When division is used to find the number of equal parts, both dividend and divisor may be considered abstract numbers. Thus, if 6 cows cost 180 dollars, 1 cow costs as many dollars as 6 is contained times in 180, which is 30. Hence one cow cost 30 dollars, if 6 cows cost 180 dollars. Questions of this kind may also be solved by fractions.

PRACTICAL PROBLEMS.

1. A man has 12000 dollars to invest in land: how many acres can he buy at 125 dollars an acre? *Ans.* 96.
2. A man owns 72640 acres of land, and desires to sell it in tracts of 320 acres each: how many tracts are there? *Ans.* 227.

3. A merchant has 1620 yards of calico which he wishes to cut in 15-yard patterns: how many patterns will he have?

Ans. 108.

4. A boat sails 1872 miles, going at the rate of 18 miles an hour: how many hours does it sail?

Ans. 104.

5. If land costs 95 dollars an acre, how much can be bought for 22515 dollars?

Ans. 237 acres.

6. There are 63 gallons in a hogshead: how many hogsheads in 1575 gallons?

Ans. 25.

7. A man earns 2920 dollars in 365 days: how much does he earn in a day?

Ans. 8 dols.

8. I sold 345 birds for 7245 cents: how much was that for each bird?

Ans. 21 cents.

9. If 4671 building-lots are worth 1985175 dollars, how much is one building-lot worth?

Ans. 425 dollars.

10. 72 houses in a city are valued at 65664 dollars: what is the average value?

Ans. 912 dols.

11. If 2498 acres of land cost 322242 dollars, how much will one acre cost?

Ans. 129 dols.

12. If 1988 hogsheads of molasses cost 115304 dollars, what will 1 hogshead cost?

Ans. 58 dols.

13. Bought 137 shares of railway-stock for 17125 dollars: how much did 1 share cost?

Ans. 125 dols.

14. A ship sailed 13932 miles in 36 days: how far did it sail in 1 day?

Ans. 387 miles.

15. Bought 76 horses for 16188 dollars: how much did 1 cost?

Ans. 213 dols.

16. Traded 16 horses for 48 cows at 43 dollars each: how much was 1 horse worth?

Ans. 129 dols.

17. A farmer desires to trade 123 acres of land at 117 dollars an acre for woodland at 39 dollars an acre: how many acres will he get?

Ans. 369.

18. A man has 8000 dollars; he buys two houses for 4500 dollars, and some land at 140 dollars an acre: how many acres of land can he buy?

Ans. 25.

19. The product of two numbers is 4312695; one of the numbers is 1205: what is the other? *Ans.* 3579.

20. The product of three numbers is 107100; one of the numbers is 42, another 34: what is the third? *Ans.* 75.

ABBREVIATED PROCESSES IN LONG DIVISION.

CASE I.

To Divide when there are Ciphers at the Right of the Divisor.

Ex. 1. Divide 87543 by 400.

SOLUTION 1. *Explanation.*—400 is contained in 875 hundreds 218 times, with a remainder of 3 hundreds. 400 is not contained in 43, hence the entire remainder is 343.

4,00)875,43

218... 343 *Rem.*

2. Divide 87425 by 3900.

SOLUTION 2. *Explanation.*—First divide both divisor and dividend by 100, which is done by cutting off the two right-hand figures in each; this leaves a remainder in the dividend of 25. Divide 874 by 39, leaving a remainder of 16 hundreds. Since the dividend 874 is hundreds, this remainder, united with 25 units, the previous remainder, gives the true remainder, 1625.

39)00)874|25(22
78
94
78
16

Rem. 1625

From the foregoing we derive the following

RULES.

1. *Cut off the ciphers at the right of the divisor, and as many places at the right of the dividend.*

2. *Divide the remaining part of the dividend by the remaining part of the divisor, and to the remainder annex the figures cut from the dividend.*

What is the value of the following:

$$3. 37845 \div 600?$$

$$4. 8743 \div 500?$$

$$5. 6849 \div 700?$$

$$6. 87496 \div 1100?$$

$$7. 87496 \div 11000?$$

$$8. 94735 \div 120?$$

9. 700000 dollars was paid to 1400 men: how much did each get? *Ans.* 500 dollars.

10. If 500 horses cost 37500 dollars, what will 1 cost ?

Ans. 75 dollars.

11. How many men at a salary of 900 dollars a year will earn 585000 dollars ?

Ans. 650.

CASE II.

To Divide by a Composite Number.

46. A Composite number is one which may be produced by multiplying together two or more numbers, called factors.

Ex. 1. Divide 10950 by 15.

SOLUTION.

$$\begin{array}{r} 3 \overline{)10950} \\ 5 \overline{)3650} \\ \underline{730} \end{array}$$

Explanation.—15 equals 3 times 5 ; 3 is contained 3650 times in 10950 ; 5 is contained 730 times in 3650. Hence, 15 is contained 730 times in 10950.

2. Divide 11412 by 36. *Ans.* 317.

3. Divide 94824 by 54. *Ans.* 1756.

4. Divide 18192 by 48. *Ans.* 379.

5. Divide 339240 by 132. *Ans.* 2570.

6. Divide 60466 by 49. *Ans.* 1234.

7. Divide 692208 by 69. *Ans.* 10032.

8. Divide 3717 by 30.

SOLUTION.

2)3717

3)1858 + 1 unit = 1

5)619 + 1 two = 2

123 + 4 sixes = 24

Remainder = 27

Explanation.— $30 = 2 \times 3 \times 5$; dividing 3717 by 2, there remains 1 unit, and the quotient becomes twos ; dividing the quotient by 3 there remains 1 two, and the quotient becomes 2×3 , or sixes ; dividing by 5 there remain 4 sixes, or 24, which added to 1 two and 1 unit equals 27, the true remainder.

9. Divide 46827 by 27. *Ans.* 1734. Rem. 9.

10. Divide 87468 by 64. *Ans.* 1366. Rem. 44.

11. Divide 97648 by 63. *Ans.* 1549. Rem. 61.

12. Divide 8742 by 125. *Ans.* 69. Rem. 117.

13. Divide 76842 by 96. *Ans.* 800. Rem. 42.

14. Divide 87648 by 81. *Ans.* 1082. Rem. 6.

PRACTICAL PROBLEMS.

1. If 17 horses cost 1802 dollars, how much will 9 horses cost?

SOLUTION.

$$\begin{array}{r} 17 \overline{)1802} \\ \underline{106} \\ 9 \\ \underline{954} \end{array}$$

Explanation.—One horse will cost as many dollars as 17 is contained times in 1802, which is 106; and 9 horses cost as much as 9×106 dollars = 954 dollars.

2. If 21 bushels of potatoes cost 1365 cents, how much will 17 bushels cost? *Ans.* 1105 cents.

3. Thirteen houses cost 16250 dollars: what will 25 houses cost? *Ans.* 31250 dollars.

4. Forty-three men earn 3225 dollars: how much will 32 men earn? *Ans.* 2400 dollars.

5. 125 cows cost 5250 dollars: what will 500 cows cost? *Ans.* 21000 dollars.

6. 1200 bushels of wheat were sold for 1500 dollars: what are 700 bushels worth at the same rate? *Ans.* 875 dollars.

7. 15 men can husk 1095 bushels of corn in a day: how many bushels can 27 men husk? *Ans.* 1971 bushels.

AVERAGE.

8. What is the average of 16, 21, 40, 23, 60?

SOLUTION.

$$\begin{array}{r} 16 \\ 21 \\ 40 \\ 23 \\ 60 \\ \hline 5 \overline{)160} \\ 32 \end{array}$$

NOTE.—To find the average of two or more numbers, add them, and divide the sum by the number of numbers.

9. What is the average of 79, 118, 97, 105, 80, 85? *Ans.* 94.

10. A man earned in 6 months the following amounts: 18 dollars, 21 dollars, 12 dollars, 27 dollars, 30 dollars and 36 dollars: what was the average? *Ans.* 24 dollars.

11. The ages of five men were as follows: 35 years, 22 years, 40 years, 29 years, 54 years: what was the average age? *Ans.* 36 years.

CHAPTER II.

UNITED STATES MONEY.

SECTION I.

DEFINITIONS AND PRINCIPLES.

47. United States Money is the legal currency of the United States. It is sometimes called *Federal Money*.

TABLE.

10 mills (<i>m.</i>)	equal 1 cent, <i>ct.</i>
10 cents	" 1 dime, <i>d.</i>
10 dimes or 100 cents	" 1 dollar, <i>\$.</i>
10 dollars	" 1 eagle, <i>E.</i>

In business the denominations in use are dollars, cents and mills.

A *quarter-dollar* is 25 cents; a *half-dollar* is 50 cents.

The *dollar* is the unit, and is denoted by the following symbol: \$, called the dollar-sign.

Dollars are separated from cents in writing by a point, called a *separatrix*. Thus, \$6.95 is read six dollars and ninety-five cents; \$62.03, 62 dollars and 3 cents. Any number of cents less than ten when written with dollars, occupies the second place to the right of the *separatrix*, and the first place is occupied by a cipher, denoting that there are no dimes.

The third place at the right denotes mills.

United States money consists of coin and paper money.

Coin is sometimes called "specie," and *paper money*, "paper currency."

The principal *gold* coins are the fifty-dollar piece, the double-eagle (\$20), the eagle (\$10), the half-eagle, the quarter-eagle and the dollar.

The principal *silver* coins are the following: dollar, half-dollar, quarter-dollar, dime, half-dime and three-cent piece.

The *nickel* coins are the five-cent piece, the three-cent piece and the cent.

The *copper* coins are the two-cent piece and the cent.

Gold coins consist of 9 parts pure gold, and 1 part composed of copper and silver. The copper and the silver are added to harden the coin.

Silver coin consists of 9 parts of silver and 1 part copper.

ORAL EXERCISE.

1. How many cents in 5 dimes?
2. How many cents in 3 dollars?
3. How many cents in 4 dollars and 15 cents?
4. How many dollars in 4 five-dollar bills?
5. How many cents in a quarter-dollar and 5 cents?
6. How many dimes are equal to 8 dollars?
7. If a book cost 10 cents, how many copies can be bought for 2 dollars?
8. How many cents in one dollar and a half?
9. How many cents in one dollar and three quarters?
10. How many pencils at 5 cents each can be bought for three-quarters of a dollar?
11. If 4 books cost 60 cents, how many can be bought for one dollar and eighty cents?

Read the following:

\$2.19	\$147.12	\$103.30	\$.255
\$4.07	\$10.00	\$301.01	\$.003
\$16.10	\$120.06	\$1004.00	\$.033
\$16.11	\$120.60	\$624.371	\$3.333

Write—

1. Four dollars and fifteen cents.
2. Sixteen dollars and twenty cents.
3. Four hundred seventeen dollars fifty-three cents and six mills.
4. Ninety-five dollars and five cents.
5. Ten dollars ten cents and one mill.
6. Two cents and two mills.
7. Three dollars three cents and three mills.
8. Forty-nine dollars and a half.
9. Seventy-five cents and five mills.
10. One hundred dollars one cent and one mill.

SECTION II.

REDUCTION OF UNITED STATES MONEY.

Since 1 ct. = 10 mills, and \$1 = 100 cents, or 1000 mills,

1. *To reduce cents to mills, multiply by 10, or annex a cipher.*
2. *To reduce dollars to cents, multiply by 100, or add two ciphers.*
3. *To reduce dollars to mills, multiply by 1000, or add three ciphers.*
4. *To reduce dollars and cents to cents, or dollars, cents and mills to mills, remove the dollar-sign and separatrix.*

Conversely,

5. *To reduce mills to cents, divide by 10; cents to dollars, divide by 100; and mills to dollars, divide by 1000, and give the sign of the denomination required.*

WRITTEN PROBLEMS.

1. How many mills in 5 cents? In 27 cents? In \$25? In \$120?
2. How many cents in \$50? In \$270? In \$100?
3. How many mills in \$.75? In \$.615?
4. How many cents in \$75.50? In \$910.27?

5. How many mills in 6 cents and 5 mills?
6. How many mills in \$.065?
7. Reduce 10000 mills to cents; to dollars.
8. Reduce 749 mills to cents and mills.
9. Reduce 15643 mills to dollars, cents and mills.

NOTE.—In business transactions when in the result a fractional part of a cent is equal to half a cent, or greater, it is considered as a cent; and when less it is usually not counted.

SECTION III.

ADDITION OF UNITED STATES MONEY.

ORAL EXERCISE.

1. A hat cost \$4.50 and a pair of boots \$7.50: what did both cost?
2. A lady bought a bonnet for \$10, a hat for \$5.50 and a pair of shoes for \$4.50: what did they all cost?
3. A farmer sold a turkey for \$1.50, a pair of ducks for \$1.25 and a pair of chickens for 75 cents: how much did he get for all?
4. An arithmetic cost \$1, a reader \$1.50, an algebra \$1.25 and a grammar 90 cents: what did all cost?
5. Bought some coffee for 40 cents, some sugar for 60 cents, some raisins for 75 cents and some rice for 50 cents: how much did they all cost?

WRITTEN PROBLEMS.

NOTE.—In solving problems in Addition of U. S. Money write the numbers, placing cents under cents, dollars under dollars and add as in simple addition; separate dollars and cents by a point and prefix the dollar-sign.

Add the following:

(1.)	(2.)	(3.)	(4.)
\$17.50	\$15.25	\$716.25	\$543.03
19.25	63.47	922.00	789.06
181.16	89.92	417.30	507.19
<u>304.20</u>	<u>127.84</u>	<u>617.03</u>	<u>1272.91</u>

5. A merchant has due him the following amounts: \$125.50, \$117.23, \$217.16, \$750.25 and \$874: how much is due him?

Ans. \$2084.14.

6. A merchant's sales for the week were as follows: \$127.15, \$62.73, \$125.50, \$374.20, \$206.19, \$572.18: what was the total amount?

Ans. \$1467.95.

7. Mr. Brown built a house; the mason's bill was \$348.25; the carpenter's, \$814.20; the plasterer's, \$427.50; the painter's, \$315.75; and the tinner's, \$127.40: what was the cost of the house?

Ans. \$2033.10.

8. Paid \$2050.75 for a house, \$1000 for a lot, \$127.50 for fencing and \$17.25 for sodding: what was the cost of the property?

Ans. \$3195.50.

9. Bought a suit of clothes for \$45.50, a hat for \$5.75, a pair of boots for \$9.50, some handkerchiefs for \$6.50 and collars for \$1.25: what was the cost of all?

Ans. \$68.50.

10. Bought coal for \$50.50; groceries, \$305.17; dry goods, \$220.30; milk, \$39.45; butter, \$95.20; and paid rent, \$225: what was the cost of all?

Ans. \$935.62.

SECTION IV.

SUBTRACTION OF UNITED STATES MONEY.

ORAL EXERCISE.

1. Bought a book for \$1.25, and sold it for \$1.50: what was the gain?

2. A merchant bought goods for \$5.75, and sold them for \$7: what was the gain?

3. I bought two chickens for \$1.25 and a turkey for \$1.50; sold them all for \$3.25: what was the gain?

4. Bought a cow for \$40, a sheep for \$7.50 and some pigs for \$23.25, and paid for them with a hundred-dollar bill: what change should I receive?

5. A man paid \$125 for a horse, \$160 for a buggy, and sold them for \$270.50: what was the loss?

6. I bought a pound of rice for 15 cts., crackers for 13 cts., raisins for 25 cts., candy for 10 cts.; gave the clerk a half-dollar and a quarter of a dollar: how much change should I get?

WRITTEN PROBLEMS.

NOTE.—In solving problems in Subtraction of United States Money, write the numbers, placing cents under cents, dollars under dollars, and proceed as in simple Subtraction; separating the dollars from the cents in the result, and prefixing the dollar-sign.

(1.)	(2.)	(3.)
From \$468.25	\$4637.25	\$5000.25
Take <u>\$394.50</u>	<u>\$3675.25</u>	<u>\$3562.75</u>

4. My house cost me \$4217.25; I sold it for \$4500: what did I gain? *Ans.* \$282.75.

5. A man's property is worth \$16845.50; he owes to different persons \$7856.75: what is he worth? *Ans.* \$8988.75.

6. I bought some horses for \$575, and sold them for \$620.25: what was my gain? *Ans.* \$45.25.

7. A man had \$15000; he gave \$5762.50 for one farm and \$7845.75 for another: how much had he remaining?

Ans. \$1391.75.

8. A merchant has store-goods worth \$15965.27, a house worth \$5000, some city lots worth \$4000: what will he gain if he sell all for \$27000? *Ans.* \$2034.73.

9. A clerk receives \$2000 for salary; he expends \$325 for rent, \$647.25 for groceries, \$265.17 for dry goods, \$27.50 for books, \$127.15 for bread and \$315.62 for other expenses: how much does he save? *Ans.* \$292.31.

SECTION V.

MULTIPLICATION OF U. S. MONEY.

ORAL EXERCISE.

1. What will 6 books cost at \$1.50 each?
2. What will 3 pairs of boots cost at \$7.50 a pair?
3. What will 30 yards of carpet cost at \$1.20 a yard?

4. What will 42 yards of calico cost at 11 cents a yard?
5. What is the cost of 12 barrels of flour at \$8.75 a barrel
6. Bought 5 barrels of flour at \$8.50 a barrel, and 6 bushels of wheat at \$1.25 a bushel: what was the cost of both?
7. What cost 2 pairs of chickens at 75 cts. a pair, and 5 pairs of ducks at 60 cts. a pair?
8. Bought 5 pounds of coffee at 35 cts. a pound, and 12 pounds of ham at 22 cts. a pound: how much change do I get from a five-dollar bill?

WRITTEN PROBLEMS.

NOTE.—Multiply in United States Money as in simple Multiplication; separate dollars from cents, and prefix the dollar-sign.

1. Multiply \$617.30 by 9.
2. Multiply \$741.25 by 20.
3. Multiply \$1247.75 by 43.
4. A farmer sold 175 acres of land at \$37.50 an acre: how much did he get for the land? *Ans.* \$6562.50.
5. A miller sold 525 barrels of flour at \$6.71 a barrel: how much did he receive for all of it? *Ans.* \$3522.75.
6. What are 27 horses worth, at \$127.25 apiece? *Ans.* \$3435.75.
7. A man pays \$12 a month for car-fare, \$10 for postage and \$25 for room-rent: how much does he spend for all these in 36 months? *Ans.* \$1692.
8. A drover bought 25 cows at \$37.25 each, 15 sheep at \$6.15 each and 12 oxen at \$72.20 each: what was the cost of all? *Ans.* \$1889.90.
9. A merchant has an income of \$5000; he pays \$125 a month for clerk-hire, \$27.25 a month rent, and \$1110.15 for other expenses for the year: how much does he save? *Ans.* \$2062.85.
10. Mr. Good bought 15 hogsheads of molasses, containing 63 gallons each, at 95 cents a gallon, and sold it at \$1.10 a gallon: what was his gain? *Ans.* \$141.75.

SECTION VI.

DIVISION OF UNITED STATES MONEY.

ORAL EXERCISE.

1. How many turkeys at \$1.50 apiece can I buy for \$9?
2. If 4 cows cost \$126, how much will 1 cost?
3. If 7 hens cost \$2.17, what will 1 cost?
4. At 6 cents each, how many oranges can be bought for \$5.58?
5. Eight tons of coal cost \$44: what is the cost of 1 ton?
6. At 25 cts. a dozen, how many eggs can be bought for \$5.50?
7. If a man earn \$45 in 6 days, how much can he earn in 9 days?
8. A barrel of flour cost \$6.25: how many barrels can be bought for \$50?
9. A yard of calico is worth 12 cents: how much change will I get if I give a two-dollar bill for 15 yards?
10. I bought 3 yards of cloth at \$2.50 a yard, and a pair of boots at \$8; I gave the clerk \$20: how many hats at 75 cents each can I get for the change due me?

WRITTEN PROBLEMS.

NOTE 1.—When finding how often one sum of money is contained in another, reduce both sums to the same denomination and divide as in simple numbers.

2. When finding equal parts, divide as in simple numbers, placing the separatrix between dollars and cents.

EX. 1.—CASE I. At 21 cents each, how many ducks may be bought for \$3.36?

SOLUTION.

21)336(16 ducks.

21
126
126

Explanation.—If 1 duck cost 21 cents, for \$3.36, or 336 cents, as many can be bought as 21 is contained times in 336, or 16 ducks.

Ex. 2—CASE II. If 5 barrels of flour cost \$52.50, what will 1 barrel cost?

SOLUTION. *Explanation.*—If 5 barrels of flour cost \$52.50, or 5250 cents, 1 barrel will cost as many cents as 5 is contained times \$10.50 in 5250, or 1050 cents, which is equal to \$10.50.

3. If a man earn \$360 a year, how much does he earn in a month? *Ans.* \$30.

4. If 63 gallons of molasses are worth \$78.75, what is 1 gallon worth? *Ans.* \$1.25.

5. If 45 acres of land cost \$1010.25, what is the cost of 1 acre? *Ans.* \$22.45.

6. If 37 tons of coal cost \$146.15, what will 1 ton cost? *Ans.* \$3.95.

7. A man paid \$2336.65 for 17 acres of land: what was the cost of 1 acre? *Ans.* \$137.45.

8. If 93 oranges cost \$5.58, what will 75 oranges cost? *Ans.* \$4.50.

9. If 15 dozen of eggs cost \$1.875, what will 25 dozen cost? *Ans.* \$3.125.

10. A hat cost \$3.75: how many could I buy for \$82.50? *Ans.* 22.

11. A farmer paid \$5546.25 for 45 acres of land: what was the price per acre? *Ans.* \$123.25.

12. How many sheep can be bought for \$302.95 at \$4.15 each? *Ans.* 73.

13. A grocer bought 250 pounds of coffee for \$82.50, and sold it at 37 cents a pound: what was his gain? *Ans.* \$10.

14. A drover bought 47 cows for \$1750.75, and sold them at \$40.50 each: what did he gain? *Ans.* \$152.75.

15. What will 53000 bricks cost at \$7.25 per thousand? *Ans.* \$384.25.

16. Exchanged hats at \$6 each for 48 yards of cloth at \$2.50 a yard: how many hats did I get? *Ans.* 20.

17. A worked 36 days at \$1.25 a day, and was paid in wheat at \$1.50 a bushel: how many bushels of wheat did he get? *Ans.* 30.

18. Bought a house for \$3453, and paid for it in installments of \$575.50 each: how many payments did I have to make?
Ans. 6.

SECTION VII.

BILLS AND ACCOUNTS.

48. A **Bill of Goods**, or **Invoice**, is a written statement of goods sold, giving quantity and price of each article and total cost; also the date of the transaction, with the names of the buyer and the seller.

49. An **Account** is a record of business transactions between two parties, giving both debits and credits. The party who owes is called a *Debtor*, and the one to whom a debt is owed is called a *Creditor*.

50. A bill is said to be receipted when the words *Received payment* are placed at the bottom, and the name of creditor signed either by himself or by his agent or authorized clerk.

51. A **Statement** of an account is a bill of the items of an account.

In bills @ signifies *at*; C, hundred or hundreds; and M or **M**, thousands.

BILLS.

Make out each of the following bills in proper form, with date, and receipt it.

(1.)

BOSTON, MASS., June 6, 1875.

MESSRS. PHELPS & CO.,

Bought of BARBER & PERKINS.

15 reams Paper	@	\$4.00	\$60.00
4 " Note Paper	@	2.00	8.00
3 " " "	@	1.75	5.25
2 M Envelopes	@	2.70	5.40

Ans. \$78.65.

(2.)

PHILADELPHIA, PA., Jan. 1, 1876.

MR. WM. SMITH,

Bought of JONES & Co.

50 lbs. Sugar	@	11¢	\$ 5.50
275 " Starch	@	12¢	33.00
25 boxes Soap	@	\$2.45	61.25
40 sacks Salt	@	2.15	86.00
65 gals. Vinegar	@	18¢	11.70
45 lbs. Cheese	@	15¢	6.75
423 " Sugar	@	9¢	38.07
			<u>\$242.27</u>

Received Payment,

JONES & Co.

(3.)

CINCINNATI, O., Sept. 4, 1877.

MR. H. KELLY,

Bought of L. M. CHAMBERLIN.

25 yds. Calico	@	12¢
12 spools Cotton	@	6¢
16 yds. Alpaca	@	68¢
17 " Muslin	@	15¢
2 bolts Tape	@	10¢

Ans. \$17.35.

(4.)

NEW PROVIDENCE, PA., Nov. 5, 1876.

MR. HIRAM PEOPLES,

Bought of HELM & RAUB.

12 lbs. Coffee	@	35¢
23 " Raisins	@	22¢
15 " Prunes	@	16¢
12 " Crackers	@	15¢
10 " Rice	@	13¢
41 " Sugar	@	13¢

Ans. \$20.09.

(5.)

WILLIAMSPORT, PA., Nov. 11, 1876.

MESSRS. FURST & SONS,

Bought of HERR & ALLISON.

61 bus. Wheat	@	\$1.25
75 " "	@	1.21
85 " "	@	1.27
65 " Corn	@	65¢
71 " "	@	72¢
95 " "	@	68¢
175 " Oats	@	53¢
140 " "	@	49¢

Ans. \$594.27.

(6.)

NEWARK, N. J., Oct. 6, 1875.

MESSRS. RUSSELL & JONES,

To SYKES & Bro., *Dr.*

1875.

Jan. 2.	To 15 yds. Muslin	@	15¢
" "	" 23 " Calico	@	9¢
" 15.	" 7 " Cloth	@	\$4.25
" "	" 18 - " Linen	@	53¢
Feb. 7.	" 6 prs. Boots	@	\$5.75
" "	" 13 doz. Buttons	@	22¢

1875.

Cr.

Jan. 20.	By Cash		\$35.00
Feb. 1.	" 15 lbs. Butter	@	33¢
" "	" 5 prs. Chickens	@	75¢

Balance due, \$37.27.

7. Mar. 1, 1876, The Carlisle Shoe Factory sold to Jacob Kamp 27 pairs Calfskin boots @ \$6.75, 96 pairs Gaiters @ \$2.37, 126 pairs Overshoes @ 73 cents, 18 pairs Slippers @ \$1.06, and 75 pairs heavy boots @ \$3.81.

Amount due, \$806.58.

Make out a bill and receipt it properly.

8. Hood, Bonbright & Co. sold, Apr. 3, 1875, to John Lane, 95 yds. Cloth @ \$2.73, 1600 yds. Calico @ 9¢, 1200 yds. Muslin @ 13¢, 1200 yds. Muslin @ 16¢, 650 yds. Cassimere at \$2.17, and 563 yds. Merino @ 73¢.

Amount due, \$2572.84.

Make out a receipted bill.

9. A merchant sold to a farmer the following: 27 yds. Calico @ 13¢, 45 yds. Muslin @ 18¢, 16 yds. Linen @ 45¢, 17 yds. Cambric @ 15¢, and 9 handkerchiefs @ 45¢; and took in exchange 12 bus. Potatoes @ 65¢, 3 bbls. Apples @ \$3.25, 13 lbs. Butter @ 35¢, and the remainder in cash: how much cash was paid?

Ans. \$3.31.

Make out a receipted bill.

CHAPTER III.

PROPERTIES OF NUMBERS.

SECTION I.

DEFINITIONS AND PRINCIPLES.

52. A **Divisor** of a number is any number that will exactly divide it.

53. A **Factor** of a number is one of its exact divisors. Thus, 2, 3 and 4 are factors of 12.

Neither a unit nor the number itself is regarded as a factor.

54. A **Prime** number is one that has no factors, and therefore has no exact divisor.

55. A **Composite** number is one that may be divided, and always is the product of two or more factors.

2, 3, 5, 7, 11, 13 are prime numbers.

4, 6, 8, 10, 12, etc. are composite numbers.

56. When any number is a factor of two or more numbers,

it is called their **Common Factor**. Thus, 3 is a common factor of 6, 9, 12.

57. A Prime Factor is a factor that cannot be divided.

58. An Even Number is one whose right-hand figure is 0, 2, 4, 6 or 8. All even numbers are divisible by 2.

An **Odd Number** is one whose right-hand figure is 1, 3, 5, 7 or 9.

Since any number of hundreds is divisible by 4, any number whose tens and units are divisible by 4 is also divisible by 4.

Any number whose right-hand figure is 0 or 5 is divisible by 5.

Any number is divisible by 3 or 9 the sum of whose digits is divisible by 3 or 9.

Any number is divisible by 6 when it is even and the sum of its digits is divisible by 3.

Any number is divisible by 8 when the number expressed by the 3 right-hand digits is divisible by 8.

SECTION II.

FACTORING.

59. Factoring is the process of finding the factors of a composite number.

It depends on the following

PRINCIPLES.—1. A composite number is the product of all its prime factors.

2. Every number is divisible by its prime factors or any product of them, and by no other numbers.

ORAL EXERCISE.

1. What are the factors of 10, 16, 21, 28, 50, 100?
2. What are the factors of 33, 30, 80, 75, 48, 64?
3. What are the prime factors of 12, 16, 15, 18?

4. What are the prime factors of 20, 22, 24, 26, 28, 30?
5. What are the prime factors of 21, 25, 27, 32, 33, 34?
6. Name the prime factors of 35, 36, 38, 39, 40.
7. Name the prime factors of 42, 45, 48, 52, 54, 55.
8. Name the prime numbers from 1 to 50.
9. Name the composite numbers from 2 to 30; from 30 to 65; from 65 to 100.
10. What prime factor is found in 6 and 9?
11. What prime factor is found in 20 and 25?
12. What prime factor is common to 12 and 30? to 21 and 28?
13. What prime factor is common to 35 and 50? 14 and 70? 33 and 99? 42 and 48? 26 and 39?

WRITTEN PROBLEMS.

CASE I.—FACTORS.

Ex. 1.—What are the prime factors of 420?

SOLUTION.

$$\begin{array}{r} 3 \overline{)420} \\ 2 \overline{)140} \\ 2 \overline{)70} \\ 5 \overline{)35} \\ 7 \end{array}$$

Explanation.—Dividing 420 by 3, a prime divisor, we have 140; dividing 140 by 2, a prime divisor, we have 70; dividing 70 by 2, a prime divisor, we have 35; and dividing 35 by 5, a prime divisor, we have 7. Hence the prime factors of 420 are 3, 2, 2, 5, 7.

Find the prime factors of the following:

- | | |
|-----------|-------------------------------------|
| 2. 160. | <i>Ans.</i> 2, 2, 2, 2, 5. |
| 3. 330. | <i>Ans.</i> 3, 2, 5, 11. |
| 4. 432. | <i>Ans.</i> 2, 2, 2, 2, 3, 3, 3. |
| 5. 325. | <i>Ans.</i> 5, 5, 13. |
| 6. 378. | <i>Ans.</i> 3, 3, 3, 2, 7. |
| 7. 594. | <i>Ans.</i> 3, 3, 3, 2, 11. |
| 8. 800. | <i>Ans.</i> 2, 2, 2, 2, 2, 5, 5. |
| 9. 576. | <i>Ans.</i> 2, 2, 2, 2, 2, 3, 3. |
| 10. 784. | <i>Ans.</i> 2, 2, 2, 2, 7, 7. |
| 11. 1280. | <i>Ans.</i> 2, 2, 2, 2, 2, 2, 2, 5. |
| 12. 1140. | <i>Ans.</i> 2, 2, 5, 3, 19. |
| 13. 1155. | <i>Ans.</i> 5, 3, 7, 11. |

14. 2310. *Ans.* 2, 5, 3, 7, 11.

15. 45045. *Ans.* 3, 3, 5, 7, 11, 13.

What prime factors are common to—

16. 50 and 70? *Ans.* 5, 2.

17. 81 and 96? *Ans.* 3.

18. 63 and 147? *Ans.* 3, 7.

19. 500 and 650? *Ans.* 5, 5, 2.

20. 125 and 375? *Ans.* 5, 5, 5.

21. 288 and 180? *Ans.* 2, 2, 3, 3.

CASE II.—DIVISORS.

NOTE.—To find the divisors of a number, resolve it into its prime factors; these or the product of any two or more of them are the divisors.

Thus, the prime factors of 330 are 3, 2, 5, 11; hence, 3×2 , or 6; 3×5 , or 15; 5×2 ; 5×11 ; 2×11 , etc. are all divisors of 330.

1. Find the divisors of 160. *Ans.* 4, 8, 10, 16, 20, 40, 80.

2. Of 378. *Ans.* 9, 6, 14, 27, 42, etc.

3. Of 420. *Ans.* 10, 42, 84, 30, 210, etc.

4. Of 9009. *Ans.* 9, 21, 55, 33, 39, etc.

SECTION III.

CANCELLATION.

60. Cancellation is the process of abbreviating operations in division by rejecting or cancelling the same or equal factors common to both the dividend and the divisor.

Ex. 1.—Divide the product of $6 \times 7 \times 5 \times 3$ by $3 \times 6 \times 5$.

SOLUTION. *Explanation.*—Dividing both dividend and divisor by the same number does not alter the value of the quotient; hence, dividing both by 6, 3 and 5 by cancelling, we obtain the quotient 7.

$$\frac{6 \times 7 \times 5 \times 3}{3 \times 6 \times 5} = 7.$$

2. What is the value of $\frac{3 \times 5 \times 5 \times 7 \times 11}{5 \times 5 \times 7}$?

$$3. \text{ Of } \frac{440}{220} = \frac{11 \times 5 \times 2 \times 2 \times 2}{11 \times 5 \times 2 \times 2} ?$$

4. Divide $7 \times 5 \times 11 \times 3$ by 7×11 .

Ans. 15.

5. Divide 4×30 by 12.

Ans. 10.

SOLUTION 1.

$$4 \times \frac{30}{12} = \frac{\cancel{2} \times \cancel{2} \times \cancel{3} \times 2 \times 5}{\cancel{3} \times \cancel{2} \times \cancel{2}} = 10.$$

Explanation.—The factors of 4×30 are 2, 2, 3, 2, 5; and of 12 they are 2, 2 and 3; cancelling the like factors, 2, 2 and 3, in both divisor and dividend, there remains 2×5 , or 10.

SOLUTION 2.

$$4 \times \frac{\overset{10}{\cancel{30}}}{\underset{3}{\cancel{12}}}.$$

Explanation.—Cancelling the like factors 4 and 3 from both divisor and dividend, there remains the quotient 10.

In a similar manner solve the following:

6. Divide $3 \times 13 \times 14$ by 42.

Ans. 13.

7. Divide $33 \times 22 \times 7 \times 2$ by 11×11 .

Ans. 84.

8. Divide $4 \times 35 \times 8$ by 10×7 .

Ans. 16.

9. Divide $31 \times 9 \times 8 \times 3$ by 2×36 .

Ans. 93.

10. Divide $145 \times 2 \times 180$ by 24×75 .

Ans. 29.

11. Divide $15 \times 3 \times 84$ by 45×12 .

Ans. 7.

12. Divide $5 \times 24 \times 3$ by 18.

Ans. 20.

13. Divide $400 \times 18 \times 30 \times 42$ by $270 \times 20 \times 30 \times 14$.

Ans. 4.

14. Divide 2954 by 14.

Ans. 211.

15. Divide 2520 by 210.

Ans. 12.

16. How many dozen of eggs at 30 cts. a dozen will pay for 6 yds. of cassimere at \$2.10 a yard?

Ans. 42.

17. How many bushels of potatoes at 75 cts. a bushel must a farmer give for 36 yds. of carpet worth \$1.25 a yard?

Ans. 60.

18. A farmer bought 65 yds. of muslin at 15 cts. a yard, and paid for it with chickens at 75 cts. a pair: how many pairs of chickens did it take?

Ans. 13.

19. A miller bought 12 loads of wheat, each containing 130 bushels, at \$1.25 a bushel, and gave in exchange 8 loads of

flour at \$6.25 a barrel : how many barrels were there to the load ? *Ans.* 39.

20. Exchanged apples at \$1.50 per bushel for 25 days' labor at \$1.20 per day : how many bushels of apples did it take ?

Ans. 20.

21. A tailor bought 12 pieces of goods, each containing 22 yards, worth \$2.25 a yard ; he makes 27 suits of clothes : how much must he get per suit so as not to lose ? *Ans.* \$22.

SECTION IV.

GREATEST COMMON DIVISOR.

61. A Common Divisor of two or more numbers is a number that will exactly divide each of them. Thus, 6 is a common divisor of 18, 24 and 60.

62. The Greatest Common Divisor of two or more numbers is the greatest number that will exactly divide each of them. Thus, 10 is the greatest common divisor of 30, 40 and 60, because it is the greatest number that will exactly divide each of them.

ORAL EXERCISE.

What is a common divisor—

- | | |
|------------------|--------------------|
| 1. Of 27 and 24? | 3. Of 120 and 160? |
| 2. Of 36 and 48? | 4. Of 48 and 72? |

What is the greatest common divisor—

- | | |
|------------------|--------------------|
| 5. Of 12 and 9? | 8. Of 24 and 72? |
| 6. Of 24 and 12? | 9. Of 72 and 96? |
| 7. Of 24 and 36? | 10. Of 72 and 144? |

PRINCIPLES.

There are two methods of finding the greatest common divisor : 1st, By factors ; 2d, By continued division.

PRINCIPLES.—1. Any factor common to two or more numbers is a common divisor of those numbers.

2. The product of the common factors of two or more numbers is the greatest common divisor of those numbers.

3. A common divisor of two or more numbers is also a divisor of their sum or difference.

WRITTEN PROBLEMS.

Ex. 1. Find the greatest common divisor of 30, 40 and 60.

SOLUTION.

$$30 = 2 \times 5 \times 3$$

$$40 = 2 \times 5 \times 2 \times 2$$

$$60 = 2 \times 5 \times 2 \times 3$$

or,

$$\begin{array}{r} 2) 30, 40, 60 \\ \hline 5) 15, 20, 30 \\ \hline 3, 4, 6 \end{array}$$

$$5) 15, 20, 30$$

$$3, 4, 6$$

Explanation.—By examining the solutions, we find the only prime factors common to 30, 40 and 60 are 2 and 5; hence the greatest common divisor of these numbers is 2×5 , or 10.

From the principles and the foregoing solutions we derive the following

RULE.

Find the factors common to all the numbers, and take the product of these common factors for the greatest common divisor.

Find the greatest common divisor—

- | | |
|----------------------------|------------------|
| 2. Of 15, 45, 75. | <i>Ans.</i> 15. |
| 3. Of 16, 48, 80. | <i>Ans.</i> 16. |
| 4. Of 24, 96, 240. | <i>Ans.</i> 24. |
| 5. Of 84, 126, 210. | <i>Ans.</i> 42. |
| 6. Of 120, 240, 600. | <i>Ans.</i> 120. |
| 7. Of 210, 360, 150. | <i>Ans.</i> 30. |
| 8. Of 96, 120, 264. | <i>Ans.</i> 24. |
| 9. Of 44, 154, 110. | <i>Ans.</i> 22. |
| 10. Of 270, 135, 162. | <i>Ans.</i> 27. |
| 11. Of 108, 270, 432. | <i>Ans.</i> 54. |
| 12. Of 180, 300, 900. | <i>Ans.</i> 60. |
| 13. Of 160, 96, 320. | <i>Ans.</i> 32. |
| 14. Of 75, 300, 450. | <i>Ans.</i> 75. |
| 15. Of 360, 288, 720, 648. | <i>Ans.</i> 72. |

SECOND METHOD.

Ex. 1. Find the greatest common divisor of 91 and 52.

SOLUTION.

52)91(1

52

39)52(1

39

G. C. D. = 13)39(3

39

Explanation.—Dividing 91 by 52, and 52 by the first remainder, 39, and 39 by the remainder 13, we find the last divisor, 13, contained an exact number of times; it is, therefore, the greatest common divisor of the given numbers.

Since 13 is a divisor of 39, it is the greatest common divisor of 13 and 39; it is also a divisor of $39 + 13$, or 52, their sum, according to Principle 3; and since the greatest common divisor of two numbers is also a divisor of their difference, 13 is the greatest common divisor of 39 and 52. According to Principle 3, 13 is a divisor of $39 + 52$, or 91, their sum; and since the greatest common divisor of two numbers is a divisor of their difference, 13 is the greatest common divisor of 91 and 52.

From this solution and the foregoing principles we derive the following

RULES.

1. Divide the larger number by the smaller, and this divisor by the remainder, continuing the division by dividing each successive remainder into the last divisor, until a divisor is found which is contained an exact number of times; this last divisor is the greatest common divisor of the numbers.

2. To find the greatest common divisor of more than two numbers, first find the greatest common divisor of two of them, then of this divisor and one of the remaining numbers, and so on to the last. The last greatest common divisor is the greatest common divisor of all the numbers.

- | | |
|--------------------------------------|-----------|
| 2. Find the G. C. D. of 308 and 630. | Ans. 14. |
| 3. Of 16, 20, 24. | Ans. 4. |
| 4. Of 66, 77, 264. | Ans. 11. |
| 5. Of 120, 210, 345. | Ans. 15. |
| 6. Of 112, 140, 560. | Ans. 28. |
| 7. Of 42, 126, 210. | Ans. 42. |
| 8. Of 165, 231, 385. | Ans. 11. |
| 9. Of 252, 630, 1134. | Ans. 126. |
| 10. Of 200, 125, 350. | Ans. 25. |

11. A land-owner has three fields, containing 24 acres, 18 acres and 42 acres. He wishes to cut them into smaller fields of an equal number of acres each: how large will the fields be?
Ans. 6 acres.

12. Three men living on a new street own land fronting as follows: A, 600 ft.; B, 720 ft.; and C, 900 ft. They wish to cut their land into lots of an equal width: how wide will the lots be, and how many will each have?

Ans. 60 ft. A, 10 lots; B, 12 lots; C, 15 lots.

13. A man has 4 tracts of land containing 420 acres, 350 acres, 490 acres and 560 acres, respectively. He wishes to divide them into farms of an equal number of acres each: how large will the farms be, and how many will there be?

Ans. 70 acres; 26 farms.

14. A man has a lot of the following dimensions: 450 ft. long on the north side, 540 ft. on the south side, 420 ft. on the east side, and 495 ft. on the west side: what is the greatest length of his panels of fence if they are equal, and how many rails and posts will be required to fence it if the fence is 5 rails high?

Ans. Panels 15 ft.; 635 rails; 127 posts.

SECTION V.

LEAST COMMON MULTIPLE.

63. A Multiple of a number is a number that is exactly divisible by that number.

64. A Common Multiple of two or more numbers is a number that is exactly divisible by each. Thus, 72 is a common multiple of 36, 18, 12.

65. The Least Common Multiple of two or more numbers is the least number exactly divisible by each. Thus, 36 is the least common multiple of 36, 18, 12.

ORAL EXERCISE.

1. What is a multiple—
Of 3? Of 10? Of 12? Of 20? Of 48? Of 60?
2. What is a common multiple—
Of 3 and 6? Of 6 and 9? Of 10 and 20? Of 5 and 10?
Of 18 and 24? Of 7 and 21? Of 25 and 75? Of 50 and 100?
3. What is the least common multiple—
Of 2 and 3? Of 3 and 5? Of 3 and 6? Of 5 and 4?
Of 4 and 7? Of 6 and 5? Of 8 and 3? Of 10 and 12?
Of 12 and 15? Of 20 and 30? Of 40 and 20? Of 40
and 50?

PRINCIPLES.—1. Every multiple of a number contains the prime factors of that number.

2. A common multiple of two or more numbers contains all the prime factors of each of those numbers.

3. The least common multiple of two or more numbers contains all the prime factors of each of the numbers, and no other factors.

4. The least common multiple of two or more numbers contains each of their prime factors the greatest number of times it occurs in either number.

WRITTEN PROBLEMS.

Ex. 1. Find the least common multiple of 12, 20, 33.

SOLUTION:

$$12 = 2 \times 2 \times 3$$

$$20 = 2 \times 2 \times 5$$

$$33 = 3 \times 11$$

$$L. C. M. = 2 \times 2 \times 3 \times 5 \times 11 = 660$$

Explanation.—The least common multiple of the given numbers must contain the factors 2, 2 and 3 to be divisible by 12; it must contain the factors 2, 2 and 5 to be divisible by

20; it must contain the factors 3 and 11 to be divisible by 33. Since the number 660 contains all these factors, and no others, it is the least common multiple of 12, 20 and 33.

This method may be simplified as follows:

SOLUTION. *Explanation.*—Placing the numbers side by side, and

$$\begin{array}{r} 2) 12, 20, 33 \\ \hline 2) 6, 10, 33 \\ \hline 3) 3, 5, 33 \\ \hline 1, 5, 11 \end{array}$$
dividing, we find that 2 is a factor of 12 and 20; it is therefore a factor of the *least common multiple*; writing the quotients and 33, which is not divisible by 2, in the second line, and dividing again, it is found that 2 is a factor of 6

and 10, and is therefore a second factor of the *least common multiple*; writing the quotients as before and dividing by 3, it is found that 3 is a factor of 3 and 33, and is therefore a factor of the *least common multiple*; the quotient 5 is a factor of 20, and 11 is a factor of 33. The factors of the *least common multiple* are therefore 2, 2, 3, 5 and 11, equal to 660, the *least common multiple*.

From the foregoing we derive the

RULES.

1. *Resolve the numbers into their prime factors, and take the product of these factors, using each the greatest number of times it occurs in any of the given numbers.*

2. *Place the given numbers in a horizontal line, divide by any prime number that is a factor of two or more of them, and write the quotients and the undivided numbers beneath. Divide these as before, and so continue until the quotients and the undivided numbers contain no common factor. The product of the divisors and the last quotients is the least common multiple of the given numbers.*

What is the least common multiple—

- | | |
|--|------------------------|
| 2. Of 15, 16, 24? | <i>Ans.</i> 240. |
| 3. Of 9, 18, 27, 54? | <i>Ans.</i> 54. |
| 4. Of 18, 36, 72, 108? | <i>Ans.</i> 216. |
| 5. Of 27, 36, 45, 90? | <i>Ans.</i> 540. |
| 6. Of 12, 48, 36, 70? | <i>Ans.</i> 5040. |
| 7. Of 18, 21, 24, 27, 30? | <i>Ans.</i> 7560. |
| 8. Of 49, 42, 63, 70, 91? | <i>Ans.</i> 57330. |
| 9. Of 25, 75, 100, 150? | <i>Ans.</i> 300. |
| 10. Of 2, 3, 4, 5, 6, 7, 8, 9? | <i>Ans.</i> 2520. |
| 11. Of 8, 16, 21, 28? | <i>Ans.</i> 336. |
| 12. Of 14, 18, 22, 24? | <i>Ans.</i> 5544. |
| 13. Of \$240, \$270, \$180, \$150? | <i>Ans.</i> \$10800. |
| 14. What is the smallest tract of land that may be divided into fields of 12, 15 or 18 acres each? | <i>Ans.</i> 180 acres. |
| 15. A farmer desires to purchase sheep at \$8, hogs at \$12, or cows at \$30 each: what is the least amount of money | |

for which he can purchase any of these at the prices named? *Ans.* \$120.

16. What is the smallest sum of money for which I can hire workmen at 12, 14, 16, 18 or 20 dollars a week?

Ans. \$5040.

17. A can hoe a row of corn in 20 minutes, B in 22 minutes, C in 15 minutes, and D in 18 minutes: if all start together, how long will it be till they all finish a row at the same time, and how many rows will each have finished?

Ans. 1980 minutes, or 33 hours; A, 99 rows; B, 90 rows; C, 132 rows; D, 110 rows.

CHAPTER IV.

FRACTIONS.

SECTION I.

DEFINITIONS AND PRINCIPLES.

If a unit be divided into *two* equal parts, one of these parts is called a *half*; if into *three* equal parts, each part is called a *third*; into *four*, a *fourth*; into *five*, a *fifth*, and so on.

These parts of a unit, one-half, one-third, one-fourth, etc., are called *fractions*.

66. A Fraction is one or more of the equal parts of a unit. It is usually expressed by writing the numbers one above the other, with a short horizontal line between them. Thus, four-fifths is written $\frac{4}{5}$.

67. The number below the line is called the *Denominator*, and shows into how many equal parts the unit is divided.

68. The number above the line is called the *Numerator*, and shows how many of the equal parts are taken. Thus, in $\frac{4}{5}$ the denominator, 5, shows that the unit is divided into

6 equal parts, and 5, the numerator, shows that 5 of these parts are taken.

69. The numerator and the denominator are called the *Terms* of a fraction.

EXERCISE.

Read— $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{1}{10}$, $1\frac{1}{2}$, $1\frac{2}{3}$, $1\frac{3}{4}$, $1\frac{4}{5}$.

Write—five-sixths, two-thirds, four-ninths, seven-tenths, nine-thirteenths, seventy-one seventy-fifths, eighty ninety-seconds, forty-five fiftieths, fifty fifty-fifths.

70. A **Common Fraction** is one in which the numerator and the denominator are both expressed by figures.

Common fractions are either *Proper* or *Improper*.

71. A **Proper Fraction** is one whose numerator is less than its denominator; as, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$, etc.

72. An **Improper Fraction** is one whose numerator is equal to or greater than its denominator; as, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{8}{3}$, etc.

Fractions are *Simple*, *Compound* or *Complex*.

73. A **Simple Fraction** is one not united with another, and both of whose terms are integers; as, $\frac{2}{3}$, $\frac{9}{10}$, etc.

74. A **Compound Fraction** is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{5}{6}$, $\frac{7}{8}$ of $1\frac{1}{2}$, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{9}{7}$, etc.

75. A **Complex Fraction** is one having a fraction in one or both of its terms; as, $\frac{\frac{2}{3}}{6}$, $\frac{7}{\frac{1}{2}}$, $\frac{\frac{8}{9}}{\frac{7}{8}}$, etc.

76. A **Mixed Number** is one consisting of an integer and a fraction; as, $1\frac{1}{2}$, $6\frac{7}{8}$, etc.

77. The **Reciprocal** of a number is 1 divided by that number; thus, the reciprocal of 6 is $\frac{1}{6}$; of 3, $\frac{1}{3}$, etc.

Tell whether the following fractions are simple, complex, or compound.

1. $\frac{1}{2}$; $\frac{2}{3}$ of $\frac{9}{7}$; $\frac{3}{8}$ of 9.
2. $1\frac{1}{2}$ times $\frac{2}{3}$ of 7.
3. $\frac{8}{10}$; $\frac{9}{11}$; $\frac{1}{2}$ of $\frac{1}{3}$.
4. $\frac{1}{2}$ of $3\frac{3}{4}$; $\frac{2}{3}$ of $3\frac{1}{4}$.

5. $\frac{5}{6}$ of $17\frac{1}{2} \times 3$.
6. $\frac{1}{3}$ of $\frac{2}{4}$ of $\frac{9}{7}$ of 8.
7. $\frac{\frac{2}{3} \text{ of } \frac{9}{7}}{\frac{1}{2} \text{ of } 6\frac{1}{2}}$.

SECTION II.

REDUCTION OF FRACTIONS.

CASE I.

To Reduce Whole or Mixed Numbers to Fractions.**ORAL EXERCISE.**

1. How many halves in 1 apple? How many thirds?
2. How many fourths in 1 apple? How many fifths?
3. How many halves in 5 apples?

SOLUTION.—In 1 apple there are 2 halves, and in 5 apples there are 5 times 2 halves, or 10 halves.

4. How many halves in 6? In 10? In 13? In 40?
5. How many thirds in 3? In 4? In 6? In $5\frac{1}{3}$?
6. How many fourths in 4? In 5? In $6\frac{3}{4}$?
7. How many fifths in 3? In 6? In $7\frac{1}{5}$? In $9\frac{4}{5}$?
8. How many sixths in 2? In 3? In $5\frac{5}{6}$? In $12\frac{5}{6}$?
9. How many sevenths in 4? In $3\frac{3}{7}$? In $10\frac{5}{7}$?
10. How many eighths in 4? In 5? In 6? In $6\frac{3}{8}$? In $7\frac{1}{8}$?
11. How many ninths in 3? In 5? In 7? In $8\frac{1}{9}$? In $7\frac{8}{9}$?
12. How many tenths in 7? In $6\frac{3}{10}$? In $7\frac{2}{10}$? In $11\frac{4}{10}$?

WRITTEN PROBLEMS.

1. Reduce 117 to 4ths; $115\frac{3}{4}$ to 4ths.

SOLUTION 1.

$$\begin{array}{r} 117 \\ 4 \\ \hline 468 \\ 4 \end{array}$$

Explanation.—In 1 there are $\frac{1}{4}$, and in 117 there are $117 \times \frac{1}{4} = \frac{468}{4}$.

SOLUTION 2.

$$\begin{array}{r} 115\frac{3}{4} \\ 4 \\ \hline 460 \\ 3 \\ \hline 463 \\ 4 \end{array}$$

Explanation.—In 1 there are $\frac{1}{4}$, and in 115 there are $115 \times \frac{1}{4} = \frac{460}{4}$; $\frac{460}{4} + \frac{3}{4}$ are $\frac{463}{4}$.

From the foregoing solutions we derive the following

RULE.

Multiply the whole number by the denominator, and to this add the numerator, writing the product over the given denominator.

2. Reduce 125 to 7ths; 34 to 12ths.

Reduce to improper fractions—

3. $5\frac{3}{7}$.	Ans. $\frac{38}{7}$.	9. $12\frac{3}{8}$.	Ans. $\frac{99}{8}$.
4. $7\frac{5}{8}$.	Ans. $\frac{59}{8}$.	10. $9\frac{1}{3}$.	Ans. $\frac{28}{3}$.
5. $7\frac{3}{11}$.	Ans. $\frac{80}{11}$.	11. $34\frac{9}{11}$.	Ans. $\frac{383}{11}$.
6. $5\frac{1}{3}$.	Ans. $\frac{16}{3}$.	12. $83\frac{1}{3}$.	Ans. $\frac{250}{3}$.
7. $10\frac{5}{8}$.	Ans. $\frac{85}{8}$.	13. $51\frac{5}{9}$.	Ans. $\frac{464}{9}$.
8. $6\frac{7}{10}$.	Ans. $\frac{67}{10}$.	14. $173\frac{9}{10}$.	Ans. $\frac{1739}{10}$.

CASE II.

To Reduce Improper Fractions to Whole or Mixed Numbers.

ORAL EXERCISE.

1. How many ones in $\frac{12}{3}$?

SOLUTION.—Since 1 equals $\frac{3}{3}$, in $\frac{12}{3}$ there are as many ones as $\frac{12}{3}$ is contained times in $\frac{3}{3}$, or 4.

2. How many ones in $\frac{12}{3}$? In $\frac{8}{3}$? In $\frac{15}{3}$?

3. How many ones in $\frac{20}{4}$? In $\frac{28}{4}$? In $\frac{35}{4}$? In $\frac{30}{4}$?

4. How many ones in $\frac{18}{6}$? In $\frac{28}{6}$? In $\frac{35}{6}$? In $\frac{42}{6}$?

5. How many ones in $\frac{48}{8}$? In $\frac{64}{8}$? In $\frac{81}{8}$? In $\frac{100}{8}$?

6. Reduce $\frac{8}{3}$ to a mixed number.

7. Reduce $\frac{15}{4}$ to a mixed number.

8. Reduce $\frac{32}{9}$ to a mixed number.

9. Reduce $\frac{15}{6}$ to a mixed number.

10. Reduce $\frac{31}{10}$ to a mixed number.

11. Reduce $\frac{194}{4}$ to a mixed number.

WRITTEN PROBLEMS.

1. How many dollars in $\$2\frac{3}{4}$?

SOLUTION. *Explanation.*—Since there are $\frac{4}{4}$ in 1 dollar, in $\frac{3}{4}$ dollars there are as many dollars as $\frac{3}{4}$ are contained times in $\frac{4}{4}$, or $\frac{3}{4}$ dollars.

$$\begin{array}{r} 4)23 \\ 20 \\ \hline 3 \end{array}$$

From the foregoing we derive the following

RULE.

Divide the numerator of the improper fraction by the denominator ; the quotient will be the whole or mixed number.

Reduce to whole or mixed numbers—

2. $\frac{27}{9}$.	Ans. 3.	8. $\frac{54\frac{3}{2}}{2}$.	Ans. $24\frac{3}{2}$.
3. $\frac{8\frac{9}{2}}{1\frac{1}{2}}$.	Ans. $7\frac{5}{2}$.	9. $\frac{18\frac{3}{8}}{8}$.	Ans. $3\frac{1}{8}$.
4. $\frac{7\frac{3}{2}}{7}$.	Ans. $11\frac{1}{7}$.	10. $\frac{87\frac{9}{8}}{1\frac{1}{8}}$.	Ans. $67\frac{9}{8}$.
5. $\frac{4\frac{1}{2}}{1\frac{1}{2}}$.	Ans. $4\frac{1}{2}$.	11. $\frac{6\frac{1}{8}}{8}$.	Ans. $6\frac{1}{8}$.
6. $\frac{73\frac{3}{2}}{7}$.	Ans. $104\frac{1}{7}$.	12. $\frac{518\frac{4}{5}}{5}$.	Ans. $143\frac{4}{5}$.
7. $\frac{1391}{9}$.	Ans. $144\frac{5}{9}$.	13. $\frac{921\frac{2}{21}}{21}$.	Ans. $31\frac{2}{21}$.

CASE III.

To Reduce Fractions to Equivalent Fractions having Higher Terms.

ORAL EXERCISE.

1. How many sixths in $\frac{2}{3}$?

SOLUTION.—In 1 there are $\frac{6}{6}$, in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{6}{6}$, or $\frac{2}{6}$, and in $\frac{2}{3}$ two times $\frac{2}{6}$, or $\frac{4}{6}$.

2. How many eighths in $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$?

3. How many tenths in $\frac{1}{2}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

4. How many twelfths in $\frac{1}{3}$? $\frac{1}{4}$? $\frac{2}{3}$? $\frac{3}{4}$?

5. How many sixteenths in $\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{8}$? $\frac{7}{8}$?

6. How many eighteenthths in $\frac{1}{3}$? $\frac{1}{6}$? $\frac{2}{3}$? $\frac{5}{6}$?

7. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{10}$ to twentieths.

8. Reduce $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{4}{10}$ to thirtieths.

9. Reduce $\frac{3}{4}$, $\frac{1}{10}$, $\frac{2}{5}$, $\frac{3}{20}$, $\frac{4}{5}$ to fortieths.

10. Reduce $\frac{2}{3}$, $\frac{1}{10}$, $\frac{3}{5}$, $\frac{1}{12}$, $\frac{8}{15}$ to sixtieths.

PRINCIPLE.—The multiplication of both terms of a fraction by the same number does not alter the value of the fraction.

WRITTEN PROBLEMS.

1. Reduce $\frac{9}{8}$ to 81sts.

SOLUTION. *Explanation.*—To reduce 9ths to 81sts it is necessary to multiply the denominator by 9; but to preserve the value of the fraction, the numerator must also be multiplied by the same number; hence, $\frac{9}{8} = \frac{81}{72}$.

From the solution we derive the following

RULE.

Multiply both numerator and denominator by that number which will produce the required denominator.

2. Reduce $\frac{2}{3}$, $\frac{5}{14}$, $\frac{7}{8}$ to 14ths. *Ans.* $\frac{4}{7}$, $\frac{5}{14}$, $\frac{12}{14}$.
3. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ to 60ths. *Ans.* $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$.
4. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{10}$ to 60ths. *Ans.* $\frac{40}{60}$, $\frac{48}{60}$, $\frac{50}{60}$, $\frac{54}{60}$.
5. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ to 168ths. *Ans.* $\frac{84}{168}$, $\frac{112}{168}$, $\frac{126}{168}$, $\frac{140}{168}$, $\frac{144}{168}$.

CASE IV.

To Reduce Fractions to their Lowest Terms.

ORAL EXERCISE.

1. How many fourths in $\frac{3}{8}$?

SOLUTION.—Since $\frac{1}{4} = \frac{2}{8}$, there are as many fourths in $\frac{3}{8}$ as $\frac{3}{8}$ is contained times in $\frac{2}{8}$, or $\frac{3}{4}$; hence, in $\frac{3}{8}$ there are $\frac{3}{4}$.

2. How many halves in $\frac{3}{8}$, $\frac{1^2}{8}$, $\frac{1^2}{6}$?
3. How many 3ds in $\frac{8}{6}$, $\frac{8}{12}$, $\frac{1^2}{6}$, $\frac{2^4}{18}$?
4. How many 5ths in $\frac{8}{10}$, $\frac{9}{15}$, $\frac{1^2}{20}$, $\frac{2^4}{30}$?
5. How many 7ths in $\frac{9}{21}$, $\frac{2^1}{28}$, $\frac{1^0}{35}$, $\frac{1^2}{14}$?
6. How many 8ths in $\frac{4}{16}$, $\frac{1^2}{16}$, $\frac{2^1}{24}$, $\frac{3^2}{32}$?
7. How many 10ths in $\frac{1^0}{10}$, $\frac{2^0}{20}$, $\frac{4^0}{40}$, $\frac{7^2}{60}$?
8. Reduce $\frac{5}{15}$ to 3ds; $\frac{1^2}{2}$ to 5ths.
9. Reduce $\frac{3^0}{6}$ to 4ths; $\frac{9^0}{90}$ to 7ths.
10. Reduce $\frac{1^2}{2}$ to 7ths; $\frac{2^1}{4}$ to 9ths.
11. Reduce $\frac{1^5}{6}$ to halves; $\frac{2^5}{8}$ to 12ths.

NOTE.—When a fraction cannot be reduced to one having a less denominator, it is said to be in its lowest terms.

12. Reduce $\frac{1^0}{2}$, $\frac{1^0}{3}$, $\frac{1^5}{6}$ to their lowest terms.
13. Reduce $\frac{1}{2}$, $\frac{1^2}{4}$, $\frac{2^2}{5}$, $\frac{3^0}{6}$ to their lowest terms.
14. Reduce $\frac{1^2}{6}$, $\frac{1^5}{10}$, $\frac{2^0}{5}$, $\frac{3^5}{6}$ to their lowest terms.

PRINCIPLE.—The division of both terms of a fraction by the same number does not change its value.

WRITTEN PROBLEMS.

1. Reduce
- $\frac{19\frac{2}{3}}{13\frac{1}{3}}$
- to its lowest terms.

SOLUTION.

$$19\frac{2}{3} = \frac{58}{3} = \frac{7}{1}$$

Or,

$$19\frac{2}{3} \div \frac{2}{3} = \frac{58}{3} \div \frac{2}{3} = \frac{58}{2} = 29$$

Explanation.—Dividing both terms of $\frac{19\frac{2}{3}}{13\frac{1}{3}}$ by 5, it reduces to $\frac{3\frac{4}{5}}{2\frac{2}{5}}$; dividing both terms of this fraction by 3, it reduces to $\frac{1\frac{1}{5}}{\frac{2}{5}}$. Since 7 and 13 do not have a common divisor, the lowest terms of the fraction $\frac{19\frac{2}{3}}{13\frac{1}{3}}$ is $\frac{7}{13}$.

From the solution we derive the following to reduce a fraction to its lowest terms:

RULE.

Divide both terms of the fraction by a common divisor, and the result again by a common divisor, and so on till the terms have no common divisor.

Or, Divide both terms of the fraction by their greatest common divisor.

Reduce—

2. $\frac{1\frac{1}{2}}{3}$ to 3ds.	Ans. $\frac{2}{3}$.	5. $\frac{4\frac{8}{9}}{10\frac{8}{9}}$ to 9ths.	Ans. $\frac{4}{9}$.
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3. $\frac{3\frac{2}{3}}{4}$ to 3ds.	Ans. $\frac{2}{3}$.	6. $\frac{1\frac{2}{3}}{7\frac{2}{3}}$ to 17ths.	Ans. $\frac{1\frac{2}{3}}{7\frac{2}{3}}$.
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4. $\frac{1\frac{8}{9}}{5}$ to 5ths.	Ans. $\frac{8}{9}$.	7. $\frac{7\frac{2}{3}}{8\frac{2}{3}}$ to 12ths.	Ans. $\frac{1\frac{1}{2}}{12}$.
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Reduce to their lowest terms—

8. $\frac{8\frac{1}{2}}{18\frac{1}{2}}$.	Ans. $\frac{2}{9}$.	13. $\frac{5\frac{6}{7}}{7\frac{6}{7}}$.	Ans. $\frac{5}{7}$.
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9. $\frac{3\frac{2}{3}}{7\frac{2}{3}}$.	Ans. $\frac{5}{8}$.	14. $\frac{4\frac{8}{9}}{12\frac{8}{9}}$.	Ans. $\frac{5}{8}$.
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10. $\frac{1\frac{2}{3}}{2\frac{2}{3}}$.	Ans. $\frac{1}{2}$.	15. $\frac{5\frac{7}{8}}{11\frac{7}{8}}$.	Ans. $\frac{5}{11}$.
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11. $\frac{5\frac{7}{8}}{18\frac{7}{8}}$.	Ans. $\frac{4}{9}$.	16. $\frac{9\frac{0}{10}}{37\frac{0}{10}}$.	Ans. $\frac{5}{37}$.
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12. $\frac{8\frac{2}{3}}{10\frac{2}{3}}$.	Ans. $\frac{1\frac{2}{3}}{10\frac{2}{3}}$.		
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CASE V.

To Reduce Compound Fractions to Simple Ones.

ORAL EXERCISE.

1. What is
- $\frac{1}{3}$
- of
- $\frac{1}{4}$
- ?

SOLUTION.—Since $\frac{1}{4} = \frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{1}{4}$, or $\frac{1}{12}$.

2. What is
- $\frac{1}{2}$
- of
- $\frac{1}{3}$
- ?
- $\frac{1}{3}$
- of
- $\frac{1}{2}$
- ?

3. What is
- $\frac{1}{4}$
- of
- $\frac{1}{3}$
- ?
- $\frac{1}{4}$
- of
- $\frac{1}{3}$
- ?

4. Having
- $\frac{1}{2}$
- of a dollar, I gave away
- $\frac{1}{4}$
- of what I had: how much did I give away?

5. A pair of chickens is worth $\frac{1}{2}$ of a dollar, what is $\frac{1}{2}$ a pair worth?

6. C owned $\frac{1}{4}$ of a farm, and sold $\frac{1}{2}$ of his share: how much did he sell?

7. Bought $\frac{1}{3}$ of a mill, and sold $\frac{1}{2}$ of my share: how much did I sell?

8. I had $\frac{1}{2}$ of a hogshead of sugar, and sold $\frac{1}{2}$ of it: how much did I sell?

9. I had $\frac{1}{2}$ a ton of coal, and gave my neighbor $\frac{1}{2}$ of it; he gave to his brother $\frac{1}{2}$ of his share: how much did his brother get?

10. What is $\frac{2}{3}$ of $\frac{4}{5}$?

SOLUTION.— $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$; $\frac{1}{2}$ of $\frac{4}{5}$ is $4 \times \frac{1}{10}$, or $\frac{4}{10}$; and $\frac{2}{3}$ of $\frac{4}{5}$ is $2 \times \frac{4}{15}$, or $\frac{8}{15}$; hence $\frac{2}{3}$ of $\frac{4}{5}$ is $\frac{8}{15}$.

11. What is $\frac{3}{4}$ of $\frac{5}{12}$?

12. What is $\frac{2}{3}$ of $\frac{3}{10}$?

13. What is $\frac{5}{6}$ of $\frac{1}{2}$?

14. What is $\frac{2}{3}$ of $\frac{7}{8}$?

15. What is $\frac{4}{5}$ of $\frac{3}{8}$?

16. What is $\frac{7}{8}$ of $\frac{4}{5}$?

17. What is $\frac{5}{6}$ of $\frac{1}{2}$?

18. What is $\frac{2}{3} \times \frac{1}{2}$?

19. A man owning $\frac{3}{4}$ of a store, sold $\frac{1}{2}$ of his share: how much did he sell?

20. Some boys owned $\frac{3}{4}$ of a boat; they sold $\frac{1}{2}$ of their share: how much did they sell?

21. If a man own $\frac{3}{4}$ of a store, and sell $\frac{1}{2}$ of his share, how much does he sell?

22. Having $\frac{3}{4}$ of a bushel of potatoes, I gave away $\frac{1}{2}$ of what I had: what part of a bushel had I remaining?

23. A boy had $\frac{5}{10}$ of a dollar, and spent $\frac{3}{5}$ of it: how much did he spend, and how much had he remaining?

24. George had a barrel of cider; $\frac{1}{2}$ of it leaked away, and he sold $\frac{2}{3}$ of the remainder: how much still remained?

WRITTEN PROBLEMS.

Ex. 1. What is $\frac{2}{3}$ of $\frac{4}{5}$?

SOLUTION.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15} = \frac{8}{15}$$

Or,

$$\frac{3}{4} \text{ times } \frac{4}{5} = \frac{3}{5}$$

ANALYSIS.

$$\frac{1}{2} \text{ of } \frac{4}{5} = \frac{4}{10},$$

$$\frac{1}{2} \text{ of } \frac{4}{5} = 4 \times \frac{1}{10} = \frac{4}{10},$$

$$\text{and } \frac{2}{3} \text{ of } \frac{4}{5} = 3 \times \frac{4}{15} = \frac{8}{15}, \text{ or } \frac{8}{15}.$$

What is—

- | | | | |
|--------------------------------------|------------------------------|--|------------------------------|
| 2. $\frac{1}{2}$ of $\frac{1}{10}$? | <i>Ans.</i> $\frac{1}{20}$. | 6. $\frac{7}{8}$ of $\frac{1}{8}$? | <i>Ans.</i> $\frac{7}{64}$. |
| 3. $\frac{5}{8}$ of $\frac{1}{4}$? | <i>Ans.</i> $\frac{5}{32}$. | 7. $\frac{2}{3}$ of $\frac{3}{4}$? | <i>Ans.</i> $\frac{1}{2}$. |
| 4. $\frac{1}{7}$ of $\frac{1}{6}$? | <i>Ans.</i> $\frac{1}{42}$. | 8. $\frac{4}{5}$ of $\frac{7}{10}$? | <i>Ans.</i> $\frac{7}{25}$. |
| 5. $\frac{3}{5}$ of $\frac{2}{3}$? | <i>Ans.</i> $\frac{2}{5}$. | 9. $\frac{1}{5}$ of $\frac{1}{3}$ of $\frac{3}{5}$? | <i>Ans.</i> $\frac{1}{75}$. |

Reduce to a simple fraction—

- | | |
|---|--------------------------------|
| 10. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{8}$ of $\frac{1}{12}$. | <i>Ans.</i> $\frac{1}{30}$. |
| 11. $\frac{2}{3}$ of $\frac{1}{2}$ of $3\frac{1}{4}$ of $\frac{3}{8}$. | <i>Ans.</i> $\frac{1}{16}$. |
| 12. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{8}$ of $\frac{1}{12}$. | <i>Ans.</i> $\frac{1}{30}$. |
| 13. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{3}{8}$ of $\frac{1}{12}$. | <i>Ans.</i> $\frac{1}{240}$. |
| 14. A man had $\frac{3}{4}$ of a ton of hay; he sold $\frac{7}{8}$ of it: how much did he sell? | <i>Ans.</i> $\frac{7}{8}$. |
| 15. John having $\frac{2}{3}$ of a quart of nuts, divided them equally among 16 boys: what part of a quart did each receive? | <i>Ans.</i> $\frac{1}{24}$. |
| 16. Henry having $\frac{2}{3}$ of a peck of apples, shared them equally with 12 playmates: how much did each receive? | <i>Ans.</i> $\frac{2}{45}$ pk. |
| 17. Smith sold $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of $\frac{1}{3}$ of 300 acres of land: how much did he sell? | <i>Ans.</i> 90 acres. |
| 18. A store cost 6000 dollars, and a house $\frac{2}{5}$ of $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{1}{2}$ as much: how much did the house cost? | <i>Ans.</i> \$960. |

COMMON DENOMINATOR.

ORAL EXERCISE.

1. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to 12ths.

SOLUTION.—Since there are $\frac{1}{3}$ in 1, in $\frac{1}{4}$ there are $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{12}$, and in $\frac{2}{3}$, twice $\frac{1}{12}$, or $\frac{2}{12}$; in $\frac{1}{4}$ there are $\frac{1}{4}$ of $\frac{1}{3}$, or $\frac{1}{12}$, and in $\frac{3}{4}$, three times $\frac{1}{12}$, or $\frac{3}{12}$; hence in $\frac{2}{3}$ and $\frac{3}{4}$ there are $\frac{2}{12}$ and $\frac{3}{12}$.

- How many 20ths in $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{10}$?
- How many 15ths in $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{5}$?
- How many 30ths in $\frac{5}{6}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{3}$?
- How many 40ths in $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{1}{10}$?
- How many 24ths in $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$?

NOTE.—When fractions have the same denominator, they are said to have a *common denominator*.

7. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having a common denominator.

8. Reduce $\frac{2}{3}$ and $\frac{4}{5}$ to fractions having a common denominator.

9. Reduce $\frac{5}{6}$, $\frac{4}{5}$ and $\frac{1}{10}$ to fractions having a common denominator.

10. Reduce $\frac{1}{2}$, $\frac{7}{15}$ and $\frac{3}{5}$ to fractions having a common denominator.

11. Reduce $\frac{2}{15}$, $\frac{1}{5}$ and $\frac{1}{3}$ to fractions having a common denominator.

12. Reduce $1\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{1}{5}$ to fractions having a common denominator.

WRITTEN PROBLEMS.

1. Reduce $\frac{5}{6}$, $\frac{4}{5}$, $\frac{1}{10}$ to fractions having a common denominator.

SOLUTION 1.

$$\begin{aligned}\frac{5}{6} \times \frac{5 \times 10}{5 \times 10} &= \frac{250}{300} \\ \frac{4}{5} \times \frac{6 \times 10}{6 \times 10} &= \frac{240}{300} \\ \frac{1}{10} \times \frac{6 \times 5}{6 \times 5} &= \frac{30}{300}\end{aligned}$$

Explanation.—Multiplying each denominator by the others, we find the common denominator to be 300; multiplying both terms of $\frac{5}{6}$ by the denominators 5 and 10, it equals $\frac{250}{300}$; multiplying both terms of $\frac{4}{5}$ by the denominators 6 and 10, it equals $\frac{240}{300}$; multiplying both terms of $\frac{1}{10}$ by the denominators 6 and 5, it equals $\frac{30}{300}$.

SOLUTION 2.

Denom. $6 \times 5 \times 10 = 300$.

$$\begin{aligned}\frac{5}{6} &= \frac{250}{300} \\ \frac{4}{5} &= \frac{240}{300} \\ \frac{1}{10} &= \frac{30}{300}\end{aligned}$$

Explanation.—We find the common denominator 300 by multiplying together the given denominators 6, 5 and 10, and reduce each fraction to 300ths.

From these solutions we derive the following for reducing fractions to equivalent ones having a common denominator:

RULE.

Multiply both terms of each fraction by all the denominators except its own.

Reduce to equivalent fractions having a common denominator—

2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$.

Ans. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

- | | |
|---|--|
| 3. $\frac{5}{8}, \frac{5}{8}, \frac{9}{10}.$ | <i>Ans.</i> $\frac{300}{480}, \frac{400}{480}, \frac{432}{480}.$ |
| 4. $\frac{5}{8}, \frac{7}{15}, \frac{11}{30}.$ | <i>Ans.</i> $\frac{2750}{2700}, \frac{1760}{2700}, \frac{990}{2700}.$ |
| 5. $\frac{2}{7}, \frac{5}{14}, \frac{8}{5}.$ | <i>Ans.</i> $\frac{140}{490}, \frac{175}{490}, \frac{588}{490}.$ |
| 6. $\frac{6}{10}, \frac{12}{10}, \frac{9}{12}, \frac{8}{12}.$ | <i>Ans.</i> $\frac{72}{120}, \frac{144}{120}, \frac{90}{120}, \frac{80}{120}.$ |
| 7. $\frac{5}{8}, \frac{2}{4}, \frac{4}{15}.$ | <i>Ans.</i> $\frac{300}{480}, \frac{240}{480}, \frac{128}{480}.$ |
| 8. $\frac{1}{8}, \frac{3}{10}, \frac{5}{6}, \frac{7}{12}.$ | <i>Ans.</i> $\frac{720}{2160}, \frac{648}{2160}, \frac{1800}{2160}, \frac{1260}{2160}.$ |
| 9. $\frac{1}{2}, \frac{1}{8}, \frac{1}{4}, \frac{1}{6}, \frac{1}{6}.$ | <i>Ans.</i> $\frac{240}{240}, \frac{30}{240}, \frac{60}{240}, \frac{40}{240}, \frac{40}{240}.$ |
| 10. $\frac{3}{4}, \frac{5}{8}, \frac{7}{8}.$ | <i>Ans.</i> $\frac{144}{144}, \frac{160}{144}, \frac{126}{144}.$ |

LEAST COMMON DENOMINATOR.

78. The **Least Common Denominator** of two or more fractions is the least denominator to which they can all be reduced. It is also the least common multiple of their denominators.

WRITTEN PROBLEMS.

1. Reduce $\frac{5}{8}, \frac{5}{9}, \frac{7}{12}$ to equivalent fractions having their least common denominator.

SOLUTION.

$$\begin{array}{r} 3 \overline{) 6, 9, 12} \\ 2 \overline{) 2, 3, 4} \\ 1, 3, 2 \end{array}$$

$$3 \times 2 \times 3 \times 2 = 36 = \text{L. C. D.}$$

$$\frac{5}{8} = \frac{225}{36}$$

$$\frac{5}{9} = \frac{200}{36}$$

$$\frac{7}{12} = \frac{210}{36}$$

Explanation.—The least common multiple of 6, 9, 12, the given denominators, is 36, which is therefore the least common denominator. By analysis, $\frac{1}{2} = \frac{6}{12}$, and $\frac{5}{8} = 5 \times \frac{1}{8} = \frac{5}{8}$; $\frac{1}{3} = \frac{4}{12}$, and $\frac{5}{9} = 5 \times \frac{1}{9} = \frac{5}{9}$; and $\frac{7}{12} = 7 \times \frac{1}{12} = \frac{7}{12}$.

Hence the rule for reducing fractions to equivalent ones having their least common denominator:

RULE.

Find the least common multiple of the denominators for the least common denominator. Divide this common denominator by each denominator, and multiply both terms of the fraction by the quotient.

NOTE.—Reduce mixed numbers and compound fractions to simple ones, and these to their lowest terms, before proceeding to find the least common denominator.

Reduce the following fractions to their least common denominator:

- | | |
|---|--|
| 2. $\frac{8}{9}, \frac{7}{12}, \frac{5}{6}.$ | <i>Ans.</i> $\frac{88}{108}, \frac{42}{108}, \frac{54}{108}.$ |
| 3. $\frac{1}{15}, \frac{2}{20}, \frac{3}{5}.$ | <i>Ans.</i> $\frac{4}{120}, \frac{12}{120}, \frac{48}{120}.$ |
| 4. $\frac{7}{8}, \frac{9}{14}, \frac{1}{6}.$ | <i>Ans.</i> $\frac{105}{840}, \frac{54}{840}, \frac{14}{840}.$ |
| 5. $\frac{4}{5}, \frac{7}{9}, \frac{2}{3}, \frac{1}{12}.$ | <i>Ans.</i> $\frac{176}{180}, \frac{154}{180}, \frac{128}{180}, \frac{11}{180}.$ |
| 6. $\frac{7}{12}, \frac{5}{14}, \frac{8}{15}.$ | <i>Ans.</i> $\frac{245}{840}, \frac{120}{840}, \frac{224}{840}.$ |
| 7. $\frac{7}{8}, \frac{1}{10}, \frac{1}{16}, 9\frac{1}{8}.$ | <i>Ans.</i> $\frac{315}{80}, \frac{8}{80}, \frac{5}{80}, \frac{729}{80}.$ |
| 8. $\frac{1}{4}, \frac{2}{5}, \frac{3}{8}, 5\frac{1}{4}.$ | <i>Ans.</i> $\frac{15}{40}, \frac{16}{40}, \frac{15}{40}, \frac{215}{40}.$ |
| 9. $\frac{7}{12}, \frac{5}{6}$ of $\frac{1}{8}, \frac{1}{6}$ of $\frac{1}{5}.$ | <i>Ans.</i> $\frac{105}{120}, \frac{125}{120}, \frac{18}{120}.$ |
| 10. $\frac{1}{11}, \frac{2}{3}$ of $\frac{1}{6}, \frac{1}{2}$ of $\frac{1}{3}$ of $4\frac{1}{2}.$ | <i>Ans.</i> $\frac{12}{110}, \frac{147}{110}, \frac{12}{110}.$ |

SECTION III.

ADDITION OF FRACTIONS.

ORAL EXERCISE.

- How many are $\frac{1}{4}, \frac{2}{5}$ and $\frac{3}{10}$?
- How many are $\frac{2}{3}, \frac{3}{5}$ and $\frac{1}{15}$?
- How many are $\frac{1}{12}, \frac{2}{15}$ and $\frac{5}{12}$?
- How many are $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$?

What is the sum—

- Of $\frac{1}{2}$ and $\frac{1}{3}$?

SOLUTION.— $\frac{1}{2}$ equals $\frac{2}{4}$, and $\frac{1}{3}$ equals $\frac{1}{3}$; $\frac{2}{4}$ and $\frac{1}{3}$ are $\frac{5}{6}$; hence the sum of $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{5}{6}$.

What is the sum—

- | | |
|--|--|
| 6. Of $\frac{1}{4}$ and $\frac{1}{5}$? | 16. Of $7\frac{1}{2}$ and $8\frac{3}{4}$? |
| 7. Of $\frac{1}{3}$ and $\frac{1}{4}$? | 17. Of $4\frac{2}{3}$ and $9\frac{3}{4}$? |
| 8. Of $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{12}$? | 18. Of $2\frac{1}{3}$ and $5\frac{2}{3}$? |
| 9. Of $\frac{2}{3}$ and $\frac{4}{5}$? | 19. Of $5\frac{2}{3}$ and $9\frac{2}{3}$? |
| 10. Of $\frac{2}{3}$ and $\frac{3}{4}$? | 20. Of $3\frac{2}{3}$ and $7\frac{1}{3}$? |
| 11. Of $\frac{3}{4}, \frac{2}{14}$ and $\frac{1}{2}$? | 21. Of $2\frac{1}{2}, 3\frac{1}{2}$ and $5\frac{1}{2}$? |
| 12. Of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$? | 22. Of $9\frac{1}{2}, 3\frac{1}{2}$ and $2\frac{1}{2}$? |
| 13. Of $\frac{3}{8}, \frac{1}{12}$ and $\frac{5}{8}$? | 23. Of $6, 3\frac{1}{2}$ and $9\frac{1}{10}$? |
| 14. Of $\frac{7}{8}$ and $\frac{6}{15}$? | 24. Of $7\frac{1}{2}, 8\frac{1}{2}$ and $5\frac{1}{2}$? |
| 15. Of $2\frac{1}{2}$ and $3\frac{1}{3}$? | 25. Of $6\frac{1}{2}, 12\frac{1}{2}$ and $18\frac{3}{4}$? |

26. A man bought some eggs for $\frac{1}{2}$ of a dollar, and some butter for $\frac{3}{4}$ of a dollar: how much did he pay for both?

27. Paid $\frac{2}{3}$ of a dollar for calico, $\frac{1}{2}$ of a dollar for muslin, and $\frac{1}{4}$ of a dollar for ribbon: how much did I pay for all?

28. A farmer sold $\frac{1}{2}$ of his grain to one man, $\frac{1}{4}$ to another and $\frac{1}{8}$ to another: how much did he sell to all?

29. A pair of boots cost $5\frac{1}{2}$ dollars, and a hat $2\frac{3}{4}$ dollars: how much did both cost?

30. If a man walk $12\frac{3}{4}$ miles one day and $9\frac{1}{2}$ miles the next, how far does he walk in both days?

31. John has $\frac{2}{3}$ of a quart of chestnuts; Mary, $\frac{1}{3}$ of a quart; and Samuel, $\frac{1}{6}$ of a quart: how many have they all?

32. If $\frac{2}{3}$ of a bushel of apples plus $\frac{1}{4}$ of a bushel equals 235 apples, how many apples are there in a bushel?

33. If $\frac{2}{3}$ of my money be increased by $\frac{1}{4}$ of my money, the sum will equal \$136: how much money have I?

DEFINITIONS AND PRINCIPLES.

79. Addition of Fractions is the process of finding the sum of two or more fractions.

80. Similar Fractional Units are those having the same name.

All abstract fractions are like or similar.

PRINCIPLES.—1. Only similar fractional units can be added.

2. Fractions to be added must have a common denominator.

WRITTEN PROBLEMS.

1. Find the sum of $\frac{2}{3}$ and $\frac{1}{4}$.

SOLUTION.

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}, \text{ or } 1\frac{1}{12}$$

Explanation.—The common denominator of 6ths and 9ths is 18. $\frac{2}{3} = \frac{8}{18}$, and $\frac{1}{4} = \frac{4.5}{18}$; $\frac{8}{18} + \frac{4.5}{18}$ are $\frac{12.5}{18}$, or $1\frac{1}{12}$.

From the foregoing principles and solution we derive for addition of fractions the following

RULES.

1. Reduce the fractions to a common denominator, add the numerators, and write the result over the common denominator.

2. To add mixed numbers, add the integers and the fractions separately, and then add the results.

NOTE 1.—Reduce each fraction to its lowest terms before adding.

2. Reduce fractional results to their lowest terms, and improper fractions to whole or mixed numbers.

Find the sum of the following :

- | | |
|--|-------------------------------|
| 2. $\frac{3}{8}, \frac{7}{12}, \frac{5}{6}.$ | <i>Ans.</i> $2\frac{1}{2}.$ |
| 3. $\frac{5}{8}, \frac{1}{2}, 1\frac{5}{8}.$ | <i>Ans.</i> $2\frac{7}{4}.$ |
| 4. $\frac{7}{12}, \frac{1}{4}, 1\frac{1}{6}.$ | <i>Ans.</i> $1\frac{1}{2}.$ |
| 5. $\frac{3}{8}, \frac{2}{5}, 1\frac{1}{2}.$ | <i>Ans.</i> $1\frac{1}{2}.$ |
| 6. $1\frac{1}{2}, 1\frac{1}{4}, \frac{3}{4}.$ | <i>Ans.</i> $1\frac{3}{2}.$ |
| 7. $\frac{3}{8}, \frac{2}{5}, 1\frac{1}{6}, 1\frac{1}{6}.$ | <i>Ans.</i> $3\frac{7}{5}.$ |
| 8. $1\frac{2}{3}, 1\frac{1}{5}, 1\frac{2}{3}, 2\frac{1}{3}.$ | <i>Ans.</i> $2\frac{7}{5}.$ |
| 9. $\frac{1}{4}, 1\frac{2}{5}, \frac{3}{4}, \frac{1}{5}, \frac{1}{5}.$ | <i>Ans.</i> $1\frac{7}{5}.$ |
| 10. $7\frac{1}{2}, 8\frac{2}{3}, 4\frac{2}{3}.$ | <i>Ans.</i> $20\frac{1}{3}.$ |
| 11. $4\frac{5}{8}, 1\frac{3}{8}, 2\frac{3}{4}.$ | <i>Ans.</i> $8\frac{7}{2}.$ |
| 12. $7\frac{1}{3}, 9\frac{1}{3}, 8\frac{2}{3}.$ | <i>Ans.</i> $25\frac{1}{3}.$ |
| 13. $6\frac{1}{2}, 8\frac{3}{8}, 1\frac{1}{8}.$ | <i>Ans.</i> $16\frac{1}{2}.$ |
| 14. $6\frac{1}{2}, 12\frac{7}{22}, 9\frac{8}{11}.$ | <i>Ans.</i> $28\frac{1}{11}.$ |

NOTE.—In mixed numbers the whole numbers and the fractions may be added separately, and the results then added. Thus, in the preceding problem, $6 + 12 + 9 = 27$; $\frac{1}{2} + \frac{7}{22} + \frac{8}{11} = \frac{11}{22} + \frac{7}{22} + \frac{16}{22} = \frac{34}{22} = 1\frac{1}{11}$; $27 + 1\frac{1}{11} = 28\frac{1}{11}$.

15. Find the sum of $12\frac{3}{8}, 28\frac{5}{8}, 15\frac{3}{8}.$ *Ans.* $57\frac{1}{8}.$
16. Find the sum of $45\frac{2}{3}, 56\frac{2}{3}, 38\frac{1}{3}.$ *Ans.* $140\frac{5}{6}.$
17. Find the sum of $\frac{1}{3}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $\frac{1}{3}$, and $9\frac{1}{4}.$ *Ans.* $9\frac{1}{2}.$
18. Find the sum of $\frac{2}{3}$ of $\frac{4}{5}$, $\frac{1}{2}$ of $6\frac{1}{2}$, and $10\frac{3}{8}.$ *Ans.* $14\frac{3}{8}.$
19. Find the sum of $28\frac{1}{4}, 39\frac{3}{8}$ and $\frac{4}{5}$ of $19\frac{1}{2}.$ *Ans.* $93\frac{1}{2}.$
20. A man has 3 fields; the first contains $16\frac{1}{2}$ acres; the second, $19\frac{1}{4}$ acres; and the third, $21\frac{3}{8}$ acres: how many acres in the 3 fields? *Ans.* $57\frac{3}{8}$ acres.
21. Four boys have the following sums of money: the first, $\$1\frac{1}{5}$; the second, $\$6\frac{2}{5}$; the third, $\$18\frac{1}{2}$; and the fourth, $\$2\frac{4}{5}$: how much have they together? *Ans.* $\$28\frac{3}{5}.$
22. A merchant sold $3\frac{3}{4}$ yards of cloth for $\$17\frac{1}{2}, 5\frac{1}{2}$ yards

for \$23 $\frac{1}{2}$, and 8 $\frac{1}{2}$ yards for \$27 $\frac{1}{2}$: how many yards did he sell, and how much money did he receive?

Ans. 17 $\frac{3}{8}$ yards for \$67 $\frac{23}{40}$.

SECTION IV.

SUBTRACTION OF FRACTIONS.

ORAL EXERCISE.

1. What is the difference between $\frac{9}{12}$ and $\frac{3}{12}$?
2. How much is $\frac{7}{8}$ less $\frac{3}{8}$? $\frac{8}{8}$ less $\frac{5}{8}$?
3. How much is $\frac{9}{10}$ less $\frac{4}{10}$? $\frac{8}{8}$ less $\frac{3}{8}$?
4. How much is $\frac{3}{4}$ less $\frac{2}{8}$?

SOLUTION.— $\frac{3}{4}$ equals $\frac{6}{8}$, and $\frac{2}{8}$ equals $\frac{2}{8}$; $\frac{6}{8}$ less $\frac{2}{8}$ is $\frac{4}{8}$. Hence, $\frac{3}{4}$ less $\frac{2}{8}$ is $\frac{4}{8}$.

Subtract—

- | | |
|---------------------------------------|--|
| 5. $\frac{2}{8}$ from $\frac{3}{4}$. | 10. $\frac{1}{8}$ from $\frac{1}{2}$. |
| 6. $\frac{3}{8}$ from $\frac{5}{8}$. | 11. $\frac{7}{8}$ from $\frac{3}{4}$. |
| 7. $\frac{3}{4}$ from $\frac{5}{8}$. | 12. $\frac{5}{8}$ from $\frac{3}{4}$. |
| 8. $\frac{3}{4}$ from $\frac{5}{8}$. | 13. $\frac{4}{8}$ from $\frac{5}{8}$. |
| 9. $\frac{1}{2}$ from $\frac{5}{8}$. | 14. $\frac{3}{10}$ from $\frac{7}{12}$. |

15. What is the difference between $2\frac{1}{2}$ and $1\frac{3}{4}$?

Suggestion.—Reduce the mixed numbers to improper fractions before subtracting.

What is the difference—

- | | |
|---|---|
| 16. Between $4\frac{1}{2}$ and $2\frac{1}{2}$? | 19. Between $7\frac{1}{2}$ and $2\frac{3}{8}$? |
| 17. Between $5\frac{1}{2}$ and $3\frac{1}{4}$? | 20. Between $9\frac{1}{2}$ and $8\frac{3}{8}$? |
| 18. Between $5\frac{3}{8}$ and $2\frac{1}{4}$? | |

What is the value—

- | | |
|---|--|
| 21. Of $\frac{1}{2} + \frac{1}{4} - \frac{1}{8}$? | 26. Of $5\frac{3}{8} + 2\frac{1}{8} - 3\frac{1}{4}$? |
| 22. Of $\frac{2}{8} + \frac{3}{4} - \frac{5}{8}$? | 27. Of $4\frac{3}{4} - 2\frac{1}{2} + 1\frac{1}{8}$? |
| 23. Of $\frac{1}{2} + \frac{7}{8} - \frac{1}{16}$? | 28. Of $14 + 6\frac{1}{2} - 9\frac{3}{4}$? |
| 24. Of $\frac{8}{8} - \frac{3}{15} + \frac{1}{2}$? | 29. Of $2\frac{3}{4} - 5\frac{1}{2} + 3\frac{7}{8}$? |
| 25. Of $\frac{3}{8} + \frac{2}{8} - \frac{2}{8}$? | 30. Of $6\frac{3}{4} + 3\frac{1}{8} - 6\frac{1}{10}$? |

31. A boy spent $\frac{1}{2}$ of his money for a coat, $\frac{1}{4}$ for a hat and $\frac{1}{8}$ for shoes: what part of his money remained?

32. James is $9\frac{1}{2}$ years old, and Mary $6\frac{1}{2}$ years: how much older is James than Mary?

33. I have \$36, and wish to pay $\$9\frac{1}{2}$: how much money will I have remaining?

34. A man has \$40; he owes $\$7\frac{1}{2}$ to one man, and $\$18\frac{1}{2}$ to another: if he pay his debts, how much money will he have remaining?

35. A ton of coal costs $\$5\frac{1}{2}$, and a barrel of flour $\$6\frac{1}{2}$: how much change should I get if I pay for these with a ten-dollar bill and a five-dollar bill?

36. A piece of carpet contains $65\frac{1}{2}$ yards: if I sell to one person $22\frac{1}{2}$ yards, and to another $27\frac{1}{2}$ yards, how much remains?

37. A farmer sold $\frac{1}{3}$ of his corn to one man, $\frac{2}{3}$ to another, and had 50 bushels remaining: how much corn had he at first?

38. Charles spent $\frac{1}{4}$ of his money for candy, and $\frac{3}{4}$ for oranges, and had 21 cents remaining: how much money had he at first?

39. B spent $\frac{3}{4}$ of his fortune, and found that \$600 was $\frac{1}{4}$ of the remainder: how much money had he at first?

40. C lost $\frac{3}{4}$ of $\frac{1}{2}$ of his money, and found that \$260 was $\frac{1}{4}$ of the remainder: how much had he at first?

DEFINITIONS AND PRINCIPLES.

81. Subtraction of Fractions is the process of finding the difference between two fractions.

PRINCIPLES.—1. Two fractions can be subtracted only when their fractional units are similar.

2. Fractions must have a common denominator before their difference can be found.

NOTE.—Reduce compound fractions to simple ones, and each fraction as well as the resulting difference to its lowest terms.

WRITTEN PROBLEMS.

1. Subtract $\frac{1}{4}$ from $\frac{1}{2}$.

SOLUTION.

$$\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$$

Explanation.— $\frac{1}{2} = \frac{2}{4}$, and $\frac{1}{4} = \frac{1}{4}$; $\frac{2}{4}$ less $\frac{1}{4}$ is $\frac{1}{4}$.

The following is the rule for subtraction of fractions:

RULE.

Reduce the fractions to a common denominator, subtract the numerators, and write the result over the common denominator.

How much is—

2. $\frac{2}{10} - \frac{2}{10} ?$	<i>Ans.</i> $\frac{0}{10}$.	6. $\frac{2}{18} - \frac{2}{18} ?$	<i>Ans.</i> $\frac{0}{18}$.
3. $\frac{3}{4} - \frac{1}{4} ?$	<i>Ans.</i> $\frac{2}{4}$.	7. $\frac{7}{8} - \frac{4}{8} ?$	<i>Ans.</i> $\frac{3}{8}$.
4. $\frac{2}{9} - \frac{1}{9} ?$	<i>Ans.</i> $\frac{1}{9}$.	8. $\frac{10}{11} - \frac{8}{11} ?$	<i>Ans.</i> $\frac{2}{11}$.
5. $\frac{4}{15} - \frac{1}{15} ?$	<i>Ans.</i> $\frac{3}{15}$.	9. $\frac{9}{15} - \frac{6}{15} ?$	<i>Ans.</i> $\frac{3}{15}$.

10. From $6\frac{2}{3}$ take $3\frac{1}{3}$.

SOLUTION 1.

$$6\frac{2}{3} - 3\frac{1}{3} =$$

$$2\frac{2}{3} - \frac{1}{3} =$$

$$1\frac{2}{3} - \frac{1}{3} = \frac{1}{3}, \text{ or } 2\frac{1}{3}$$

NOTE.—Let the pupil give the explanation.

SOLUTION 2.

$$\text{From } 6\frac{2}{3} = 6\frac{4}{6}$$

$$\text{Take } 3\frac{1}{3} = 3\frac{2}{6}$$

$$\text{Remainder} = 2\frac{2}{6}$$

Explanation.—Since $\frac{2}{3}$ cannot be taken from $\frac{1}{3}$, convert 1 of the 6 units to $\frac{2}{6}$, thus giving $\frac{4}{6} + \frac{2}{6}$, or $\frac{6}{6}$; $\frac{6}{6} - \frac{2}{6} = \frac{4}{6}$; and 5 units -- 3 units = 2 units, etc.

11. From $24\frac{1}{2}$ take $8\frac{1}{2}$.	<i>Ans.</i> $16\frac{1}{2}$.
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12. From $42\frac{1}{2}$ take $6\frac{1}{2}$.	<i>Ans.</i> $36\frac{1}{2}$.
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13. From $54\frac{1}{2}$ take $21\frac{1}{2}$.	<i>Ans.</i> $32\frac{1}{2}$.
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14. From $55\frac{1}{2}$ take $38\frac{1}{2}$.	<i>Ans.</i> $17\frac{1}{2}$.
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15. From $36\frac{2}{3}$ take $18\frac{2}{3}$.	<i>Ans.</i> $17\frac{2}{3}$.
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16. From $39\frac{2}{3}$ take $33\frac{1}{3}$.	<i>Ans.</i> $6\frac{1}{3}$.
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17. From $\frac{2}{3}$ of $\frac{5}{6}$ take $\frac{2}{3}$ of $\frac{1}{6}$.	<i>Ans.</i> $\frac{2}{9}$.
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18. From $\frac{4}{5}$ of $3\frac{1}{5}$ take $\frac{1}{5}$ of $2\frac{2}{5}$.	<i>Ans.</i> $\frac{5}{5}$.
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19. From $\frac{2}{3} + \frac{1}{3} + \frac{1}{3}$ take $\frac{2}{3}$ of $2\frac{1}{3}$.	<i>Ans.</i> $\frac{2}{3}$.
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20. A man having $95\frac{1}{2}$ acres of land, sold to A $21\frac{1}{2}$ acres, to B $19\frac{1}{2}$ acres, and to C $24\frac{1}{2}$ acres: how much land remained?

Ans. $29\frac{1}{2}$ acres.

21. I have two rolls of carpet, each containing $125\frac{1}{4}$ yards; from one piece I sell $26\frac{7}{8}$ yards, and from the other $47\frac{1}{2}$ yards: how much remains? *Ans.* $176\frac{1}{8}$ yds.

22. A boy gave $18\frac{3}{4}$ cents for a slate, $62\frac{1}{2}$ cents for a book, and $31\frac{1}{4}$ cents for some paper: how much change would he get out of a two-dollar bill? *Ans.* $87\frac{1}{2}$ cents.

23. A, B, C and D bought a house; A paid $\$107\frac{1}{2}$; B, $\$215\frac{2}{5}$; C, $\$317\frac{1}{2}$; and D, $\$245\frac{3}{4}$; the house was to cost $\$1000$: how much do they yet owe? *Ans.* $\$113\frac{1}{8}$.

24. I had $\frac{2}{3}$ of $\frac{1}{4}$ of $\$1000$, and lost $\frac{5}{8}$ of $\frac{3}{4}$ of $\$630$: how much have I remaining? *Ans.* $\$150$.

SECTION V.

MULTIPLICATION OF FRACTIONS.

CASE I.

To Multiply a Fraction by an Integer.

ORAL EXERCISE.

1. What cost 6 yards of cloth at $\frac{2}{3}$ of a dollar a yard?

SOLUTION.—If 1 yard of cloth cost $\frac{2}{3}$ of a dollar, 6 yards will cost $6 \times \frac{2}{3}$, or $\frac{12}{3}$ of a dollar, equal to 4 dollars.

2. At $\frac{1}{2}$ dollar a pair, what will 9 pairs of chickens cost?

3. If 1 fish weigh $\frac{3}{8}$ of a pound, how much will 10 weigh if they are of the same size?

4. A man earns $\frac{2}{3}$ of a dollar in one day: how much can he earn at the same rate in 12 days?

5. If a yard of delaine cost $\frac{2}{3}$ of a dollar, how much will 9 yards cost?

6. If 1 horse can eat $\frac{3}{8}$ of a ton of hay in a month, how much can 13 horses eat in the same time?

7. What will 25 pounds of tea cost at $\$1\frac{1}{2}$ a pound?

8. How many are 6 times $\frac{7}{8}$? $\frac{2}{10}$? $\frac{1}{12}$? $\frac{3}{8}$?

9. How many are 9 times $\frac{2}{10}$? $\frac{5}{14}$? $\frac{1}{11}$? $\frac{12}{16}$?

10. How many are 4 times $\frac{5}{8}$? $\frac{5}{8}$? $\frac{8}{14}$? $\frac{10}{8}$?
 11. How many are 8 times $\frac{1}{2}$? $\frac{8}{9}$? $\frac{12}{8}$? $\frac{9}{12}$?
 12. How many are 10 times $1\frac{3}{10}$? $2\frac{3}{4}$? $6\frac{1}{8}$? $31\frac{7}{8}$?

PRINCIPLE.—A fraction is multiplied by multiplying its numerator or by dividing its denominator.

WRITTEN PROBLEMS.

1. Multiply $1\frac{1}{3}$ by 6.

SOLUTION 1.

$$1\frac{1}{3} \times 6 = 1\frac{1}{3} = 1\frac{4}{3}, \text{ or } 4\frac{2}{3}.$$

NOTE.—Let the pupil explain the solution.

Or 2.

$$1\frac{1}{3} \times 6 = 1\frac{4}{3}, \text{ or } 4\frac{2}{3}.$$

Explanation.—Since dividing the denominator of a fraction multiplies the value, $6 \times 1\frac{1}{3}$ equals $1\frac{4}{3}$, or $4\frac{2}{3}$.

RULES.

1. To multiply a fraction by an integer, multiply the numerator or divide the denominator.
 2. To multiply a mixed number by an integer, multiply the whole number and the fraction separately, and add the results.

Multiply—

- | | | | |
|---------------------------|------------------------|----------------------------|--------------------------|
| 2. $1\frac{1}{3}$ by 9. | Ans. $7\frac{1}{3}$. | 7. $18\frac{3}{4}$ by 10. | Ans. $187\frac{1}{4}$. |
| 3. $1\frac{1}{3}$ by 40. | Ans. $37\frac{1}{3}$. | 8. $9\frac{5}{8}$ by 21. | Ans. $206\frac{1}{8}$. |
| 4. $1\frac{1}{3}$ by 49. | Ans. $37\frac{1}{3}$. | 9. $8\frac{3}{8}$ by 24. | Ans. $213\frac{3}{8}$. |
| 5. $1\frac{1}{4}$ by 91. | Ans. $71\frac{1}{4}$. | 10. $63\frac{7}{8}$ by 56. | Ans. 3560. |
| 6. $1\frac{1}{8}$ by 125. | Ans. 75. | 11. $45\frac{1}{8}$ by 60. | Ans. $2743\frac{1}{8}$. |

CASE II.

To Multiply an Integer by a Fraction.

ORAL EXERCISE.

1. If 1 yard of muslin cost 16 cents, what will $\frac{1}{4}$ of a yard cost?
 2. A man had 18 chickens, and sold $\frac{1}{3}$ of them: how many did he sell?
 3. If 1 ton of hay cost \$24, what will $\frac{3}{4}$ of a ton cost?
 4. John has 60 cents, and Mary has $\frac{2}{3}$ as many: how many cents has Mary?

5. A father is 48 years old ; his son is $\frac{2}{3}$ as old : how much older is the father than the son ?

6. $\frac{2}{3}$ of 60 dollars is 3 times what a shawl cost : what was the cost of the shawl ?

7. What will $\frac{3}{4}$ of 70 pounds of coffee cost at 35 cents a pound ?

8. If 2 hats cost \$14, what will $\frac{3}{4}$ of 42 hats cost ?

9. A is worth \$3200 ; $\frac{3}{4}$ of his fortune is 6 times B's : what is B's fortune ?

10. $\frac{5}{8}$ of 91 are how many times 7 ?

11. A horse cost \$120, and a wagon $\frac{2}{3}$ as much : what did the wagon cost ?

12. How many yards of muslin can you buy for \$1.44, if $\frac{2}{3}$ of 24 yards cost \$3.20 ?

13. $\frac{3}{4}$ of 63 yards of calico are worth \$3.24 : what are $\frac{5}{8}$ of 16 yards worth ?

14. $\frac{3}{4}$ of \$81 is \$40 less than $\frac{1}{2}$ of what a building-lot cost : what was the cost of the lot ?

15. A farmer had 600 bushels of wheat ; at one time he sold $\frac{2}{3}$, and at another $\frac{1}{4}$ of it : how many bushels remained ?

16. $\frac{3}{4}$ of 81 are how many times $\frac{1}{2}$ of 18 ?

17. $\frac{2}{3}$ of 56 are how many times $\frac{1}{4}$ of 10 ?

18. $\frac{2}{3}$ of 121 are how many times $\frac{1}{4}$ of 44 ?

19. $\frac{2}{3}$ of 120 are how many times $\frac{1}{4}$ of 77 ?

20. $\frac{1}{2}$ of 288 are how many times $\frac{1}{4}$ of 64 ?

21. What is $\frac{1}{4}$ of 8 ?

SOLUTION.— $\frac{1}{4}$ of 1 is $\frac{1}{4}$, and $\frac{1}{4}$ of 8 is 8 times $\frac{1}{4}$, or $\frac{1}{2}$; hence, $\frac{1}{4}$ of 8 is $\frac{1}{2}$.

22. What is $\frac{1}{4}$ of 10 ? $\frac{1}{4}$ of 13 ?

23. What is $\frac{1}{4}$ of 22 ? $\frac{1}{4}$ of 21 ?

24. What is $\frac{1}{6}$ of 25 ? $\frac{1}{6}$ of 27 ?

25. What is $\frac{2}{3}$ of 7 ? $\frac{2}{3}$ of 8 ?

26. What is $\frac{2}{3}$ of 10 ? $\frac{2}{3}$ of 14 ?

27. What is $\frac{2}{3}$ of 12 ? $\frac{2}{3}$ of 20 ?

28. What is $\frac{1}{6}$ of 6 ? $\frac{1}{6}$ of 12 ?

29. $\frac{2}{10}$ of \$6 is $\frac{3}{15}$ of what a hat cost: what did the hat cost?

30. One part of a pole is 12 feet long, which lacks 4 feet of being $\frac{4}{11}$ of the length of the other part: how long is the pole?

31. John has 20 cents, and James has $5\frac{1}{4}$ times as much: how many cents has James?

32. Mary has 16 dollars, and $6\frac{1}{4}$ times her number is 5 times Emma's number: how many has Emma?

PRINCIPLE.—The product of an integer by a fraction equals the fraction of the integer.

WRITTEN PROBLEMS.

1. Multiply 66 by $\frac{2}{3}$.

SOLUTION.— $66 \times \frac{2}{3} = \frac{2}{3} \times 66 = \frac{2 \times 66}{3} = \frac{132}{3}$, or 44.

Multiply—

2. 57 by $\frac{1}{2}$. *Ans.* $47\frac{1}{2}$. | 5. 75 by $\frac{1}{3}$. *Ans.* $47\frac{1}{2}$.

3. 88 by $\frac{3}{4}$. *Ans.* 66. | 6. 90 by $\frac{9}{10}$. *Ans.* $40\frac{1}{2}$.

4. 124 by $\frac{7}{8}$. *Ans.* $54\frac{1}{4}$. | 7. 91 by $\frac{1}{8}$. *Ans.* $38\frac{1}{2}$.

8. Multiply 91 by $6\frac{2}{3}$.

SOLUTION.

$\begin{array}{r} 91 \\ 6\frac{2}{3} \\ \hline 546 \\ 36\frac{2}{3} \\ \hline 582\frac{2}{3} \end{array}$	<i>Explanation.</i>—6 times 91 are 546, and $\frac{2}{3}$ of 91 are $36\frac{2}{3}$; adding the two results, we have $582\frac{2}{3}$.
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The following are the rules for the multiplication of integers by fractions:

RULES.

1. Multiply the integer by the numerator, and divide the result by the denominator.

Or, Divide the integer by the denominator, and multiply the result by the numerator.

2. When the multiplier is a mixed number, multiply by the integer and the fraction separately, and add the results.

9. Multiply 29 by $3\frac{1}{3}$. *Ans.* $103\frac{1}{3}$.

Multiply—

10. 60 by $7\frac{1}{2}$. *Ans.* 428 $\frac{1}{2}$. | 12. 426 by $15\frac{1}{2}$. *Ans.* 6560 $\frac{1}{2}$.
 11. 127 by $12\frac{3}{4}$. *Ans.* 1571 $\frac{3}{4}$. | 13. 125 by $\frac{2}{3}$ of $\frac{5}{8}$. *Ans.* 69 $\frac{1}{2}$.
 14. 220 by $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{6}{7}$. *Ans.* 103 $\frac{1}{2}$.
 15. 990 by $1\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. *Ans.* 565 $\frac{5}{6}$.
 16. A has \$325, B has $6\frac{2}{3}$ times, and C $18\frac{3}{4}$ times as much:
 how much has each, and how much have they all?
Ans. B, \$2210; C \$6093 $\frac{3}{4}$; all, \$8628 $\frac{3}{4}$.

CASE III.

To Multiply a Fraction by a Fraction.

ORAL EXERCISE.

1. What is $\frac{2}{3}$ of $\frac{5}{8}$?

SOLUTION.— $\frac{1}{3}$ of $\frac{1}{8}$ is $\frac{1}{24}$; $\frac{1}{3}$ of $\frac{5}{8}$ is 5 times $\frac{1}{24}$, or $\frac{5}{24}$; and $\frac{2}{3}$ of $\frac{5}{8}$ is 2 times $\frac{5}{24}$, or $\frac{10}{24}$; hence, $\frac{2}{3}$ of $\frac{5}{8}$ is $\frac{5}{12}$.

2. John found $\frac{3}{4}$ of a dollar, and gave his brother $\frac{2}{3}$ of it: what part of a dollar did John keep?

3. A farmer having $\frac{3}{4}$ of a ton of hay, gave away $\frac{1}{3}$ of $\frac{2}{3}$ of it: how much did he give away?

4. If I have $4\frac{3}{4}$ bushels of wheat, and sell $\frac{2}{3}$ of it, how many bushels have I remaining?

5. A bought a cow for $\frac{2}{3}$ of 66 $\frac{2}{3}$ dollars: how much did he pay?

6. Three-fourths of $18\frac{3}{4}$ years is $\frac{5}{16}$ of my age: how old am I?

7. If 12 pairs of shoes cost \$28 $\frac{1}{2}$, what will 5 pairs cost?

8. Two and three-quarters pounds of rice will cost how much, at 8 $\frac{1}{2}$ cents a pound?

9. How much will $3\frac{1}{2}$ dozen of eggs cost at 12 $\frac{1}{2}$ cents a dozen?

10. What will $3\frac{1}{4}$ yards of cassimere cost at \$3 $\frac{1}{4}$ a yard?

11. What will 12 $\frac{1}{2}$ yards of calico cost at 6 $\frac{1}{4}$ cents a yard?

12. What is the cost of $3\frac{1}{8}$ boxes of soap at \$6 $\frac{3}{8}$ a box?

PRINCIPLE.—The product of a fraction by a fraction equals the fraction of that fraction.

Remark.—The process of multiplying fractions may be shortened by cancellation.

WRITTEN PROBLEMS.

1. What is the product of $\frac{2}{3}$ of $5\frac{1}{2}$ by $\frac{2}{3}$?

SOLUTION 1.

$$\frac{2}{3} \times 5\frac{1}{2} \text{ by } \frac{2}{3} = \frac{2}{3} \times \frac{11}{2} \times \frac{2}{3} = \frac{44}{9}, \text{ or } 4\frac{8}{9}.$$

SOLUTION 2.

$$\frac{2}{3} \times \frac{11}{2} \times \frac{2}{3} = \frac{44}{9}, \text{ or } 4\frac{8}{9}.$$

Note.—Let the pupils give the explanations.

The following is the rule for multiplying fractions by fractions:

RULE.

Multiply the numerators together for the numerator, and the denominators for the denominator of the product.

Suggestion 1.—Reduce mixed numbers first to improper fractions.

Suggestion 2.—Cancel common factors in the numerators and the denominators.

Remark.—This case is practically the same as finding the fractional part of a fraction.

Multiply—

- | | | | |
|--|---|--|--------------------------------|
| 2. $1\frac{2}{3}$ by $\frac{9}{10}$. | <i>Ans.</i> $1\frac{2}{5}$. | 7. $1\frac{1}{2}$ by $\frac{3}{10}$ by $1\frac{7}{12}$. | <i>Ans.</i> $7\frac{1}{4}$. |
| 3. $1\frac{2}{3}$ by $1\frac{1}{4}$. | <i>Ans.</i> $2\frac{7}{6}$. | 8. $3\frac{1}{2}$ by $\frac{2}{3}$ by $\frac{2}{3}$. | <i>Ans.</i> $2\frac{2}{3}$. |
| 4. $\frac{9}{10}$ by $\frac{5}{21}$ by $\frac{7}{10}$. | <i>Ans.</i> $\frac{3}{10}$. | 9. $\frac{2}{3}$ by $1\frac{1}{2}$ by $6\frac{2}{3}$. | <i>Ans.</i> $4\frac{2}{3}$. |
| 5. $\frac{2}{3}$ by $1\frac{1}{2}$ by $1\frac{1}{3}$. | <i>Ans.</i> $\frac{8}{3}$. | 10. $1\frac{1}{2}$ of $\frac{2}{3}$ by $21\frac{1}{2}$. | <i>Ans.</i> $28\frac{1}{2}$. |
| 6. $1\frac{7}{12}$ by $\frac{2}{3}$ by $1\frac{7}{12}$. | <i>Ans.</i> $2\frac{7}{12}$. | 11. $9\frac{2}{3}$ by $35\frac{7}{12}$. | <i>Ans.</i> $348\frac{1}{2}$. |
| 12. $1\frac{1}{4}$ of $2\frac{1}{2}$ by $1\frac{1}{8}$ of $\frac{7}{8}$. | <i>Ans.</i> $1\frac{5}{8}$. | | |
| 13. $\frac{1}{4}$ of $3\frac{1}{2}$ by $\frac{2}{3}$ of $5\frac{1}{2}$. | <i>Ans.</i> $1\frac{2}{3}$. | | |
| 14. A has $\frac{2}{3}$ of \$375, B has $\frac{2}{3}$ as much, and C $\frac{2}{3}$ as much as both: how many dollars has each, and how many have they all? | <i>Ans.</i> A, \$281 $\frac{1}{3}$; B, \$225; C, \$303 $\frac{1}{3}$; all, \$810. | | |

SECTION VI.

DIVISION OF FRACTIONS.

CASE I.

To Divide a Fraction by an Integer.

ORAL EXERCISE.

1. If 4 apples cost $\frac{3}{10}$ of a dime, how much will 1 cost?

SOLUTION.—If 4 apples cost $\frac{3}{10}$ of a dime, 1 apple, which is $\frac{1}{4}$ of 4 apples, will cost $\frac{1}{4}$ of $\frac{3}{10}$, or $\frac{3}{40}$ of a dime; hence, if 4 apples cost $\frac{3}{10}$ of a dime, 1 apple will cost $\frac{3}{40}$ of a dime.

2. If 5 hens cost $\frac{1}{5}$ of a dollar, what will 1 hen cost?
 3. If 6 oranges cost $\frac{3}{10}$ of a dollar, what will 1 orange cost?

SOLUTION.—If 6 oranges cost $\frac{3}{10}$ of a dollar, one orange will cost $\frac{1}{6}$ of $\frac{3}{10}$, or $\frac{1}{20}$ of a dollar, equal to $\frac{1}{20}$ of a dollar; hence, if 6 oranges cost $\frac{3}{10}$ of a dollar, 1 orange will cost $\frac{1}{20}$ of a dollar.

4. What will 1 yard of muslin cost if 5 yards cost $\frac{2}{3}$ of a dollar?

5. If 8 men can do $\frac{3}{4}$ of a piece of work, how much can 1 man do?

6. If 7 horses can eat $\frac{2}{3}$ of a load of hay in a day, how much can 1 horse eat in the same time?

7. If $\frac{1}{12}$ of a bushel be divided into 6 equal parts, how large will each part be?

8. A man shares $\frac{3}{4}$ of a ton of coal among 5 poor neighbors: how much does each get?

9. If I have $\$4\frac{1}{2}$, and share it equally with 3 companions, how much does each get?

10. A boy has 10 quarts of blackberries: if he sell $\frac{2}{3}$ of them at 6 cents a quart, how much does he receive?

11. What is $\frac{1}{2}$ of $6\frac{1}{2}$? $\frac{1}{3}$ of $7\frac{1}{3}$? $\frac{1}{4}$ of $\frac{5}{8}$?

12. How often is 7 contained in $\frac{3}{4}$? 4 in $\frac{5}{8}$? 8 in $\frac{2}{3}$?

PRINCIPLE.—A fraction is divided by dividing its numerator or by multiplying its denominator.

WRITTEN PROBLEMS.

1. Divide
- $\frac{1}{5}$
- by 5.

SOLUTION. *Explanation.*— $\frac{1}{5}$ divided by $1 = \frac{1}{5}$: hence, $\frac{1}{5}$ divided $\frac{1}{5} \div 5 = \frac{1}{25}$. by 5 = $\frac{1}{5}$ of $\frac{1}{5}$, or $\frac{1}{25}$.

2. Divide
- $\frac{1}{3}$
- by 5.

SOLUTION. *Explanation.*— $\frac{1}{3}$ divided by $1 = \frac{1}{3}$; therefore, $\frac{1}{3}$ divided $\frac{1}{3} \div 5 = \frac{1}{15}$ by 5 = $\frac{1}{5}$ of $\frac{1}{3}$, or $\frac{1}{15}$. Hence, $\frac{1}{3}$ divided by 5 is $\frac{1}{15}$.

Divide—

3. $\frac{1}{5}$ by 5.	<i>Ans.</i> $\frac{1}{25}$.	8. $\frac{7}{8}$ by 17.	<i>Ans.</i> $\frac{7}{136}$.
4. $\frac{2}{5}$ by 7.	<i>Ans.</i> $\frac{2}{35}$.	9. $7\frac{1}{2}$ by 6.	<i>Ans.</i> $1\frac{1}{4}$.
5. $\frac{3}{4}$ by 9.	<i>Ans.</i> $\frac{3}{36}$.	10. $63\frac{3}{4}$ by 11.	<i>Ans.</i> $5\frac{3}{44}$.
6. $\frac{7}{12}$ by 5.	<i>Ans.</i> $\frac{7}{60}$.	11. $117\frac{1}{2}$ by 8.	<i>Ans.</i> $14\frac{1}{16}$.
7. $\frac{9}{11}$ by 8.	<i>Ans.</i> $\frac{9}{88}$.	12. $233\frac{1}{3}$ by 14.	<i>Ans.</i> $16\frac{2}{3}$.

CASE II.

To Divide an Integer by a Fraction.

ORAL EXERCISE.

1. At
- $\$ \frac{2}{3}$
- a yard, how many yards of cloth can be bought for \$6?

SOLUTION.—If 1 yard of cloth cost $\$ \frac{2}{3}$, for \$6 as many yards can be bought as $\$ \frac{2}{3}$ is contained times in \$6; \$6 is equal to $\$ \frac{12}{2}$, and $\$ \frac{2}{3}$ is contained in $\$ \frac{12}{2}$, 10 times. Hence, at $\frac{2}{3}$ of a dollar a yard 10 yards of cloth may be bought for \$6.

2. If 1 bottle hold
- $\frac{1}{5}$
- of a gallon, how many bottles will be required to hold 5 gallons?

3. How many baskets holding
- $\frac{3}{4}$
- of a bushel each will be required to hold 15 bushels of peaches?

4. If a yard of cloth cost
- $\$ \frac{1}{5}$
- , how many yards can be bought for \$10?

5. If
- $\frac{7}{8}$
- yards of gingham make 1 apron, how many can be made out of 14 yards? 7 yards? 21 yards?

6. At
- $\$ \frac{1}{3}$
- a dozen, how many dozen of eggs can I buy for \$3? \$5? \$8?

7. How many times is
- $\frac{1}{2}$
- contained in 6? In 10? In 12?

8. How many times is
- $\frac{2}{3}$
- contained in 3? In 9? In 12?

9. How many times is $\frac{2}{10}$ contained in 2? In 3? In 4?
 10. How many times is $\frac{2}{3}$ contained in 5? In 12? In 15?

WRITTEN PROBLEMS.

1. Divide 12 by $\frac{2}{3}$.

SOLUTION. *Explanation.*—Since 12 divided by 1 = 12, 12 divided by $\frac{2}{3}$ = 6 $\times$ 12, and divided by $\frac{2}{3}$ = $\frac{1}{3}$ of 6 $\times$ 12, or 12 $\times$ $\frac{3}{2}$ = $14\frac{1}{2}$, or $14\frac{1}{2}$ $\frac{2}{3}$ of 12, equal to $14\frac{1}{2}$, or $14\frac{1}{2}$.

The following is the rule for dividing an integer by a fraction:

RULE.

Multiply the integer by the denominator, and divide by the numerator of the given fraction.

Or, Multiply the integer by the divisor inverted.

Divide—

2. 10 by $\frac{2}{3}$.	<i>Ans.</i> 14.	6. 88 by $1\frac{1}{2}$.	<i>Ans.</i> $94\frac{2}{3}$.
3. 12 by $\frac{2}{3}$.	<i>Ans.</i> 32.	7. 125 by $3\frac{1}{2}$.	<i>Ans.</i> 35.
4. 60 by $\frac{2}{3}$.	<i>Ans.</i> $77\frac{1}{2}$.	8. 118 by $5\frac{1}{2}$.	<i>Ans.</i> $22\frac{2}{3}$.
5. 95 by $1\frac{1}{2}$.	<i>Ans.</i> $135\frac{1}{2}$.	9. 745 by $21\frac{1}{2}$.	<i>Ans.</i> 35.

CASE III.

To Divide a Fraction by a Fraction.

ORAL EXERCISE.

1. At $\$ \frac{2}{10}$ each, how many ducks can be bought for $\$ \frac{2}{3}$?

SOLUTION 1.—If 1 duck cost $\$ \frac{2}{10}$, for $\$ \frac{2}{3}$ or $\$ 1\frac{1}{3}$ as many ducks can be bought as $\frac{2}{10}$ is contained times in $1\frac{1}{3}$, or 6. Hence, at $\$ \frac{2}{10}$ each, 6 ducks can be bought for $\$ \frac{2}{3}$.

SOLUTION 2.—1 is contained in $\frac{3}{2}$, $\frac{3}{2}$ times; $\frac{1}{10}$ is contained $10 \times \frac{3}{2}$ times, or $\frac{30}{2}$ times; and $\frac{2}{10}$ is contained $\frac{1}{2}$ of $\frac{30}{2}$, or $\frac{15}{2}$ times, equal to 6. Hence, etc.

2. How many pounds of butter, at $\$ \frac{1}{4}$ per pound, can be bought for \$2?

3. If 1 quire of paper cost $\$ \frac{2}{10}$, how many quires can I get for $\$ \frac{1}{2}$?

4. When apples are worth $\$ \frac{2}{3}$ per bushel, how many bushels can be bought for $\$ \frac{2}{3}$?

5. A man had $\$4\frac{4}{5}$: how many dozen of eggs could he buy at $\$2\frac{2}{5}$ a dozen?

6. At $\$7\frac{1}{8}$ per yard, how many yards of silk can be bought for $\$4\frac{1}{2}$?

7. If 3 bushels of corn cost $\$3\frac{3}{4}$, how many bushels can I buy for $\$15$?

8. If 4 hats cost $\$5\frac{1}{2}$, how many can I buy for $\$8\frac{1}{4}$?

9. If $5\frac{1}{2}$ apples cost $11\frac{1}{2}$ cents, how much will 8 apples cost?

10. How many times is $\frac{2}{3}$ contained in $\frac{4}{3}$? In $\frac{8}{3}$?

11. How many times is $\frac{2}{3}$ contained in $\frac{5}{3}$? In $\frac{7}{3}$?

12. How many times is $\frac{2}{3}$ contained in $2\frac{1}{3}$? In $3\frac{1}{3}$?

13. How many times is $\frac{2}{3}$ contained in $7\frac{1}{4}$? In $33\frac{1}{8}$?

Divide—

14. $\frac{7}{8}$ by $\frac{2}{3}$; $\frac{2}{5}$ by $\frac{3}{4}$.

15. $\frac{1}{3}$ of $\frac{1}{2}$ by $\frac{2}{3}$; $\frac{1}{4}$ of $\frac{1}{3}$ by $\frac{1}{5}$.

16. $\frac{4}{15}$ by $3\frac{1}{3}$; $\frac{1}{4}$ of $\frac{1}{3}$ by $\frac{1}{2}$ of $\frac{1}{5}$.

17. $9\frac{1}{2}$ by $\frac{1}{3}$; $16\frac{1}{4}$ by $3\frac{1}{4}$.

18. $22\frac{2}{3}$ by $2\frac{1}{3}$; $\frac{3}{4}$ of $\frac{1}{2} \times 28$ by $\frac{2}{3}$ of 7.

WRITTEN PROBLEMS.

1. Divide $\frac{8}{9}$ by $\frac{7}{12}$.

SOLUTION. *Explanation.*—One is contained in $\frac{8}{9}$, $\frac{9}{8}$ times; $\frac{1}{12}$ is contained $12 \times \frac{8}{9}$ times, and $\frac{7}{12}$ is contained $\frac{1}{7}$ of $12 \times \frac{8}{9}$ times, or $\frac{1}{7}$ of $\frac{8}{3}$ times, equal to $\frac{8}{21}$, or $1\frac{1}{3}$. Hence, $\frac{8}{9}$ divided by $\frac{7}{12}$ is $1\frac{1}{3}$.

The following are the rules for division of fractions by fractions:

RULES.

1. Divide the dividend by the numerator of the divisor, and multiply by its denominator.

2. Invert the terms of the divisor, and multiply the numerators together, also the denominators.

3. Reduce the fractions to a common denominator, and divide the numerator of the dividend by the numerator of the divisor.

Divide—

- | | | | |
|---|---|---|-------------------------------|
| 2. $\frac{1\frac{1}{2}}{7}$ by $\frac{7}{11}$. | <i>Ans.</i> $\frac{5\frac{5}{11}}{7}$. | 8. $9\frac{1}{2}$ by $\frac{2}{10}$. | <i>Ans.</i> $10\frac{1}{2}$. |
| 3. $\frac{1\frac{1}{2}}{8}$ by $\frac{1}{11}$. | <i>Ans.</i> $1\frac{1}{8}$. | 9. $9\frac{1}{2}$ by $3\frac{1}{2}$. | <i>Ans.</i> $2\frac{1}{2}$. |
| 4. $\frac{1\frac{1}{2}}{7}$ by $\frac{1}{10}$. | <i>Ans.</i> $\frac{5}{7}$. | 10. $5\frac{1}{2}$ by $3\frac{1}{2}$. | <i>Ans.</i> $1\frac{1}{2}$. |
| 5. $\frac{1\frac{1}{2}}{7}$ by $\frac{1}{10}$. | <i>Ans.</i> $1\frac{5}{14}$. | 11. $38\frac{1}{2}$ by $\frac{3}{11}$. | <i>Ans.</i> $66\frac{1}{2}$. |
| 6. $\frac{1\frac{1}{2}}{8}$ by $\frac{1}{11}$. | <i>Ans.</i> $\frac{1}{2}$. | 12. $\frac{3}{8}$ of 20 by $9\frac{1}{2}$. | <i>Ans.</i> $\frac{1}{9}$. |
| 7. $7\frac{1}{2}$ by $\frac{3}{8}$. | <i>Ans.</i> 10. | 13. $48\frac{1}{2}$ by $17\frac{1}{2}$. | <i>Ans.</i> $2\frac{5}{11}$. |
| 14. $\frac{3}{7}$ of $24\frac{1}{2}$ by $\frac{1}{11}$. | | | <i>Ans.</i> $64\frac{1}{2}$. |
| 15. $\frac{3}{8}$ of $35\frac{1}{2}$ by $\frac{3}{7}$ of $8\frac{1}{2}$. | | | <i>Ans.</i> $1\frac{1}{2}$. |

COMPLEX FRACTIONS.

WRITTEN PROBLEMS.

1. Reduce
- $\frac{\frac{3}{8}}{\frac{1}{10}}$
- to its simplest form.

NOTE.—Examples like this may be regarded as being one form of expressing *division of fractions*. The signification is, that the fraction in the numerator is to be divided by the fraction in the denominator.

$$\text{SOLUTION.} - \frac{\frac{3}{8}}{\frac{1}{10}} = \frac{3}{8} \div \frac{1}{10} = \frac{3}{8} \times \frac{10}{1} = \frac{30}{8} = \frac{15}{4}, \text{ or } 3\frac{3}{4}.$$

NOTE.—Let the pupils give the explanation.

Reduce to its simplest form—

- | | | | |
|---|--|--|--------------------------------|
| 2. $\frac{\frac{1\frac{1}{2}}{7}}{\frac{1}{8}}$ | <i>Ans.</i> $\frac{3\frac{3}{8}}{8}$. | 9. $\frac{6\frac{1}{2}}{\frac{7}{8}}$ | <i>Ans.</i> $11\frac{1}{10}$. |
| 3. $\frac{\frac{1\frac{1}{2}}{10}}{\frac{1}{16}}$ | <i>Ans.</i> $1\frac{1}{8}$. | 10. $\frac{\frac{7}{8}}{7\frac{1}{2}}$ | <i>Ans.</i> $\frac{7}{66}$. |
| 4. $\frac{\frac{2\frac{1}{2}}{24}}{\frac{1}{16}}$ | <i>Ans.</i> $\frac{1}{8}$. | 11. $\frac{\frac{7}{8} \text{ of } \frac{7}{8}}{\frac{3}{8} \text{ of } \frac{8}{14}}$ | <i>Ans.</i> $1\frac{1}{2}$. |
| 5. $\frac{84}{\frac{7}{12}}$ | <i>Ans.</i> 144. | 12. $\frac{\frac{3}{8} \text{ of } 2\frac{1}{2}}{1\frac{1}{2}}$ | <i>Ans.</i> $\frac{7}{100}$. |
| 6. $\frac{7\frac{1}{2}}{\frac{4}{8}}$ | <i>Ans.</i> $1\frac{1}{2}$. | 13. $\frac{\frac{7}{8} + \frac{7}{8}}{\frac{7}{8} + \frac{1}{8}}$ | <i>Ans.</i> $1\frac{2}{3}$. |
| 7. $\frac{21\frac{1}{2}}{14}$ | <i>Ans.</i> $1\frac{5}{8}$. | 14. $\frac{\frac{8}{8} - \frac{8}{10}}{\frac{7}{8} - \frac{3}{8}}$ | <i>Ans.</i> $7\frac{7}{10}$. |
| 8. $\frac{20}{6\frac{1}{8}}$ | <i>Ans.</i> $3\frac{1}{8}$. | 15. $\frac{\frac{1}{8} \times \frac{10}{8}}{\frac{1}{8}}$ | <i>Ans.</i> $13\frac{1}{2}$. |

SECTION VII.

RELATION OF NUMBERS.

CASE I.

The Relation of Whole Numbers to Whole Numbers.

ORAL EXERCISE.

1. Twelve is how many times 4?
 2. What is the relation of 10 to 5? *Ans.* 10 is 2 times 5.
- What is the relation—
3. Of 16 to 4? Of 25 to 5? Of 36 to 4?
 4. Of 36 to 6? Of 36 to 9? Of 40 to 8?
 5. Of 6 to 12? Of 8 to 40? Of 7 to 49?
 6. If 4 pens cost 6 cents, how much will 20 pens cost?
- SOLUTION.—If 4 pens cost 6 cents, 20 pens, or 5 times 4 pens, will cost 5 times 6 cents, or 30 cents. Hence, if 4 pens cost 6 cents, 20 pens will cost 30 cents.
7. What will 15 slates cost if 5 slates cost 80 cents?
 8. What will 48 trees cost if 4 trees cost 75 cents?
 9. If 9 chickens cost \$2.25, how much will 18 chickens cost?
 10. If 27 oranges cost 90 cents, how much will 9 oranges cost?
 11. If \$24 will buy 15 turkeys, how many turkeys will \$8 buy?
 12. If 18 chickens cost \$4.20, how much will 3 chickens cost?
 13. A man can travel 33 miles in 9 hours: how far can he travel in 3 hours?
 14. If 12 pairs of boots cost \$40, how much will 3 pairs cost?
 15. If 12 men can earn \$36 in a day, how much can 4 men earn in the same time?
 16. If 5 men can eat 2 loaves of bread, how many loaves can 15 men eat in the same time?

WRITTEN PROBLEMS.

1. If 12 pens cost 14 cents, how much will 60 pens cost?

SOLUTION.

NOTE.—Let the pupil give explanations similar to solution of Problem No. 6, on preceding page.

$$60 = 5 \times 12$$

$$14 \times 5 = 70$$

2. If 13 horses cost \$1250, how much will 39 horses cost?

Ans. \$3750.

3. A man bought 450 sheep for \$1350: how much are 15 sheep worth?

Ans. \$45.

4. A farmer has 40 pigs; he sells to a butcher $\frac{2}{5}$ of his number: what relation is the number remaining to the number sold?

Ans. $\frac{3}{5}$.

5. A boy has \$100; his sister has $\frac{2}{3}$ as much, lacking \$10: what part of the boy's money equals his sister's?

Ans. $\frac{1}{3}$.

CASE II.

The Relation of Fractions to Whole Numbers.

ORAL EXERCISE.

1. What part of 6 is $\frac{2}{3}$?

SOLUTION 1.—1 is $\frac{1}{6}$ of 6; $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{1}{6}$ of 6, or $\frac{1}{12}$ of 6, and $\frac{2}{3}$ is 3 times $\frac{1}{12}$ of 6, or $\frac{1}{4}$ of 6. Hence, $\frac{2}{3}$ is $\frac{1}{4}$ of 6.

SOLUTION 2.—1 is $\frac{1}{6}$ of 6, and $\frac{2}{3}$ is $\frac{2}{3}$ of $\frac{1}{6}$ of 6, or $\frac{1}{9}$ of 6. Hence, etc.

2. What part of 10 is $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$?

3. What part of 8 is $\frac{5}{8}$? $\frac{3}{8}$? $\frac{6}{8}$?

4. What part of 5 is $\frac{2}{5}$ of $\frac{4}{5}$? $\frac{3}{5}$ of $\frac{4}{5}$?

5. What part of 8 is $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{3}{4}$ of $\frac{5}{8}$?

6. What part of 10 is $7\frac{1}{2}$? $3\frac{1}{2}$? $2\frac{1}{2}$?

WRITTEN PROBLEMS.

1. What is the relation of $\frac{5}{8}$ to 40?

Ans. $\frac{1}{8}$.

2. What is the relation of $\frac{2}{3}$ of $\frac{3}{4}$ to 80?

Ans. $\frac{1}{120}$.

3. What is the relation of $\frac{2}{3}$ of $\frac{4}{5}$ to 50?

Ans. $\frac{1}{75}$.

4. What is the relation of $7\frac{1}{2}$ to 25?

Ans. $\frac{3}{5}$.

5. What is the relation of $8\frac{1}{4}$ to 30?

Ans. $\frac{1}{15}$.

6. What is the relation of $\frac{2}{3}$ of $5\frac{1}{3}$ to 16?

Ans. $\frac{2}{3}$.

7. What is the relation of $\frac{1}{2}$ of $\frac{2}{3}$ of 5 to 20?

Ans. $\frac{3}{8}$.

CASE III.

The Relation of Whole Numbers to Fractions.

ORAL EXERCISE.

1. What is the relation of 8 to $\frac{2}{3}$?

SOLUTION.—8, which is $\frac{16}{2}$, is 10 times $\frac{2}{3}$.

What is the relation—

- | | | |
|-----------------------------|-----------------------------|-------------------------------|
| 2. Of 9 to $\frac{3}{8}$? | 5. Of 9 to $\frac{9}{10}$? | 8. Of 12 to $1\frac{1}{2}$? |
| 3. Of 12 to $\frac{4}{5}$? | 6. Of 20 to $\frac{5}{6}$? | 9. Of 8 to $2\frac{2}{3}$? |
| 4. Of 6 to $\frac{2}{3}$? | 7. Of 15 to $\frac{5}{6}$? | 10. Of 10 to $3\frac{1}{2}$? |

WRITTEN PROBLEMS.

1. What is the relation of 175 to $\frac{7}{8}$?

SOLUTION.

$$175 \div \frac{7}{8} =$$

NOTE.—The relation of one number to another is found by dividing the first by the second.

What is the relation—

- | | | | |
|-------------------------------|----------|---|------------------------|
| 2. Of 40 to $6\frac{2}{3}$? | Ans. 6. | 6. Of 75 to $6\frac{3}{4}$? | Ans. $11\frac{1}{2}$. |
| 3. Of 425 to $6\frac{1}{2}$? | Ans. 68. | 7. Of 180 to $12\frac{1}{2}$? | Ans. $14\frac{2}{5}$. |
| 4. Of 42 to $2\frac{1}{2}$? | Ans. 15. | 8. Of 84 to $\frac{1}{2}$ of $3\frac{3}{4}$? | A. $44\frac{2}{3}$. |
| 5. Of 128 to $6\frac{2}{3}$? | Ans. 20. | 9. Of 75 to $\frac{3}{4}$ of $\frac{5}{6}$? | A. 120. |

CASE IV.

The Relation of Fractions to Fractions.

ORAL EXERCISE.

1. What part of $\frac{2}{3}$ is $\frac{2}{3}$?

SOLUTION.— $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$; and $\frac{2}{3}$, or 1, is $4 \times \frac{1}{2}$ of $\frac{2}{3}$, or $\frac{4}{2}$ of $\frac{2}{3}$; if 1 is $\frac{4}{2}$ of $\frac{2}{3}$, $\frac{2}{3}$ is $\frac{2}{4}$ of $\frac{2}{3}$, or $\frac{1}{2}$ of $\frac{2}{3}$. Hence, $\frac{2}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$.

What part—

- | | | |
|--|---|---|
| 2. Of $\frac{5}{6}$ is $\frac{5}{6}$? | 5. Of $\frac{5}{6}$ is $1\frac{1}{2}$? | 8. Of $\frac{7}{8}$ is $\frac{3}{8}$? |
| 3. Of $\frac{7}{8}$ is $\frac{3}{8}$? | 6. Of $\frac{4}{5}$ is $\frac{2}{5}$? | 9. Of $6\frac{1}{2}$ is $\frac{2}{3}$? |
| 4. Of $\frac{2}{3}$ is $\frac{3}{4}$? | 7. Of $\frac{5}{6}$ is $\frac{2}{3}$? | 10. Of $8\frac{1}{2}$ is $2\frac{1}{2}$? |

WRITTEN PROBLEMS.

1. What is the relation of $\frac{4}{5}$ to $\frac{7}{8}$?

SOLUTION.

$$\frac{4}{5} \div \frac{7}{8} =$$

NOTE.—The relation of one fraction to another is found by dividing the first fraction by the second.

$$\frac{4}{5} \times \frac{8}{7} = \frac{32}{35}$$

What is the relation—

- | | | | |
|--|------------------------------|--|------------------------------|
| 2. Of $\frac{1}{4}$ to $\frac{1}{8}$? | <i>Ans.</i> $2\frac{3}{4}$. | 4. Of $\frac{1}{4}$ to $\frac{1}{8}$? | <i>Ans.</i> $2\frac{1}{2}$. |
| 3. Of $\frac{1}{2}$ to $\frac{1}{8}$? | <i>Ans.</i> $5\frac{1}{2}$. | 5. Of $6\frac{1}{2}$ to $7\frac{1}{2}$? | <i>Ans.</i> $\frac{1}{2}$. |
| 6. Of $\frac{2}{3}$ of $\frac{4}{5}$ to $\frac{2}{5}$ of $\frac{1}{2}$? | | | <i>Ans.</i> $\frac{4}{15}$. |
| 7. Of $\frac{2}{3}$ of $2\frac{1}{2}$ to $\frac{2}{5}$ of $2\frac{3}{4}$? | | | <i>Ans.</i> $\frac{4}{3}$. |

SECTION VIII.

ABBREVIATED PROCESSES.

CASE I.

To Multiply a Number by 25.

Multiplying a number by 25 is equivalent to multiplying it by 100 and dividing by 4; hence the

RULE.

To multiply by 25, annex two ciphers, and divide by 4.

1. Multiply 391 by 25.

PROCESS.

$$\begin{array}{r} 4)39100 \\ \underline{9775} \end{array}$$

- | | |
|-------------------------|---------------------|
| 2. Multiply 721 by 25. | <i>Ans.</i> 18025. |
| 3. Multiply 7842 by 25. | <i>Ans.</i> 196050. |
| 4. Multiply 862 by 25. | <i>Ans.</i> 21550. |

CASE II.

To Multiply a Number by $12\frac{1}{2}$.

Since $12\frac{1}{2}$ is $\frac{100}{8}$, or $\frac{1}{8}$ of 100, we have the following

RULE.

To multiply by $12\frac{1}{2}$, annex two ciphers, and divide by 8.

1. Multiply 822 by $12\frac{1}{2}$.

PROCESS.

$$\begin{array}{r} 8)82200 \\ \underline{10275} \end{array}$$

- | | |
|--------------------------------------|--------------------|
| 2. Multiply 984 by $12\frac{1}{2}$. | <i>Ans.</i> 12300. |
|--------------------------------------|--------------------|

3. Multiply 8462 by $12\frac{1}{2}$. *Ans.* 105775.

4. Multiply 10461 by $12\frac{1}{2}$. *Ans.* 130762 $\frac{1}{2}$.

NOTE.—In multiplying any number by 125, annex three ciphers and divide by 8.

5. Multiply 864 by 125. *Ans.* 108000.

6. Multiply 75 by 125. *Ans.* 9375.

CASE III.

To Multiply any Number by $33\frac{1}{3}$.

Since $33\frac{1}{3}$ is $\frac{100}{3}$, we have the following

RULE.

To multiply by $33\frac{1}{3}$, annex two ciphers and divide by 3.

1. Multiply 792 by $33\frac{1}{3}$. *Ans.* 26400.

2. Multiply 6843 by $33\frac{1}{3}$. *Ans.* 228100.

3. Multiply 7844 by $33\frac{1}{3}$. *Ans.* 261466 $\frac{2}{3}$.

4. Multiply 8941 by $33\frac{1}{3}$. *Ans.* 298033 $\frac{1}{3}$.

CASE IV.

To Multiply any Number by $16\frac{2}{3}$.

Since $16\frac{2}{3}$ equals $\frac{50}{3}$ or $\frac{100}{6}$, we have the following

RULE.

To multiply by $16\frac{2}{3}$, annex two ciphers and divide by 6.

1. Multiply 864 by $16\frac{2}{3}$. *Ans.* 14400.

2. Multiply 9764 by $16\frac{2}{3}$. *Ans.* 162733 $\frac{1}{3}$.

3. Multiply 8748 by $16\frac{2}{3}$. *Ans.* 145800.

4. Multiply 7846 by $16\frac{2}{3}$. *Ans.* 130766 $\frac{2}{3}$.

CASE V.

To Divide any Number by 25.

Since $\frac{1}{25}$ is $\frac{4}{100}$, we have the following

RULE.

To divide any number by 25, multiply by 4 and divide by 100.

1. Divide 625 by 25.

PROCESS.

$$\frac{625 \times 4}{100} = \frac{2500}{100} = 25.$$

- | | |
|------------------------|-------------------|
| 2. Divide 93775 by 25. | <i>Ans.</i> 3751. |
| 3. Divide 87550 by 25. | <i>Ans.</i> 3502. |
| 4. Divide 46825 by 25. | <i>Ans.</i> 1873. |

CASE VI.

To Divide any Number by $12\frac{1}{2}$.

Since dividing by $12\frac{1}{2}$ is equivalent to multiplying by $\frac{2}{25}$ or $\frac{8}{100}$, we have the following

RULE.

To divide any number by $12\frac{1}{2}$, multiply by 8 and divide by 100.

- | | |
|-------------------------------------|------------------|
| 1. Divide 1250 by $12\frac{1}{2}$. | <i>Ans.</i> 100. |
|-------------------------------------|------------------|

PROCESS.

$$\frac{1250 \times 8}{100} = \frac{10000}{100} = 100.$$

- | | |
|--------------------------------------|------------------|
| 2. Divide 9225 by $12\frac{1}{2}$. | <i>Ans.</i> 738. |
| 3. Divide 8375 by $12\frac{1}{2}$. | <i>Ans.</i> 670. |
| 4. Divide 10625 by $12\frac{1}{2}$. | <i>Ans.</i> 850. |

CASE VII.

To Divide any Number by $33\frac{1}{3}$.

Since $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, we have the following

RULE.

To divide any number by $33\frac{1}{3}$, multiply by 3 and divide by 100.

- | | |
|-------------------------------------|--------------------------------|
| 1. Divide 6700 by $33\frac{1}{3}$. | <i>Ans.</i> 201. |
| 2. Divide 850 by $33\frac{1}{3}$. | <i>Ans.</i> 25 $\frac{1}{2}$. |
| 3. Divide 900 by $33\frac{1}{3}$. | <i>Ans.</i> 27. |

CASE VIII.

To Divide any Number by $16\frac{2}{3}$.

Since $16\frac{2}{3}$ is $\frac{1}{6}$ of 100, we have the following

RULE.

To divide any number by $16\frac{2}{3}$, multiply by 6 and divide by 100.

- | | |
|--------------------------------------|-------------------|
| 1. Divide 150 by $16\frac{2}{3}$. | <i>Ans.</i> 9. |
| 2. Divide 1050 by $16\frac{2}{3}$. | <i>Ans.</i> 63. |
| 3. Divide 48650 by $16\frac{2}{3}$. | <i>Ans.</i> 2919. |

NOTE.—Corresponding abbreviations may be made in the case of $6\frac{1}{2}$, $8\frac{1}{2}$, 50, and other aliquot parts of 100.

SECTION IX.

GREATEST COMMON DIVISOR OF FRACTIONS.

82. The **Greatest Common Divisor** of two or more fractions is the greatest fraction that will exactly divide each of them.

PRINCIPLE.—A fraction is an exact divisor of a given fraction when its numerator is a divisor of the given numerator, and its denominator a multiple of the given denominator.

Hence, a common divisor of two or more fractions is one whose numerator is a common divisor of their numerators, and its denominator a common multiple of their denominators.

Similarly, the greatest common divisor of two or more fractions is a fraction whose numerator is the greatest common divisor of their numerators, and its denominator the least common multiple of their denominators.

NOTE.—Reduce whole and mixed numbers to improper fractions, and all fractions to their lowest terms before finding the greatest common divisor.

WRITTEN PROBLEMS.

1. Find the greatest common divisor of $\frac{3}{4}$, $\frac{1^2}{6}$, $\frac{9}{10}$. *Ans.* $\frac{3}{40}$.

PROCESS.

G. C. D. of 3, 12, 9 = 3.

L. C. M. of 4, 5, 10 = 20.

Hence, G. C. D. of $\frac{3}{4}$, $\frac{1^2}{6}$, $\frac{9}{10}$ = $\frac{3}{40}$.

Find the greatest common divisor—

- | | | | |
|---|------------------------------|---|-------------------------------|
| 2. Of $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$. | <i>Ans.</i> $\frac{1}{60}$. | 6. Of $\frac{7}{12}$, $\frac{9}{16}$, $\frac{3}{20}$, $\frac{5}{24}$. | <i>Ans.</i> $\frac{1}{240}$. |
| 3. Of $\frac{9}{10}$, $\frac{1^2}{5}$, $\frac{3}{20}$. | <i>Ans.</i> $\frac{1}{60}$. | 7. Of $6\frac{1}{2}$, $3\frac{1}{4}$, $2\frac{3}{8}$. | <i>Ans.</i> $\frac{1}{80}$. |
| 4. Of 5, $\frac{3}{5}$, $\frac{2}{3}$. | <i>Ans.</i> $\frac{1}{20}$. | 8. Of $58\frac{2}{3}$, $22\frac{1}{10}$, $12\frac{1}{15}$. | <i>Ans.</i> $\frac{1}{30}$. |
| 5. Of 10, $2\frac{1}{2}$, $\frac{3}{4}$. | <i>Ans.</i> $\frac{1}{4}$. | 9. Of $12\frac{1}{2}$, $3\frac{1}{3}$, $17\frac{1}{4}$. | <i>Ans.</i> $\frac{5}{42}$. |

SECTION X.

LEAST COMMON MULTIPLE OF FRACTIONS.

83. The **Least Common Multiple** of two or more fractions is the least number that can be exactly divided by each of them.

PRINCIPLE.—A fraction is a multiple of a given fraction when its numerator is a multiple of the given numerators, and its denominator a divisor of the given denominators.

Hence, a common multiple of two or more fractions is one whose numerator is a multiple of the given numerators, and its denominator a divisor of the given denominators.

Similarly, the least common multiple of two or more fractions is that whose numerator is the least common multiple of the given numerators, and its denominator the greatest common divisor of its denominators.

WRITTEN PROBLEMS.

1. What is the least common multiple of $\frac{3}{4}$, $\frac{5}{12}$, $\frac{15}{24}$? *Ans.* $3\frac{3}{4}$.

PROCESS.

L. C. M. of 3, 5, 15 = 15

G. C. D. of 4, 12, 24 = 4

Hence, L. C. M. of $\frac{3}{4}$, $\frac{5}{12}$, $\frac{15}{24}$ = $\frac{15}{4}$, or $3\frac{3}{4}$.

What is the least common multiple—

2. Of $\frac{2}{3}$, $\frac{7}{12}$, $\frac{5}{4}$? *Ans.* 168. 5. Of $\frac{5}{6}$, $\frac{10}{18}$, $\frac{11}{9}$? *Ans.* 20.
 3. Of $\frac{3}{4}$, $\frac{9}{12}$, $\frac{2}{14}$? *Ans.* $2\frac{1}{2}$. 6. Of $3\frac{1}{2}$, $3\frac{1}{3}$, $7\frac{1}{8}$? *Ans.* 1430.
 4. Of $\frac{4}{5}$, $\frac{3}{2}$, $\frac{2}{3}$? *Ans.* 20. 7. Of $6\frac{2}{3}$, $3\frac{2}{3}$, $2\frac{1}{3}$? *Ans.* $19\frac{2}{3}$.
 8. What is the least sum of money for which I can buy hats at $\$3\frac{1}{2}$, boots at $\$5\frac{1}{4}$, or shoes at $\$3\frac{3}{8}$? *Ans.* $\$94\frac{1}{4}$.
 9. What is the least number that will exactly contain $3\frac{1}{2}$, $5\frac{1}{3}$, $2\frac{1}{4}$, $3\frac{1}{6}$? *Ans.* 30225.

10. A can walk around a circle in $7\frac{1}{2}$ minutes, B in $6\frac{1}{4}$ minutes, C in $6\frac{3}{4}$ minutes and D in $7\frac{1}{8}$ minutes: if all start together, how long before they all come to the starting-point together, and how many times will each have passed around?

Ans. 900 minutes. A, 120 times; B, 144 times; C, 135 times; D, 125 times.

SECTION XI.

REVIEW.

ORAL EXERCISE.

1. A boy having $\$ \frac{3}{4}$, earned $\$ \frac{2}{3}$ and $\$ \frac{7}{10}$: what part of a dollar had he then?
2. John having $27 \frac{1}{2}$ acres of land, bought $16 \frac{1}{2}$ acres: how much land did he have then?
3. A merchant owning $\frac{1}{2}$ of a ship, sold $\frac{3}{8}$ of it: how much does he still own?
4. A gentleman buying a farm, paid $\frac{1}{4}$ cash, $\frac{1}{8}$ the second year and $\frac{1}{4}$ the next year: how much remains unpaid?
5. A man bought a pair of boots for $\$6 \frac{1}{2}$: how much change did he get from a ten-dollar bill?
6. A gentleman owning $\frac{3}{4}$ of a farm, sold $\frac{1}{4}$ of his share: how much did he sell?
7. Mr. Smith has 60 sheep; $\frac{5}{12}$ of his number is 10 less than Mr. Thompson's number: how many has Mr. Thompson?
8. A drover sold a horse for $\$180$, which was $\frac{3}{4}$ of its cost: what did the horse cost?
9. The difference between $\frac{3}{4}$ and $\frac{3}{8}$ of my age is 2 years: how old am I?
10. A boy after spending $\frac{3}{8}$ of $\frac{4}{7}$ of his money, has 66 cents left: how much had he at first?
11. If a boy can earn $\frac{1}{8}$ of a dollar in 3 days, how much can he earn in 1 day?
12. A wife is 40 years old, and her age is $\frac{3}{4}$ of her husband's age: how old is he?
13. $\frac{3}{4}$ of 80 is $\frac{3}{4}$ of what number?
14. A horse cost $\$120$, and $\frac{3}{4}$ of his cost is twice the cost of a cow: what is the cost of the cow?
15. A man bought a horse and a sleigh for $\$180$; the sleigh cost $\frac{2}{3}$ as much as the horse: how much did each cost?
16. If to the height of a tree you add $\frac{2}{3}$ and $\frac{1}{4}$ of the height, the sum will equal 165 feet: how high is the tree?

17. The distance from Albany to New York is 144 miles; $\frac{2}{3}$ of this distance is $1\frac{1}{2}$ of the distance from New York to Philadelphia: how far is it from New York to Philadelphia?

18. The distance from Harrisburg to Allentown is 90 miles; $\frac{3}{4}$ of this distance is $\frac{2}{3}$ of the distance from Easton to Mahanoy City: what is the distance from Easton to Mahanoy City?

19. The distance from Baltimore to Harrisburg is 85 miles; $1\frac{1}{2}$ of this distance is $1\frac{1}{2}$ of the distance from Sunbury to Lock Haven: what is the distance from Sunbury to Lock Haven?

20. The distance from Philadelphia to South Wilkesbarre is 100 miles, and $\frac{1}{10}$ of this distance is $\frac{1}{4}$ of the distance from Scranton to Northumberland: how far is it from Scranton to Northumberland?

21. The distance from Cleveland to Pittsburg is 150 miles; $\frac{2}{3}$ of this distance is 1 mile more than $\frac{3}{4}$ of the distance from Cleveland to Galion: how far is it from Cleveland to Galion?

22. $\frac{3}{4}$ is $\frac{2}{3}$ of what number?

23. $\frac{2}{3}$ of A's money equals $\frac{1}{4}$ of B's, and together they have \$380: how much has each?

24. $\frac{5}{8}$ of B's money equals $\frac{2}{10}$ of C's, and together they have \$5200: how much has each?

25. $\frac{2}{3}$ of C's age is $\frac{1}{4}$ of D's, and $\frac{3}{4}$ of D's age is 30 years: how old is each?

26. How many pounds of candy at $\frac{3}{4}$ of a dollar a pound can you buy for \$4?

27. How many pens at $3\frac{1}{2}$ cents a dozen can you buy for 70 cents?

28. If there are $2\frac{1}{2}$ bushels of potatoes in a barrel, how many barrels will contain 33 bushels?

29. A man bought geese at $1\frac{1}{2}$ dollars a pair: how many pairs could he get for \$28 at the same rate?

30. If shoes are worth $2\frac{1}{2}$ dollars a pair, and I exchange 14 bushels of clover-seed at $5\frac{1}{2}$ dollars a bushel, how many pairs of shoes can I get?

WRITTEN PROBLEMS.

1. Reduce $17\frac{4}{5}$ to an improper fraction. *Ans.* $2\frac{18}{5}$.
2. Reduce $63\frac{3}{8}$ to an improper fraction. *Ans.* $57\frac{3}{2}$.
3. Reduce $1\frac{3}{4}$ to a mixed number. *Ans.* $54\frac{3}{4}$.
4. Reduce $1\frac{1}{2}$ to a mixed number. *Ans.* $74\frac{3}{4}$.
5. Reduce $1\frac{3}{4}$ to its lowest terms. *Ans.* $\frac{7}{4}$.
6. Reduce $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{4}{5}$ to a simple fraction. *Ans.* $\frac{1}{5}$.
7. Reduce $\frac{3}{2}$ of $1\frac{1}{2}$ of $9\frac{1}{2}$ to a simple fraction. *Ans.* $4\frac{3}{4}$.
8. Reduce $\frac{7}{8}$, $\frac{2}{10}$, $\frac{5}{8}$, $\frac{1}{2}$ to their least common denominator.
Ans. $1\frac{1}{2}$, $2\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$.
9. Reduce $\frac{7}{10}$, $1\frac{1}{2}$, $1\frac{1}{8}$ to their least common denominator.
Ans. $4\frac{3}{8}$, $8\frac{5}{8}$, $8\frac{5}{8}$.
10. Add $\frac{2}{4}$, $1\frac{1}{8}$, $4\frac{5}{8}$, $15\frac{3}{8}$. *Ans.* $21\frac{3}{8}$.
11. From $44\frac{3}{8} + 9\frac{1}{2}$ take $33\frac{1}{8}$. *Ans.* $20\frac{7}{8}$.
12. Multiply $\frac{1}{4}$ of $18\frac{1}{2}$ by $\frac{4}{7}$ of $44\frac{3}{8}$. *Ans.* $142\frac{1}{8}$.
13. Divide $\frac{3}{4}$ of $3\frac{1}{2}$ by $\frac{5}{8}$ of $1\frac{5}{8}$. *Ans.* $4\frac{1}{2}$.
14. Reduce $\frac{4}{5}$ of $\frac{3}{4}$ to a simple fraction. *Ans.* $1\frac{3}{5}$.
15. Reduce $\frac{63\frac{1}{2} + \frac{3}{4} \text{ of } 7\frac{1}{2}}{\frac{5}{8} \text{ of } 16\frac{1}{2} - \frac{5}{8} \text{ of } 3\frac{1}{2}}$ to a mixed number.
Ans. $11\frac{3}{8}$.
16. If a man earn $\$53\frac{3}{4}$ a week, how much can he earn in a year of 52 weeks? *Ans.* $\$2795$.
17. A farmer bought 183 acres of land at $\$97\frac{1}{2}$ an acre, and $43\frac{1}{2}$ acres at $\$122\frac{3}{4}$ an acre: what was the cost of all his land? *Ans.* $\$23182\frac{1}{2}$.
18. A man owning $\frac{3}{4}$ of a foundry, sold $\frac{1}{4}$ of his share for $\$26320$: what was the value of the foundry? *Ans.* $\$49350$.
19. If $7\frac{1}{2}$ pounds of rice cost 90 cents, how many pounds can be bought for $\$1.10$? *Ans.* $9\frac{1}{2}$.
20. A farmer sold $197\frac{1}{2}$ bushels of wheat at $\$1.37\frac{1}{2}$ a bushel, and took in exchange flour at $\$4.93\frac{3}{4}$ per barrel: how many barrels of flour did he get? *Ans.* 55.
21. A has $\$325\frac{1}{2}$, B has $\frac{3}{4}$ as much + $\$18\frac{1}{4}$: how much has B? *Ans.* $\$235\frac{1}{4}$.
22. A horse cost $\$125$, a cow $\frac{3}{4}$ as much, less $\$21\frac{1}{2}$, a sheep

\$61 $\frac{1}{2}$ less than the cow: what was the cost of each and of all? *Ans.* Cow, \$72 $\frac{1}{2}$; sheep, \$11 $\frac{1}{2}$; all, \$208 $\frac{1}{2}$.

23. A merchant had 150 yards of carpet; he sold $\frac{1}{4}$ to A, $\frac{1}{3}$ to B, and $\frac{1}{6}$ to C: how many yards did he sell to each, and how many yards has he remaining?

Ans. A, 37 $\frac{1}{2}$ yds.; B, 50 yds.; C, 30 yds.; remainder 32 $\frac{1}{2}$ yds.

24. $\frac{2}{3}$ of my money, increased by the difference between $\frac{2}{3}$ and $\frac{1}{4}$ of my money, equals \$4551: how much have I?

Ans. \$5535.

25. A owned $\frac{2}{3}$ of a ship; he sold $\frac{1}{3}$ of his share to B, and B sold $\frac{1}{3}$ of his to C: how much does each own?

Ans. A, $\frac{1}{3}$; B, $\frac{1}{6}$; C, $\frac{1}{6}$.

26. I owned a house worth \$5000; I had it insured for $\frac{3}{4}$ of its value, but when burned the company offered to pay me $\frac{1}{4}$ of my insurance: how much did I get? *Ans.* \$3281 $\frac{1}{2}$.

27. A merchant has 137 $\frac{1}{2}$ yards of cloth, from which he wishes to cut coats, pants or vests: how many can he cut of each if the coats contain 3 $\frac{1}{2}$ yards each, the pants 2 $\frac{1}{2}$ yards each, and the vests 1 $\frac{1}{2}$ yards each?

Ans. 44 coats, 50 pairs of pants, or 110 vests.

CHAPTER V.

COMPARISON OF NUMBERS.

CASE I.

Comparison of Integers with Integers.

ORAL EXERCISE.

1. If 6 caps cost \$4 $\frac{1}{2}$, how much will 10 caps cost?

ANALYSIS.—If 6 caps cost \$4 $\frac{1}{2}$, or 2 $\frac{1}{2}$ dollars, 1 cap will cost $\frac{1}{6}$ of \$2 $\frac{1}{2}$, or $\frac{1}{2}$ of a dollar, and 10 caps will cost 10 times $\frac{1}{2}$ dollars, or 4 $\frac{1}{2}$ dollars, equal to 8 dollars.

2. If 3 yards of cloth cost \$13 $\frac{1}{2}$, how much will 7 yards cost?

3. A man earned $14\frac{3}{4}$ dollars in 4 weeks: how much could he earn at the same rate in 9 weeks?

4. If 12 acres of land produce $14\frac{3}{4}$ tons of hay, how much will 15 acres produce?

5. A pole 8 feet long casts a shadow of $26\frac{3}{4}$ feet: how long a shadow will a pole 15 feet long cast?

6. Four yards of carpet cost $3\frac{1}{2}$ dollars: if a man buy 20 yards and give a twenty-dollar bill, how much change will he get?

7. If 6 apples cost $4\frac{1}{2}$ cents, how much will 20 apples cost?

WRITTEN PROBLEMS.

1. What will 27 horses cost if 15 horses cost \$1200?

Ans. \$2160.

2. If a man earn \$14 $\frac{7}{8}$ in 7 days, how much will he earn in 24 days?

Ans. \$51.

3. What cost 117 barrels of flour if 11 barrels cost \$71.50?

Ans. \$760.50.

4. A drover exchanged 23 horses for a house: what did the house cost him if 4 horses are worth 500 dols. *Ans.* \$2875.

5. A merchant gave 12 pairs of boots and 27 yards of carpet for a horse: what did the horse cost him if 2 pairs of boots are worth 11 dollars, and 5 yards of carpet are worth 6 dollars?

Ans. \$98.40.

6. A miller paid \$105 for wheat: how many bushels did he get if \$16 buy $11\frac{1}{8}$ bushels?

Ans. $76\frac{1}{4}$.

CASE II.

Comparison of Fractions with Integers.

ORAL EXERCISE.

1. If $\frac{3}{4}$ of a yard of cloth cost 60 cents, what will 4 yards cost?

ANALYSIS.—If $\frac{3}{4}$ of a yard cost 60 cents, $\frac{1}{4}$ of a yard will cost $\frac{1}{3}$ of 60 cents, which is 20 cents; $\frac{3}{4}$, or 1 yard, will cost 4 times 20 cents, or 80 cents, and 4 yards will cost 4 times 80 cents, or 3 dollars and 20 cents.

2. If $\frac{3}{4}$ of an apple cost $\frac{1}{8}$ of a cent, what will 5 apples cost?

3. 24 is $\frac{3}{8}$ of some number: what is 4 times the number?
4. $\frac{4}{7}$ of the cost of a cow is 30 dollars: what will 6 cows cost at the same rate?
5. $\frac{2}{3}$ of the cost of a farm is \$4500: what is the value of the farm?
6. If $6\frac{1}{2}$ pounds of coffee are worth \$2.08, what are 8 pounds worth?
7. Since I became 21 years of age I have lived $\frac{2}{3}$ of my life: how old am I?

WRITTEN PROBLEMS.

1. If $27\frac{1}{2}$ acres of land cost \$3300, what will 120 acres cost?
Ans. \$14400.
2. A farmer bought 200 acres of land, paying \$1645 for $11\frac{1}{4}$ acres: what did it all cost?
Ans. \$28000.
3. A merchant bought 75 yards of cloth at the rate of 10 dollars for $7\frac{1}{2}$ yards: what did it all cost him?
Ans. \$100.
4. Bought 46 tons of coal at the rate of $6\frac{1}{2}$ tons for 35 dollars: what did it all cost?
Ans. \$257 $\frac{1}{2}$.
5. If $\frac{1}{12}$ of a ton of plaster cost \$10.50, what will 25 tons cost?
Ans. \$450.
6. When $\frac{7}{8}$ of a pound of tea cost 64 cents, what will 28 pounds cost?
Ans. \$20.48.

CASE III.

Comparison of Integers with Fractions.

ORAL EXERCISE.

1. If 8 pounds of rice cost 96 cents, how much will $\frac{3}{4}$ of a pound cost?

ANALYSIS.—If 8 pounds cost 96 cents, 1 pound will cost $\frac{1}{8}$ of 96 cents, which is 12 cents, and $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of 12 cents, which is 9 cents.

2. If 1 barrel of flour is worth 8 dollars, how much is $\frac{3}{4}$ of a barrel worth?
3. If 5 yards of muslin are worth 80 cents, how much is $\frac{5}{8}$ of a yard worth?
4. If 9 acres of land are worth \$1800, what is a lot containing $\frac{2}{3}$ of an acre worth at the same rate?

5. A grocer bought 8 barrels of flour for \$72: what did $\frac{1}{4}$ of a barrel cost?

6. If 12 dozen of buttons are worth \$1.08, what is the value of $\frac{3}{4}$ of a dozen?

7. If 7 bushels of potatoes cost $5\frac{3}{4}$ dollars, what will $\frac{7}{8}$ of a bushel cost?

WRITTEN PROBLEMS.

1. Bought 125 acres of land for \$15625; I wish to sell $37\frac{1}{2}$ acres at the price it cost me: how much should I get?

Ans. \$4687.50.

2. A roll of carpet containing 160 yards cost me \$128: if I sell $48\frac{3}{4}$ yards at cost, what do I get? *Ans.* \$39.

3. If a piece of calico containing 43 yards cost me \$3.54 $\frac{3}{4}$, what do $12\frac{1}{2}$ yards cost? *Ans.* \$1.03 $\frac{1}{4}$.

4. 27 dozen of buttons cost \$3.64 $\frac{1}{2}$: if I gain $3\frac{1}{2}$ cents a dozen, how much do I get for 7 dozen? *Ans.* \$1.19.

5. Bought 40 acres of land for \$4500; sold $32\frac{1}{2}$ acres at $12\frac{1}{2}$ dollars an acre more than I paid for it: how much did I get? *Ans.* \$4062 $\frac{1}{2}$.

6. Bought 12 barrels of sugar containing 220 pounds each for \$283.80; sold $3\frac{3}{4}$ barrels at 12 cents a pound, and the remainder at $11\frac{1}{2}$ cents a pound: what did I make?

Ans. \$23.92 $\frac{1}{2}$.

CASE IV.

Comparison of Fractions with Fractions.

ORAL EXERCISE.

1. If $\frac{2}{3}$ of a yard of muslin cost 8 cents, what will $\frac{3}{4}$ of a yard cost?

ANALYSIS.—If $\frac{2}{3}$ of a yard cost 8 cents, $\frac{1}{3}$ of a yard will cost $\frac{1}{2}$ of 8 cents, which is 4 cents, and $\frac{3}{4}$ or 1 yard will cost 3 times 4 cents, or 12 cents. If a yard of muslin cost 12 cents, $\frac{3}{4}$ of a yard will cost $\frac{3}{4}$ of 12 cents, which is 9 cents.

2. If $\frac{3}{7}$ of a yard of delaine cost 30 cents, how much will $\frac{3}{8}$ of a yard cost?

3. If $\frac{3}{4}$ of a ton of coal cost \$4.50, how much will 5 tons cost?

4. If $\frac{2}{10}$ of a man's age is 27 years, what is $\frac{3}{4}$ of his age?
5. When $\frac{2}{3}$ of a barrel of flour is worth \$4.50, what are 3 $\frac{1}{2}$ barrels worth?
6. Five-sevenths of an acre of land is worth \$60: how much is $\frac{3}{4}$ of an acre worth?
7. A man bought 3 $\frac{1}{2}$ bushels of wheat at the rate of 90 cents for $\frac{2}{3}$ of a bushel: what did he pay for it?

WRITTEN PROBLEMS.

1. If 2 $\frac{1}{2}$ bushels of wheat cost 5 dollars, what will $\frac{1}{4}$ of a bushel cost?

SOLUTION.— $2\frac{1}{2} = \frac{5}{2}$;
 If $\frac{5}{2}$ cost \$5, $\frac{1}{2}$ cost $\frac{1}{5}$ of \$5 = \$1;
 $\frac{3}{4}$ cost $2 \times \$1 = \$2$;
 $\frac{1}{4}$ cost $\frac{1}{2}$ of \$2 = \$1.

NOTE.—Let the pupil give the explanation.

2. If 5 $\frac{2}{3}$ yards of carpet cost 7 $\frac{1}{2}$ dollars, how much will 12 $\frac{3}{4}$ yards cost? *Ans.* 17 dollars.
3. \$9 $\frac{2}{10}$ will buy 5 $\frac{1}{2}$ bushels of wheat: how much will 40 $\frac{1}{2}$ dollars buy? *Ans.* 22 $\frac{1}{2}$ bushels.
4. If 6 $\frac{1}{4}$ tons of iron cost \$95, how much will 10 $\frac{2}{10}$ tons cost? *Ans.* \$165.68.
5. A man traded 7 $\frac{2}{10}$ pounds of butter for 5 $\frac{3}{4}$ yards of delaine: how much delaine could he have bought with 17 $\frac{1}{4}$ pounds of butter? *Ans.* 13 $\frac{3}{4}$ yards.
6. A coal-merchant traded 15 $\frac{1}{4}$ tons of coal for hay at the rate of 18 $\frac{2}{10}$ tons of coal for 6 $\frac{1}{2}$ tons of hay: how many tons of hay did he get? *Ans.* 5 $\frac{1}{2}$.

CASE V.

Given a Fractional Part and the Integral Remainder, to find the Whole Number.

ORAL EXERCISE.

1. A boy lost $\frac{2}{3}$ of his money, and has 6 cents remaining: how much had he at first?

ANALYSIS.—If a boy lost $\frac{2}{3}$ of his money, he has remaining $\frac{1}{3}$ — $\frac{1}{3}$, or $\frac{2}{3}$; if $\frac{2}{3}$ of his money is 6 cents, $\frac{1}{3}$ of his money is $\frac{1}{2}$ of 6 cents, which is 3 cents, and $\frac{2}{3}$, or his money, 5 times 3 cents, which is 15 cents.

2. A man after spending $\frac{3}{4}$ of his money, found that \$25 was $\frac{5}{10}$ of what he had remaining: how much had he at first?

3. A farmer sold $\frac{1}{3}$ of his sheep to one man, $\frac{1}{3}$ to another, and had 21 remaining: how many had he at first?

4. A grocer having a barrel of sugar, sold $\frac{1}{4}$ of it to one man, $\frac{1}{3}$ to another, and $\frac{3}{8}$ to another, and then had 35 pounds remaining: how many pounds were in the barrel?

5. A boy spent $\frac{2}{3}$ of $\frac{3}{4}$ of his money, and had 60 cents remaining: how much had he at first?

6. B's money, being diminished by its $\frac{1}{2}$ and $\frac{1}{3}$, equals 33 dollars: how much has he?

7. $\frac{4}{5}$ of B's money, being increased by $\frac{1}{3}$ of $\frac{1}{2}$ of his money, equals \$99: how much money has he?

8. A man lost $\frac{3}{4}$ of his money, and found $\frac{1}{3}$ of what he lost; he then had \$36: how much had he at first?

9. John found 20 cents, then lost $\frac{4}{5}$ of what he found, and had $\frac{2}{3}$ as much as at first: how much had he at first?

10. A farmer sold $\frac{1}{3}$ of his cattle, then bought $\frac{2}{3}$ as many as he sold, and had 40: how many had he at first?

11. In an orchard $\frac{1}{3}$ of the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{5}$ cherries, and the remainder, 26, bear peaches: how many trees are there in the orchard?

12. A balloon rose in the air, then fell $\frac{1}{3}$ of the distance, then rose $\frac{1}{2}$ of the distance it was from the ground, then fell $\frac{1}{3}$ of the distance, and was 400 feet from the ground: how high was it when it was at the greatest height?

WRITTEN PROBLEMS.

1. A merchant lost $\frac{1}{3}$ of his money, then made \$620, and had \$3000; how much had he at first? *Ans.* \$3570.

SOLUTION.— $\frac{1}{3} - \frac{1}{3} = \frac{2}{3}$;

$\frac{2}{3} + \$620 = \$3000$;

$\frac{2}{3} = \$3000 - \$620 = \$2380$;

$\frac{1}{3} = \$1190$;

$\frac{1}{3} = 3 \times \$1190 = \$3570$.

2. What number diminished by its $\frac{2}{3}$ and $\frac{2}{3}$ equals 42?

Ans. 245.

3. A man's expenses for the year are $\frac{3}{4}$ of $\frac{8}{10}$ of his income, and he saves \$1375: what is his income? *Ans.* \$2500.

4. $\frac{2}{5}$ of the rental of a store-room is paid for taxes, and the owner saves \$440: what rent does he get? *Ans.* \$550.

5. A man spent $\frac{1}{4}$ of his money for a store, $\frac{1}{5}$ for a house, and the remainder, \$9020, for a farm: how much did he pay for each and for all?

Ans. Store, \$4100; house, \$3280; all, \$16400.

CHAPTER VI.

DECIMAL FRACTIONS.

SECTION I.

NOTATION AND NUMERATION.

84. A Decimal Fraction is a number of *tenths, hundredths* or *thousandths*, etc.

85. A Decimal is a decimal fraction expressed without its denominator.

When the denominators are written, as, $\frac{4}{10}$, $\frac{6}{100}$, etc., decimal fractions do not differ from common fractions. When expressed as follows: .5 (five tenths); .07 (seven hundredths); or .113 (one hundred thirteen thousandths), they are called *decimals*. Ten of any order equal one of the next higher order.

The decimal notation has the same scale as the notation of integers.

Its divisions from the units' place to the right are named in the following order: tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, etc.

86. A point called the *decimal point*, or *separatrix*, is placed to the left of the tenths. Thus, 5 tenths is expressed decimally .5, five hundredths .05, etc.

87. The following table shows the relative position of whole numbers and decimals :

8	7	6	5	4	3	2	1	.	1	2	3	4	5	6	7	8
ten-millions.	millions.	hundred thousands.	ten thousands.	thousands.	hundreds.	tens.	units.		tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
Integers.									Decimals.							

Read the decimal .14.

Read as follows: 1 tenth and 4 hundredths, or 14 hundredths.

Read the following:

.8	.07	.006	.06
.7	.03	.007	.006
.3	.04	.002	.0006.

NOTE.—When the right-hand figure denotes hundredths, thousandths, etc., the whole decimal is read as denoting the same as the right-hand figure. Thus, in .1273 the figure 3 occupies the place of ten-thousandths, and the decimal is therefore read one thousand two hundred and seventy-three ten-thousandths.

Read the following :

.16	.291	.1204	8430.06
.27	.674	.06	72.808
.84	.703	.8007	.60006
.93	.809	34.806	.834681
.12	.650	700.007	7.8203
.18	.904	6.845	16.00043

NOTE.—When a fraction is expressed decimally, the units' figure of the numerator is placed in the order indicated by the decimal. Thus, in $\frac{1118}{10000}$, the 8 is placed in the ten-thousandth's place, and the other figures of the numerator in their proper connection with it; the fraction $\frac{1118}{10000}$ is expressed decimally as follows: .0118, a cipher being put in the vacant place.

Express decimally—

$\frac{2}{10}$	$\frac{27}{100}$	$\frac{118}{1000}$	$\frac{1242}{10000}$
$\frac{5}{10}$	$\frac{18}{100}$	$\frac{804}{1000}$	$\frac{8642}{10000}$
$\frac{9}{10}$	$\frac{62}{100}$	$\frac{906}{1000}$	$\frac{8008}{10000}$

Write the following decimally :

$\frac{125}{1000}$.	3062 ten-thousandths,
246 thousandths.	8007 hundred-thousandths.
246 millionths.	80043 millionths.
81 thousandths.	6 ten-millionths.
71 hundredths.	3005 hundred-millionths.
29 thousandths.	9405 ten-millionths.
709 thousandths.	5 hundredths.
816 hundred-thousandths.	8 hundred-millionths.
816 hundred-millionths.	18003 ten-millionths.
33 thousandths.	3 hundredths.
33 ten-millionths.	3 millionths.
9045 millionths.	3 hundred-thousandths.

An integer and a decimal may be written as one number. Thus, $16\frac{8}{10}$ is expressed decimally 16.03.

Write the following decimally :

6 and 16 hundredths.	$646\frac{18}{100}$.
$18\frac{6}{100}$.	$847\frac{18}{10000}$.
$19\frac{27}{1000}$.	$907\frac{908}{100000}$.

603 units and 607 thousandths.

300000 units and 4 millionths.

7005 units and 14 thousandths.

PRINCIPLES.—1. The denominator of a decimal fraction is 1 with as many ciphers annexed as there are decimal places in the numerator.

2. Moving the decimal point to the right increases the value of the decimal tenfold for every removal of one space.

3. Moving the decimal point to the left decreases the value of the decimal to one-tenth of its former value for every removal of one space.

4. Annexing a cipher to the right of a decimal, or removing a cipher from the right of a decimal, does not alter the value, as it simply multiplies or divides both numerator and denominator by the same number.

SECTION II.

REDUCTION OF DECIMALS.

CASE I.

Reduction to Higher or Lower Terms.**ORAL EXERCISE.**

1. How many tenths in 5 units? In 12 units?
2. How many hundredths in 6 tenths? In 8 tenths? In 7 tenths?
3. How many thousandths in .03? In .13? In .6?
4. How many tenths in .20? In 300?

WRITTEN PROBLEMS.

1. Reduce .12 to thousandths. *Ans.* .120.
Process.—.12 = .120.
2. Reduce .18 to thousandths.
3. Reduce .05 to hundred-thousandths.
4. Reduce 305 to millionths.
5. Reduce 6.5 to tenths; to hundredths.
6. Reduce 12 to thousandths.
7. Reduce 18.75 to ten-thousandths.
8. Reduce .06600 to hundred-thousandths.
9. Reduce .0300 to hundredths.
10. Reduce .3000 to tenths.

CASE II.

Reduction of Decimals to Common Fractions.**ORAL EXERCISE.**

1. In .6 how many fifths?
2. How many fourths in $\frac{25}{100}$? In .50? In .75?
3. How many 20ths in $\frac{15}{100}$? In .25? In .35? In .70?

WRITTEN PROBLEMS.

1. Reduce .125 to a common fraction.

PROCESS.—.125 = $\frac{125}{1000}$ = $\frac{1}{8}$.

NOTE.—Omit the decimal point, supply the denominator and reduce to the lowest terms.

Reduce the following fractions to common fractions in their lowest terms :

2. .625.	Ans. $\frac{5}{8}$.	11. 16.25.	Ans. $16\frac{1}{4}$.
3. .025.	Ans. $\frac{1}{40}$.	12. 56.375.	Ans. $56\frac{3}{8}$.
4. .404.	Ans. $\frac{101}{250}$.	13. 56.37 $\frac{1}{2}$.	Ans. $56\frac{3}{8}$.
5. .005.	Ans. $\frac{1}{200}$.	14. 56.3 $\frac{1}{4}$.	Ans. $56\frac{3}{8}$.
6. .1250.	Ans. $\frac{1}{8}$.	15. .33 $\frac{1}{3}$.	Ans. $\frac{1}{3}$.
7. .0008.	Ans. $\frac{1}{12500}$.	16. 27.66 $\frac{2}{3}$.	Ans. $27\frac{2}{3}$.
8. .648.	Ans. $\frac{81}{125}$.	17. .56 $\frac{1}{2}$.	Ans. $\frac{9}{16}$.
9. 3.25.	Ans. $3\frac{1}{4}$.	18. .075.	Ans. $\frac{3}{40}$.
10. 32.5.	Ans. $32\frac{1}{2}$.	19. .857142 $\frac{2}{7}$.	Ans. $\frac{6}{7}$.

CASE III.

Reduction of Common Fractions to Decimals.

ORAL EXERCISE.

- How many tenths in $\frac{1}{5}$, $\frac{1}{2}$, $\frac{3}{4}$?
- How many hundredths in $\frac{1}{5}$, $\frac{1}{2}$, $\frac{3}{4}$?
- How many thousandths in $\frac{1}{5}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{20}$, $\frac{1}{40}$?

WRITTEN PROBLEMS.

1. Change
- $\frac{3}{40}$
- to a decimal.
- Ans. .075.

PROCESS.

40)3.000(.075

280

200

200

Explanation.— $\frac{3}{40} = \frac{1}{40}$ of 3; $3 = \frac{3}{10}$, or $\frac{30}{100}$, or $\frac{300}{1000}$; $\frac{3}{40}$ of $\frac{300}{1000} = \frac{900}{4000} = \frac{9}{400} = \frac{225}{10000}$, or .075.

From the foregoing we have the

RULE.

Annex ciphers to the numerator, and divide by the denominator, pointing off as many decimal places as there are ciphers annexed to the numerator.

Reduce the following to decimals :

2. $\frac{1}{8}$.	<i>Ans.</i> .125.	14. $6\frac{3}{8}$.	<i>Ans.</i> 6.6.
3. $\frac{3}{4}$.	<i>Ans.</i> .75.	15. $56\frac{1}{2}$.	<i>Ans.</i> 56.75.
4. $\frac{5}{8}$.	<i>Ans.</i> .625.	16. $12\frac{3}{8}$.	<i>Ans.</i> 12.6.
5. $\frac{5}{80}$.	<i>Ans.</i> .0625.	17. $32\frac{3}{8}$.	<i>Ans.</i> 32.625.
6. $\frac{9}{40}$.	<i>Ans.</i> .225.	18. $46\frac{5}{16}$.	<i>Ans.</i> 46.3125.
7. $1\frac{11}{16}$.	<i>Ans.</i> .3525.	19. $21\frac{1}{4}$.	<i>Ans.</i> 21.2857 $\frac{1}{4}$.
8. $3\frac{1}{2}$.	<i>Ans.</i> 3.525.	20. $1\frac{1}{8}$.	<i>Ans.</i> .6875.
9. $37\frac{3}{4}$.	<i>Ans.</i> 37.75.	21. $\frac{3}{8}$.	<i>Ans.</i> .375.
10. $\frac{1}{40}$.	<i>Ans.</i> .025.	22. $15\frac{3}{8}$.	<i>Ans.</i> 15.66 $\frac{3}{8}$.
11. $\frac{3}{16}$.	<i>Ans.</i> .1875.	23. $\frac{5}{80}$.	<i>Ans.</i> .0125.
12. $\frac{7}{8}$.	<i>Ans.</i> .875.	24. $24\frac{1}{16}$.	<i>Ans.</i> 24.008.
13. $1\frac{2}{5}$.	<i>Ans.</i> .53 $\frac{1}{5}$.	25. $12\frac{1}{2}$.	<i>Ans.</i> 12.7619 $\frac{1}{11}$.

NOTE 1.—When the denominator of the common fraction contains other prime factors than 2 or 5, the division will not terminate.

2. When a sufficient number of decimal places is obtained, the remainder may be expressed by a common fraction.

3. When the quotient repeats the same figure or set of figures, it is called a circulating or repeating decimal.

SECTION III.

ADDITION OF DECIMALS.

WRITTEN PROBLEMS.

1. Add 25.75, 35.04, 1.648, 442.613.

PROCESS.

25.75
35.04
1.648
442.613
505.051

Explanation.—The decimals are so written that units of the same order stand in the same perpendicular column, and the addition is then performed the same as the addition of simple numbers.

RULE.

Write the numbers so that the decimal points shall stand in a perpendicular column, and add as in simple numbers.

NOTE.—Place the separatrix under those in the column.

2. Add 25.002, 206.31, 505.05, 1.015, 8.33.

Ans. 745.707.

3. Add 89.07, 76.79, 31.375, 403.3021. *Ans.* 600.5371.

4. Add .718, 643.5, 29.21, 114. *Ans.* 787.428.

5. Add 72.5, 7.29, 4.009, 3.275, .4. *Ans.* 87.474.

6. Add 5.316, 8.299, 33.7865, 884.13. *Ans.* 931.5315.

7. Add 127.334, 55.827, 94.8607, 7.25. *Ans.* 285.2717.

8. Add 17.02, 98.6438, 2.867, 1.077. *Ans.* 119.6078.

9. Add 18.65, 8.4139, 10.895, 87.007. *Ans.* 124.9659.

10. Add 34.883, 13.4072, 12.0734, 110.642, 8.0008.

Ans. 179.0064.

11. Four fields contain as follows: 16.375 acres, 12.6125 acres, 14.003 acres, 16.5 acres: how much do the four fields contain?

Ans. 59.4905 acres.

12. Five pieces of silver weigh as follows: .33 pounds, 1.275 pounds, .127 pounds, .5 pounds and 1.324 pounds: how much do the five pieces weigh?

Ans. 3.556 pounds.

SECTION IV.

SUBTRACTION OF DECIMALS.

WRITTEN PROBLEMS.

1. From 78.75 take 23.125.

Ans. 55.625.

Explanation.—Since annexing a cipher to the right of a decimal does not alter its value, 78.75 may be written 78.750. Writing the numbers so that the decimal points are immediately one over the other, the subtraction is performed the same as in simple numbers.

PROCESS.

$$\begin{array}{r} 78.750 \\ 23.125 \\ \hline 55.625 \end{array}$$

2. From 274.684 take 217.423.

Ans. 57.261.

3. From 186.004 take 87.621.

Ans. 98.383.

4. From 8648.3684 take 1764.8342.

Ans. 6883.5342.

5. From 621 take 3.17.

Ans. 617.83.

6. From 8.4062 take .6434.

Ans. 7.7628.

7. From 13.3175 take 9.61478.

Ans. 3.70272.

8. From 4256.85 take .00564.

Ans. 4256.84436.

- | | |
|---|-----------------------|
| 9. From 7.642 take $3\frac{1}{8}$. | <i>Ans.</i> 3.7045. |
| 10. From 296.4005 take $127\frac{1}{2}$. | <i>Ans.</i> 168.5505. |
| 11. From $87\frac{1}{2}$ take 31.642. | <i>Ans.</i> 55.558. |
| 12. From $1274\frac{1}{2}$ take 1232.625. | <i>Ans.</i> 41.5. |
| 13. From $25\frac{1}{2}$ take $18\frac{1}{2}$. | <i>Ans.</i> 7.55. |
| 14. From \$18.86 take \$13.685. | <i>Ans.</i> \$5.175. |

SECTION V.

MULTIPLICATION OF DECIMALS.

WRITTEN PROBLEMS.

1. Multiply 6.13 by 7.24. *Ans.* 44.3812.

PROCESS.

6.13	<i>Explanation</i> 1.—6.13 or $\frac{613}{100}$ multiplied by 7.24, or $\frac{724}{100}$, equals
7.24	
<u>24 52</u>	<i>Explanation</i> 2.—6.13 multiplied by 724 = 4438.12, and by 724 hundredths, one one-hundredth as much, or 44.3812.
122 6	
<u>4291</u>	
44.3812	

From the foregoing we have the rule for multiplication of decimals:

RULE.

Multiply decimals as simple numbers, and point off from the right as many decimal places as there are in the multiplier and the multiplicand.

NOTE.—If the number of figures in the product is less than the number in the two factors, prefix as many ciphers as may be necessary to make the number of decimal places in the product equal to the number in both factors.

What is the product—

- | | |
|-------------------------|--------------------------------|
| 2. Of 25.75 by 5.6? | <i>Ans.</i> 144.200, or 144.2. |
| 3. Of 167.43 by 18.606? | <i>Ans.</i> 3115.20258. |
| 4. Of 24.408 by .0008? | <i>Ans.</i> .0195264. |
| 5. Of 16.004 by 1.68? | <i>Ans.</i> 26.88672. |
| 6. Of .008 by 448.9? | <i>Ans.</i> 3.5912. |
| 7. Of 236.45 by 20.016? | <i>Ans.</i> 4732.7832. |

8. Of 75.64 by .125? *Ans.* 9.455.
 9. Of .37 by 1000? *Ans.* .370.
 10. Of 5.728 by .004? *Ans.* .022912.
 11. Of 56.25 by $17\frac{1}{2}$? *Ans.* 998.4375.
 12. Of 186.203 by $18\frac{1}{2}$? *Ans.* 3388.8946.
 13. Of $16\frac{1}{2}$ by 14.55? *Ans.* 234.61875.
 14. Of $277\frac{3}{8}$ by 12.004? *Ans.* 3329.6095.
 15. Of $874\frac{9}{16}$ by $163\frac{1}{16}$? *Ans.* 142663.38125.
 16. If a link contains 7.92 inches, how many inches are there in 75 links? *Ans.* 594.
 17. A farmer exchanged 28 bushels of wheat at \$1.25 a bushel for 200 pounds of sugar at $13\frac{1}{4}$ cents a pound, and the balance in cash: how much is due him? *Ans.* \$7.50.

SECTION VI.

DIVISION OF DECIMALS.

WRITTEN PROBLEMS.

1. Divide 578.5728 by 8.32. *Ans.* 69.54.

SOLUTION. *Explanation.*—578.5728 is equal to $\frac{5785728}{10000}$; 8.32 equals $\frac{832}{100}$; $\frac{5785728}{10000}$ divided by $\frac{832}{100} = \frac{5954}{100}$, or 69.54.

NOTE.—Since the quotient multiplied by the divisor equals the dividend, it is evident, according to the preceding Section, that the number of decimal places in the divisor, plus the number in the quotient, equals the number in the dividend; hence, the number in the dividend, less the number in the divisor, equals the number in the quotient.

From the foregoing solution and explanation we derive the following rule for division of decimals.

RULE.

Divide as in simple numbers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number of those in the divisor.

NOTE.—If the number of decimal places in the divisor exceeds the number in the dividend, first annex enough ciphers to the dividend to make the number of places equal to the number in the divisor.

- | | |
|---|--------------------|
| 2. Divide 12.5111 by 30.5. | <i>Ans.</i> .4102. |
| 3. Divide 641.28 by 38.4. | <i>Ans.</i> 16.7. |
| 4. Divide 18782.03 by 146.05. | <i>Ans.</i> 128.6. |
| 5. Divide 101.6688 by 43.08. | <i>Ans.</i> 2.36. |
| 6. Divide .3075 by .75. | <i>Ans.</i> .41. |
| 7. Divide .0189 by .025. | <i>Ans.</i> .756. |
| 8. Divide 47.362575 by 6.045. | <i>Ans.</i> 7.835. |
| 9. Divide 18.1771 by 6.7. | <i>Ans.</i> 2.713. |
| 10. Divide 9.455 by $12\frac{1}{2}$. | <i>Ans.</i> .7564. |
| 11. Divide 234.61875 by $16\frac{1}{2}$. | <i>Ans.</i> 14.55. |
| 12. Divide $998\frac{7}{8}$ by 56.25. | <i>Ans.</i> 17.75. |
| 13. Divide 1846 by 100. | <i>Ans.</i> 18.46. |
| 14. Divide 1143 by 1000. | <i>Ans.</i> 1.143. |
| 15. Divide 16 by 10000. | <i>Ans.</i> .0016. |

SECTION VII.

APPLICATION OF DECIMALS.

CASE I.

To find the Cost when the Price of 100 or 1000 and the Quantity are given.

WRITTEN PROBLEMS.

1. What cost 340 cabbages at \$4.20 a hundred? *Ans.* \$14.28.

SOLUTION.

$$\begin{array}{r} 4.20 \\ 340 \\ \hline 16800 \\ 1260 \\ \hline 14.2800 \end{array}$$

Or, $\left\{ \begin{array}{l} \frac{4.20 \times 340}{100} = 14.28. \end{array} \right.$ *Explanation.*—If cabbages cost \$4.20 per hundred, 1 will cost $\frac{1}{100}$ of \$4.20, and 340 will cost $\frac{340}{100}$ of \$4.20, which is \$14.28.

2. What will 675 ft. of boards cost at \$18.50 per thousand? *Ans.* \$12.48 $\frac{3}{4}$.

SOLUTION.

$$\begin{aligned} \$18.50 \times 675 &= \$12487.50, \\ \$12487.50 \div 1000 &= \$12.4875; \end{aligned}$$

Or,

$$\frac{\$18.50 \times 675}{1000} = \$12.4875, \text{ or } \$12.48\frac{3}{4}.$$

Explanation.—If boards cost \$18.50 per thousand feet, 1 foot will cost $\frac{1}{1000}$ of \$18.50, and 675 feet will cost 675 times $\frac{1}{1000}$ of \$18.50, or $\frac{675 \times \$18.50}{1000} = \$12.48\frac{3}{4}$, etc.

From the above processes and explanations we derive the rules to find the cost of articles sold by the hundred or thousand.

RULES.

1. *Multiply the cost per hundred by the total quantity, and cut off two places from the right hand.*

2. *Multiply the cost per thousand by the total quantity, and cut off three places from the right hand.*

3. Find the value of 17745 bricks at \$7.50 per thousand.

Ans. \$133.0875.

4. What cost 23425 feet of hemlock lumber at \$15 a thousand?

Ans. \$351.375.

5. A lumber-merchant sold 60000 lath at 30 cents a hundred, and 1250 feet of boards at \$21.50 a thousand: how much did he get for both?

Ans. \$206.875.

6. A grocer bought 320 cucumbers at 40 cents a hundred, and 115 cabbages at \$3.50 a hundred: what did he pay for both?

Ans. \$5.305.

7. What is the cost of 15520 bricks at \$7 a thousand, and 5760 at \$12.50 a thousand?

Ans. \$180.64.

8. A lumber-merchant sold the following bill:

2500 ft. boards at \$27.50 per M;

1875 ft. " at \$24 per M;

1650 laths at 32 cts. per hundred;

1520 pickets at \$15 per M;

7500 shingles at \$6.50 per M.

What did he receive for the whole amount of lumber sold?

Ans. \$190.58.

CASE II.

To find the Cost when the Price of a Ton and the Number of Pounds are given.

WRITTEN PROBLEMS.

1. At \$12 a ton what will 2315 pounds of hay cost?

$$\text{SOLUTION. } \frac{2315 \times \$12}{2000} = \$13.89.$$

$$\text{Or, } \frac{2315 \times 6}{1000} = 13.89.$$

From these solutions we derive the following

RULES.

1. *Multiply the cost of a ton by the number of pounds, and divide by 2000.*

2. *Multiply one-half the cost of a ton by the number of pounds, and divide by 1000.*

2. What will 1875 pounds of coal cost at \$6 a ton?

Ans. \$5.62 $\frac{1}{2}$.

3. What is the cost of 6340 pounds of coal at \$5.75 a ton?

Ans. \$18.2275.

4. What must I pay for 7850 pounds of stone at \$2.20 a ton?

Ans. \$8.635.

5. A farmer bought 3350 pounds of plaster at \$5.20 a ton: how much did it cost?

Ans. \$8.71.

6. When pig iron is worth \$32 a ton, what is the value of a car-load containing 15640 pounds?

Ans. \$250.24.

7. A farmer sold 2520 pounds of hay at \$22 a ton, and 1860 pounds of straw at \$16.50 a ton: how much did he get for both?

Ans. \$43.065.

8. If silver ore is worth \$190 a ton, what is the value of 10870 pounds?

Ans. \$1032.65.

9. A dealer sells 130 sacks of salt, each containing 230 pounds, at \$25 a ton: how much does he get?

Ans. \$373.75.

10. A coal and lumber dealer sold the following:

18640 pounds nut coal at \$4.25 per ton;

6350 " stove coal at \$5 per ton;

1865 feet flooring-boards at \$25 per M;

12000 lath at 45 cents a hundred.

What was the amount of the bill?

Ans. \$156.11.

SECTION VIII.

PROMISCUOUS PROBLEMS IN DECIMALS.

1. A farmer had 3 fields, containing as follows: 16.475 acres, 12.227 acres and 18.605 acres: how many acres in the three fields?

Ans. 47.307.

2. John walks 3.745 miles, and James 3.8005 miles: which walks the farthest, and how much? *Ans.* James—.0555 miles.

3. A merchant owned 64.803 acres of land; he bought 10.7045 acres, and sold all but 16.455 acres: how many acres did he sell? *Ans.* 59.0525.

4. What is the value of 7.5 tons of hay at \$18.75 a ton? *Ans.* \$140.625.

5. A farmer exchanged 12.55 tons of hay at \$17.75 per ton for 20 head of cattle at \$16.25 a head, the balance to be paid in cash: how much cash did he have to pay? *Ans.* \$102.2375.

6. If a wine-merchant sell 27.5 casks of wine, each containing 31.25 gallons, from a stock of 1000 gallons, how much remains? *Ans.* 140.625 gallons.

7. A merchant purchased oil at \$9.43 per barrel, and sold it at \$11.48 a barrel, and gained \$451: how many barrels did he have? *Ans.* 220 barrels.

8. A store was valued at \$9000; it was insured for .625 of its value: what is that man's share of the insurance who owns .45 of the stock? *Ans.* \$2531.25.

9. A store worth \$8800 was damaged by fire, but $\frac{1}{2}$ of its goods were saved; A owns .40; B, .35; and C, the remainder of the stock: what was the value of each one's share of the goods saved? *Ans.* A, \$2112; B, \$1848; C, \$1320.

10. A bricklayer furnished 17840 bricks at \$7.50 a thousand, and worked 43 days at \$2.75 a day: what is his bill? *Ans.* \$252.05.

11. A gardener paid as follows: for ploughing, \$26.50; for seed, \$2.375; for labor, \$40.625; for cultivating, \$12.50; and for hauling and marketing, \$26.25. He sold 850 heads of cabbage at \$5.25 a hundred, 1275 cucumbers at \$.75 a hundred, 840 ears of corn at 15 cts. a dozen, and 325 bushels of potatoes at \$1.75 a bushel. What was his profit for the year? *Ans.* \$525.1875.

12. A coal-merchant sold to a lumberman 5 loads of coal, as follows: 2840 pounds, 2725 pounds, 2742 pounds, 2930 pounds, 2563 pounds—all at \$5.75 a ton; and bought in ex-

change 1554 ft. boards at \$18.50 a thousand: which is in debt to the other, and how much?

Ans. The lumber-merchant owes the coal-dealer \$10.926.

13. A contractor furnished the following materials for a house:

23500 bricks at \$8.50 per **M**;
 6850 ft. lumber at \$15 per **M**;
 8040 ft. flooring at \$22.50 per **M**;
 6500 shingles at \$7.25 per **M**;
 2500 lath at 30 cts. a hundred.

He paid for labor and additional material \$742.50: how much did he make if he charged \$1800 for building the house?

Ans. \$519.475.

14. A coal-dealer sold to a farmer at three separate times coal as follows: 4235 pounds, 4285 pounds, 4080 pounds, at \$5.80 a ton; he agreed to take potatoes at 63 cents a bushel: how many bushels of potatoes does he get? *Ans.* 58.

CHAPTER VII.

DENOMINATE NUMBERS.

SECTION I.

DEFINITIONS.

88. A Denominate Number is one whose unit is named; as, 3 inches, 7 pounds, etc.

89. Numbers are of the same denomination when they have the same unit; as, 6 feet, 8 feet, etc.

90. A Compound Number is one consisting of several denominate numbers, but of the same species or measure; as, 6 yards, 2 feet, 3 inches.

91. A Measure is a unit used in estimating quantity.

92. A **Quantity** is measured by finding how often it contains the unit of measurement.

93. The principal measures are those of *Value, Extension, Weight, Time and Angles.*

SECTION II.

TABLES AND MEASURES.

UNITED STATES CURRENCY.

The currency of the United States was established by act of Congress in 1792.

TABLE.

10 mills (m.)	= 1 cent,	ct.
10 cents	= 1 dime,	d.
10 dimes	= 1 dollar,	\$.
10 dollars	= 1 eagle,	E.

The unit of United States currency is the dollar.

The currency of Canada is reckoned by the same scale as United States money, in dollars and cents.

ENGLISH CURRENCY.

English currency is the national currency of Great Britain and Ireland.

TABLE.

4 farthings (far.)	= 1 penny,	d.
12 pence	= 1 shilling,	s.
20 shillings	= 1 pound, or sovereign,	£.
21 shillings	= 1 guinea,	G.

COMPARISON OF UNITS.

1 £	= 20 s.	= 240 d.	= 960 far.
1 s.	= 12 d.	= 48 far.	
	1 d.	= 4 far.	

The unit of English currency is the pound sterling, indicated by the symbol £. The coin representing the pound sterling is called a *sovereign*. Its value in U. S. money is \$4.8665.

ORAL EXERCISE.

1. How many farthings in 6 d.? 8 d.? 12 d.?
2. How many pence in 3 s.? 4 s.? 10 s.? 8 far.? 12 far.?
3. How many shillings in 6 £? 5 £? 30 £?
4. How many shillings in 48 d.? 480 far.? 16 £? 3 £ 6 s.?
5. How many pence in 3 £? 2 £ 3 s.? 1 £ 6 s. 6 d.?
6. How many £ in 40 s.? 960 d.?

WRITTEN PROBLEMS.

1. Reduce 6 £ 15 s. to shillings.

SOLUTION.

$$\begin{array}{r} 6\text{ £ } 15\text{ s.} \\ 20 \\ \hline 120 \\ 15 \\ \hline 135\text{ s.} \end{array}$$

Explanation.—1 £ = 20 s.; 6 £ = $6 \times 20\text{ s.} = 120\text{ s.}$; 120 s. + 15 s. = 135 s. Hence, 6 £ 15 s. = 135 s.

2. Reduce 1820 d. to pounds.

SOLUTION.

Explanation.—In 1820 d. there are as many shillings as 12 is contained times in 1820, or 151 s. and 8 d.; in 151 s. there are as many pounds as 20 is contained times in 151, or 7 £ 11 s. Hence, 1820 d. = 7 £ 11 s. 8 d. = 7 £ 11 s. 8 d.

3. Reduce 16 £ 12 s. to shillings. *Ans.* 332 s.

4. Reduce 18 £ 10 s. 5 d. to pence. *Ans.* 445 d.

5. Reduce 1872 d. to £, s. and d. *Ans.* 7 £ 16 s.

6. Reduce 16842 far. to £, s., d. and far. *Ans.* 17 £ 10 s. 10 d. 2 far.

WEIGHT.

94. Weight is the quantity of matter in bodies as estimated by the force with which they are attracted to the earth's centre.

AVOIRDUPOIS WEIGHT.

95. Avoirdupois Weight is that used in weighing produce, coal, iron, groceries, etc.

TABLE.

16 drams (dr.)	- 1 ounce,	oz.
16 ounces	- 1 pound,	lb.
25 pounds	- 1 quarter,	qr.
4 quarters	- 1 hundred-weight,	cwt.
20 hund.-weight	- 1 ton,	T.

COMPARISON OF UNITS.

1 T.	= 20 cwt.	= 80 qr.	= 2000 lb.	= 32000 oz.	= 512000 dr.
	1 cwt.	= 4 qr.	= 100 lb.	= 1600 oz.	= 25600 dr.
		1 qr.	= 25 lb.	= 400 oz.	= 6400 dr.
			1 lb.	= 16 oz.	= 256 dr.
				1 oz.	= 16 dr.

1 lb. avoirdupois equals 7000 grains.

In collecting duties at the U. S. custom-houses, and in freighting and selling coal at wholesale, what is called the long ton (2240 lb.) is used. In this ton 1 qr. equals 28 lbs.; and 1 cwt. equals 112 lb.

The following units are also used :

32 lb. of oats	}	- 1 bushel.
45 " of timothy-seed		
48 " of barley		
56 " of rye		
56 " of Indian corn		
60 " of wheat		
60 " of clover-seed	}	- 1 firkin.
60 " of potatoes		
56 " of butter		
196 " of flour	}	- 1 barrel.
200 " of beef or pork		
100 " of dry fish		
100 " of nails		- 1 quintal.
		- 1 keg.

ORAL EXERCISE.

1. How many ounces in 6 lb. avoirdupois? $12\frac{1}{2}$ lb.? $10\frac{1}{2}$ lb.?
2. How many pounds in 2000 oz.? 2400 oz.?
3. How many pounds in 3 T.? 6 T. 3 cwt.?
4. How many pounds in 9 qr.? 3 cwt. 3 qr.
5. A farmer exchanged 1 T. 3 cwt. of hay at 80 cts. per cwt. for delaine at 40 cts. a yard: how many yards did he get?

WRITTEN PROBLEMS.

1. Reduce 27 T. 3 cwt. to cwt. *Ans.* 543 cwt.
2. Reduce 3 T. 5 cwt. 1 qr. to pounds. *Ans.* 6525 lb.
3. Reduce 27000 lb. to tons. *Ans.* 13 T. 10 cwt.
4. Reduce 234650 oz. to tons.
Ans. 7 T. 6 cwt. 2 qr. 15 lb. 10 oz.
5. What is the value of 2 lb. 4 oz. of butter at 30 cents a pound? *Ans.* $67\frac{1}{2}$ cents.
6. What is the value of 3 lb. 5 oz. of cheese at 22 cts. a pound? *Ans.* $72\frac{1}{4}$ cents.
7. What will 5 barrels of beef cost at $5\frac{1}{2}$ cts. a pound? *Ans.* \$55.
8. What will 19 barrels of flour cost at $3\frac{1}{2}$ cts. a pound? *Ans.* \$130.34.
9. What is the value of 16840 lb. of wheat at \$1.20 a bushel? *Ans.* \$336.80.
10. A farmer sold 64 bags of wheat, each weighing $183\frac{1}{2}$ lb., at \$1.30 per bushel: how much did he get for his wheat? *Ans.* \$254.45 $\frac{1}{2}$.

TROY WEIGHT.

96. Troy Weight is used in weighing gold, silver, gems and jewels.

TABLE.

24 grains (gr.)	= 1 pennyweight, dwt.
20 pennyweights	= 1 ounce, oz.
12 ounces	= 1 pound, lb.

COMPARISON OF UNITS.

1 lb. = 12 oz. = 240 dwt. = 5760 gr.

1 oz. = 20 dwt. = 480 gr.

1 dwt. = 24 gr.

The *Troy pound* = 5760 gr. The *Troy ounce* = 480 gr.

The *avoirdupois pound* = 7000 gr. The *avoirdupois ounce* = $437\frac{1}{4}$ gr.

APOTHECARIES' WEIGHT.

97. Apothecaries' Weight differs from Troy weight in the division of the ounce. It is used in measuring medicines.

TABLE.

20 grains (gr.) = 1 scruple, $\mathfrak{z}$.

3 scruples = 1 dram, $\mathfrak{ss}$.

8 drams = 1 ounce, $\mathfrak{z}$.

12 ounces = 1 pound, $\mathfrak{lb}$.

COMPARISON OF UNITS.

1 $\mathfrak{lb}$ = 12 $\mathfrak{z}$ = 96 $\mathfrak{ss}$ = 288 $\mathfrak{gr}$ = 5760 gr.

1 $\mathfrak{z}$ = 8 $\mathfrak{ss}$ = 24 $\mathfrak{gr}$ = 480 gr.

1 $\mathfrak{ss}$ = 3 $\mathfrak{gr}$ = 60 gr.

1 $\mathfrak{gr}$ = 20 gr.

ORAL EXERCISE.

1. How many ounces in 9 lb. Troy? In $10\frac{1}{2}$ lb.? In 12 lb. 3 oz.?

2. How many pounds in 60 oz.? In 180 oz.? In 100 oz.?

3. How many pennyweights in 6 oz.? In 3 lb.? In 5 lb. 3 oz.?

4. How many ounces in 40 dwt.? In 960 gr.? In $12\frac{1}{2}$ lb.?

5. How many scruples in 1 $\mathfrak{lb}$? In 6 $\mathfrak{z}$ 3 $\mathfrak{ss}$? In 2 $\mathfrak{lb}$ 2 $\mathfrak{z}$ 3?

6. How much heavier is an ounce of silver than an ounce of salt?

7. How much heavier is a pound of feathers than a pound of gold?

WRITTEN PROBLEMS.

1. Reduce 7 lb. 6 oz. Troy to ounces. *Ans.* 90 oz.
2. Reduce 12 lb. 5 oz. 10 dwt. to pennyweights.
Ans. 2990 dwt.
3. Reduce 6400 dwt. to pounds. *Ans.* 26 lb. 8 oz.
4. Reduce 84150 gr. to pounds.
Ans. 14 lb. 7 oz. 6 dwt. 6 gr.
5. How many grains in $13\frac{3}{4}$ lb. Troy? *Ans.* 79200.
6. What is a silver bar weighing $9\frac{1}{2}$ lb. Troy worth at $\$.06\frac{3}{4}$ a pennyweight? *Ans.* \$150.48.
7. What is the value of a gold chain weighing 1 oz. 5 dwt. at 85 cents a pennyweight? *Ans.* \$21.25.
8. What is a $4\frac{3}{8}$ bottle of medicine worth at 2 cents a scruple? *Ans.* \$1.92.

EXTENSION.

98. **Extension** is that which has one or more of the dimensions—length, breadth and thickness.

99. A **Line** has length only.

100. A **Surface** has length and breadth.

101. A **Volume** has length, breadth and thickness.

The standard unit of all measurements of extension is the yard.

LONG MEASURE.

102. **Long Measure** is used in measuring length and distance.

TABLE.

12 inches (in.)	= 1 foot,	ft.
3 feet	= 1 yard,	yd.
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod,	rd.
40 rods	= 1 furlong,	fur.
8 furlongs	= 1 mile,	mi.
3 miles	= 1 league,	lea.

COMPARISON OF UNITS.

1 mi. = 8 fur. = 320 rd. = 1760 yd. = 5280 ft. = 63360 in.

1 fur. = 40 rd. = 220 yd. = 660 ft. = 7920 in.

1 rd. = $5\frac{1}{2}$ yd. = $16\frac{1}{2}$ ft. = 198 in.

1 yd. = 3 ft. = 36 in.

1 ft. = 12 in.

The inch is generally divided into halves, quarters, etc.

Civil engineers and others divide it into tenths, etc.

The following denominations or measures come under the head of Long Measure:

3 barleycorns or sizes = 1 inch—used by shoemakers.

4 inches = 1 hand—used to measure the height of horses.

9 inches = 1 span—used by sailors.

8 spans, or 6 feet = 1 fathom, “ “

120 fathoms = 1 cable-length.

21.888 inches = 1 cubit.

3 feet = 1 pace.

1.16 common miles = 1 geographical mile—used to measure distance at sea.

3 geog. miles = 1 league.

60 geog miles, or
69.16 statute miles } = 1 degree { of latitude on the meridian.
of longitude at the equator.

360 degrees = 1 circumference of the earth.

1 knot or geog. mile = $\frac{1}{60}$ of $\frac{1}{360}$, or $\frac{1}{21600}$ of the circumference of the earth.

Cloth Measure is now out of use. Goods are now sold by the yard, quarter, etc.

In custom-houses the yard is divided decimally in estimating duties.

ORAL EXERCISE.

1. How many inches in 2 ft? In 4 ft.? In 8 ft.? In 27 ft.?
2. How many feet in 108 in.? In 144 in.? In 180 in.?
3. How many feet in 6 yd.? In 25 yd.? In 120 yd.?

4. How many yards in 24 ft.? In 72 ft.? In 144 in.? In 1728 in.?

5. How many yards in 4 rd.? In $6\frac{1}{2}$ rd.? In 40 rd.? In 80 rd.?

6. How many rods in 11 yd.? In 44 yd.? In 66 ft.? In 132 ft.?

7. How many inches in 2 ft. 10 in.? 2 yd.? 6 ft. 8 in.?

8. If a man travel 20 leagues a day by ship, how long will it take to travel 2400 miles?

9. At 60 cents a foot how much will 4 rods of fence cost?

10. In 60 rods how many feet? How many inches in 60 rods?

WRITTEN PROBLEMS.

1. Reduce 6 ft. 3 in. to inches. *Ans.* 75 in.

SOLUTION.

6 ft. 3 in. =

$\frac{12}{72}$ in.

$\frac{3}{75}$ in.

Let the pupil explain the solution.

2. Reduce 1000 in. to yards. *Ans.* 27 yd. 2 ft. 4 in.

SOLUTION.

Explanation. = Since 12 in. = 1 ft., 1000 in. = $1\frac{2}{3}$ ft., or 83 ft. 4 in.; and since 3 ft. = 1 yd., 83 ft. = $27\frac{2}{3}$ yd., or 27 yd. 2 ft. Hence, 1000 in. = 27 yd. 2 ft. 4 in.

3. Reduce 19 ft. 7 in. to inches. *Ans.* 235 in.

4. Reduce 6 yd. 2 ft. 3 in. to inches. *Ans.* 243 in.

5. Reduce 40 rd. to feet; to inches. *Ans.* 660 ft.

6. Reduce 800 in. to yards and feet. *Ans.* 22 yd. 8 in.

7. Reduce 640 ft. to rods. *Ans.* 38 rd. 4 yd. 1 ft.

8. Reduce 1200 in. to rods. *Ans.* 6 rd. 1 ft.

9. If a horse is $15\frac{1}{2}$ hands high, how many feet and inches is he high? *Ans.* 5 ft. 2 in.

10. What is the difference in length between a boot No. 6 and a boot No. 11? *Ans.* ———.

11. How many feet deep is water 10 fathoms deep?

SURVEYORS' MEASURE.

103. Surveyors' Measure is used in measuring land and boundaries.

TABLE.

7.92 inches (in.)	= 1 link, li.
25 links	= 1 rod, r.
4 rods	= 1 chain, ch.
80 chains	= 1 mile, mi.

COMPARISON OF UNITS.

1 mile	= 80 ch.	= 320 r.	= 8000 li.	= 63360 in.
1 ch.	=	4 r.	= 100 li.	= 792 in.
		1 r.	= 25 li.	= 198 in.
			1 li.	= 7.92 in.

104. A Gunter's chain, 66 feet long and containing 100 links, is the unit of measure.

Engineers generally use a tape or a chain 100 feet long.

Surveyors record their measurements in chains and hundredths.

WRITTEN PROBLEMS.

1. Reduce 6 ch. to inches. *Ans.* 4752 in.
2. Reduce 3 ch. to yards. *Ans.* 66 yd.
3. In the side of a field 10 ch. long, how many feet? *Ans.* 660.

4. A field's boundaries are as follows : north side, 16 chains; west side, 14 chains; south side, 15.25 chains; east side, 12.33 $\frac{1}{3}$ chains: how many feet in each side, and how far around the field? *Ans.* N., 1056 ft.; W., 924 ft.; S., 1006 ft. 6 in.; E., 814 ft.; around, 3800 ft. 6 in.

SURFACE OR SQUARE MEASURE.

105. Square Measure is used in measuring areas or surfaces.

A **Surface** has length and breadth only.

The area of a surface is expressed by the product of the two numbers representing the two dimensions.

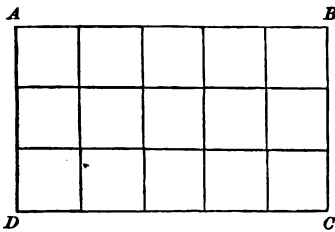
106. A Rectangle is a figure having four sides, each of which is perpendicular to two of the other sides.

A slate, a book and a sheet of paper are examples of rectangles.

107. A Square is a rectangle whose four sides are equal.

A square inch is a square whose sides are each one inch long.

The unit of measure in surface measure is a square whose length is known, as an inch or a foot.



In the rectangle A B C D, five feet long and three feet wide, the unit of measurement is the square foot, and the area is equal to the number of units contained in the rectangle, which is found by actual count to

be 15, or 5×3 . From this and similar processes we derive the

RULE.

To find the area of a rectangle, multiply the length by the breadth; the product is the area.

TABLE.

144 square inches (sq. in.)	= 1 square foot,	sq. ft.
9 " feet	= 1 " yard,	sq. yd.
30 $\frac{1}{4}$ " yards }	= 1 square rod or perch, rd. or P.	
or 272 $\frac{1}{4}$ " feet }		
160 " rods	= 1 acre	A.
640 acres	= 1 square mile,	sq. mi.

The rood, now but seldom used, is equal to 40 square rods.

COMPARISON OF UNITS.

1 A.	= 160 sq. rd.	= 4840 sq. yd.	= 43560 sq. ft.	= 6272640 sq. in.
	1 sq. rd.	= 30 $\frac{1}{4}$ sq. yd.	= 272 $\frac{1}{4}$ sq. ft.	= 39204 sq. in.
		1 sq. yd.	= 9 sq. ft.	= 1296 sq. in.
			1 sq. ft.	= 144 sq. in.

ORAL EXERCISE.

1. How many sq. in. in 3 sq. ft.? In 1 sq. yd.?
2. How many sq. ft. in 6 sq. yd.? In 288 sq. in.?
3. How many sq. yd. in 10 sq. rd.? In 81 sq. ft.?
4. How many rods in 10 A.? In $\frac{1}{4}$ A.? In $\frac{5}{8}$ A.?
5. At 10 cts. a sq. ft. how much will 1 A. cost?

WRITTEN PROBLEMS.

1. Reduce 871200 sq. ft. to acres. *Ans.* 20 A.
2. How many acres in a township 6 miles square?
Ans. 23040.
3. A rectangular piece of land is 40 rd. long and 12 rd. wide: how many acres does it contain? *Ans.* 3.
4. A rectangular piece of land is 14 rd. 6 ft. long and 12 rd. 3 ft. wide: how many acres does it contain?
Ans. 1 A. 14 sq. rd. 29 sq. yd. 4 sq. ft. 72 sq. in.
5. Bought 12 acres of land at \$200 an acre, and sold it at 8 cts. a sq. ft.: what was the gain? *Ans.* \$39417.60.
6. What part of an acre is a town-lot 60 ft. by 160 ft?
Ans. $\frac{80}{863}$.
7. What part of an acre is a town-lot 25 ft. by 120 ft?
Ans. $\frac{25}{863}$.

MEASURES OF VOLUME.

108. Cubic Measure is used in computing the contents of volumes or solids.

109. A Volume or Solid has three dimensions—length, breadth and thickness.

The volume of a body is expressed by the product of the numbers representing its three dimensions.

110. A Cube is a volume bounded by six equal squares, which are called its faces. The three dimensions of a cube are equal.

A cubic inch is a cube each side of which is one inch in length.

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot, cu. ft.
27 cubic feet	= 1 cubic yard, cu. yd.

COMPARISON OF UNITS.

1 cu. yd. = 27 cu. ft. = 46656 cu. in.

1 cu. ft. = 1728 cu. in.

WOOD MEASURE.

TABLE.

16 cubic feet	= 1 cord foot,	cd. ft.
8 cord feet, or	} = 1 cord,	cd.
128 cubic feet		
24½ cubic feet	= 1 perch of stone or masonry, pch.	

ORAL EXERCISE.

1. How many cu. ft. in 2 cu. yd.? In 5 cu. yd.?
2. How many cu. ft. in 3 cords? In 5 cords? In ½ cord?
3. How many cu. ft. in 2 pch. of stone? In 6 pch.?
4. How many cords in 256 cu. ft.? In 26 cd. ft.?

WRITTEN PROBLEMS.

1. How many cu. ft. in 16 pch. of stone? *Ans.* 396.
2. In a 13-inch cube how many cu. in.? *Ans.* 2197.
3. In ¾ of a cd. how many cu. ft.? *Ans.* 96.
4. In a pile of wood 20 ft. long, 4 ft. wide and 6 ft. high, how many cords? *Ans.* 3½.
5. What is the difference between a 6-inch cube and 6 cu. in.? *Ans.* 210 cu. in.
6. How many cu. ft. in a block 6 ft. long, 4 ft. wide and 3 ft. high? *Ans.* 72 cu. ft.
7. How many 2-foot cubes could be cut from a block 6 ft. high, 8 ft. long and 4 ft. wide? *Ans.* 24.
8. How many perch of stone in a wall 20 ft. long, 6 ft. high and 3 ft. wide? *Ans.* 14⅓.

MEASURES OF CAPACITY.

111. Measures of Capacity are used to determine the quantity of fluids and of many dry substances.

They are therefore of two kinds—*Measures of Liquids* and *Measures of Dry Substances*.

The unit of Liquid Measure is the *gallon*, and of Dry Measure the *bushel*.

LIQUID MEASURE.

112. Liquid Measure is used in measuring liquids.

TABLE.

4 gills (gi.)	= 1 pint,	pt.
2 pints	= 1 quart,	qt.
4 quarts	= 1 gallon,	gal. = 231 cu. in.

COMPARISON OF UNITS.

1 gal.	= 4 qt. = 8 pt. = 32 gi.
1 qt.	= 2 pt. = 8 gi.
1 pt.	= 4 gi.

The ale gallon contains 282 cu. in., but is now rarely used.

The barrel and the hogshead are no longer fixed measures, but vary, and their contents are estimated by gallons.

In estimating the capacity of cisterns, reservoirs, etc.,

1 barrel (bbl.)	= $31\frac{1}{2}$ gallons.
1 hogshead (hhd.)	= 63 gallons.

ORAL EXERCISE.

1. How many gills in 6 pt.? In 2 qt.?
2. How many pints in 20 gi.? In 6 gal.?
3. How many quarts in 40 gal.? In 200 gi.?
4. If a gill of wine cost 5 cents, how much will a gallon cost?
5. How many pints in 6 gal. 3 qt.?
6. If 1 gal. of oil cost 80 cents, how much will 5 gal. and 1 pt. cost?
7. How many five-gallon kegs can be filled from a hogshead of 63 gal.?

DRY MEASURE.

113. Dry Measure is used in measuring grain, fruit, salt, soft coal, etc.

TABLE.

2 pints	= 1 quart, qt.
8 quarts	= 1 peck, pk.
4 pecks	= 1 bushel, bu.

COMPARISON OF UNITS.

1 bu.	= 4 pk.	= 32 qt.	= 64 pt.
1 pk.	= 8 qt.	= 16 pt.	
1 qt.	= 2 pt.		

The Winchester bushel, which is the unit of measure, contains 2150.42 cubic inches.

A chaldron = 36 bushels.

1 pint dry measure = nearly $1\frac{1}{8}$ pt. liquid measure.

In measuring small seeds, beans, grain, etc. the measure must be *even* full. Four *heaped* measures are equal to five *even* measures.

A *heaped* bushel equals 2688 cu. in., or $\frac{5}{4}$ of an *even* bushel.

ORAL EXERCISE.

1. How many pints in 7 pk.? In 3 bu.?
2. How many pints in 1 bu. 3 pk.?
3. How many quarts in 3 bu. 3 pk.?
4. How many pecks in 27 pt.? In 18 qt.?
5. How many pints in 3 bu. 3 pk. 3 qt.?

WRITTEN PROBLEMS.

1. How many pints in 6 gal. 1 qt. 1 pt.? *Ans.* 51.
2. How many pints in 18 bu. 3 pk.? *Ans.* 1200.
3. How many bushels in 17632 pt. of beans? *Ans.* $275\frac{1}{2}$.
4. What is the difference between a quart of wine and a quart of clover-seed in cubic inches? *Ans.* 9.45 cu. in.
5. Bought 5 bushels of chestnuts at \$4 a bushel, and sold them at 15 cents a quart: how much was the gain?

Ans. \$4.

6. Bought 3 bushels of nuts at \$2 a bushel, and sold them at 10 cents a quart, liquid measure: how much was the gain?

Ans. \$5.17.

APOTHECARIES' FLUID MEASURE.

114. This measure is used in measuring liquids in preparing medicines.

TABLE.

60 minims or drops (m.)	= 1 fluid drachm, f 3.
8 fluid drachms	= 1 fluid ounce, f 3.
16 fluid ounces	= 1 pint, O.
8 pints	= 1 gallon, Cong.

The minim is about equal to a drop of water. A pint of water weighs nearly a pound.

Cong., from the Lat. *congius*, means *gallon*; and *O.*, from the Latin *octarius*, means *one-eighth*.

Physicians write their prescriptions in the Roman notation, using the small j for small i when final. Thus, 1 scruple is written ʒi.; 7 drams, ʒvij.; 13 ounces, ʒxiiij.

TIME.

115. Time is a measured portion of duration.

TABLE.

60 seconds	= 1 minute,	min.
60 minutes	= 1 hour,	hr.
24 hours	= 1 day,	d.
365 days	= 1 year	yr.
366 days	= 1 leap-year.	
100 years	= 1 century,	cen.

Also,

7 days	= 1 week,	wk.
4 weeks	= 1 month,	mo.
52 weeks	= 1 year,	yr.

In business transactions 30 days are considered a month, and 12 months a year.

116. The **Solar Day** is the interval of time between two successive passings of the sun across any given meridian.

117. The **Mean Solar Day** is the average length of all the solar days of the year. It is equal to 24 hours.

118. The **Civil Day** corresponds to the mean solar day, but begins and ends at 12 midnight.

119. **A. M.** denotes before noon; **M.**, noon, and **P. M.**, after noon.

120. The **Solar Year** is 365 days 5 hours 48 minutes 49.7 seconds in length.

The months of April, June, September and November have 30 days each. February has 28 days, except in leap-year, when it has 29. The other months have 31 days each.

Every year whose number is divisible by 4, except the centennial years not divisible by 400, is a leap-year.

Since the common year is almost 6 hours greater than 365 days, every fourth year is made a leap-year; but this makes more than the proper time, hence every hundredth year is considered a common year; this again makes less than the proper time, hence every centennial year divisible by 400 is made a leap-year; this makes a difference so small that almost 4000 years will elapse before the difference becomes a full day.

ORAL EXERCISE.

1. In 6 min. how many seconds? In $\frac{1}{2}$ min.? In $\frac{1}{4}$ min.?
2. How many minutes in 3 hr. 11 min.? In $1\frac{1}{2}$ hr.? In 480 sec.?
3. How many hours in a week? In 3 d. 11 hr.?
4. How many weeks in 49 d.? In 90 d.?
5. How many days from Jan. 1 to Feb. 28, inclusive?
6. How many days from March 28 to Aug. 16, inclusive?
7. What part of a day are 6 hr.? 10 hr.?

WRITTEN PROBLEMS.

1. Reduce 60000 sec. to hours, etc. *Ans.* 16 hr. 40 min.
2. How many seconds in a week? *Ans.* 604800.

3. How many days from Jan. 1, 1875, to May 1, 1876, inclusive? *Ans.* 487.

4. A man was born Jan. 2, 1836, and he died Dec. 9, 1874: how old was he? *Ans.* 38 yr. 11 mo. 7 d.

5. If a note is given Jan. 1, 1876, for 90 days, when does it fall due? *Ans.* Mar. 31.

6. Point out the leap-years in the following: 1872, 1875, 1876, 1900, 1906, 1908, 2000, 1840, 1600, 1700, 1800, 1820?

7. How much time passes from 20 min. before 9 o'clock to 15 min. after 11 o'clock? *Ans.* 2 hr. 35 min.

8. How much time passes from 15 min. before 8 A. M. to 20 min. past 2 P. M.? *Ans.* 6 hr. 35 min.

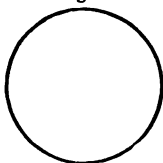
9. How many leap-years in the nineteenth century?

CIRCULAR MEASURE.

121. Circular or Angular Measure is used in measuring angles and arcs of circles, and in determining latitude and longitude, including the location of places and vessels at sea.

The unit of this measure is the *degree*, equal to $\frac{1}{360}$ part of the circumference of a circle.

Fig. 1.



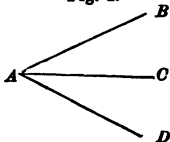
122. A Circle is a plane figure bounded by a curved line, every part of which is equally distant from a point within called the centre.

123. The bounding line is called the *Circumference*.

circumference.

124. An Arc is any part of the cir-

Fig. 2.

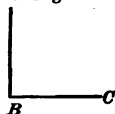


125. A Degree is $\frac{1}{360}$ part of the circumference.

126. An Angle is the difference in the direction of two lines meeting at a common point called the vertex.

Thus, D A C and C A B are angles.

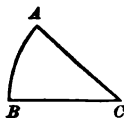
A Fig. 3.



127. A *Right Angle* is one in which the two lines are perpendicular to each other, as in Fig. 3.

The measure of an angle is the arc of a circle included between its two sides. Thus, in A B C the arc

A B is the measure of the angle C.



128. Any straight line extending from the centre to the circumference of a circle is called a *Radius*; and any straight line passing through the centre and limited by the circumference is called a *Diameter*.

TABLE.

60 seconds (")	= 1 minute, (')
60 minutes	= 1 degree, (°)
30 degrees	= 1 sign, S.
12 signs or 360°	= 1 circle, C.
90°	= 1 quadrant or right angle.

ORAL EXERCISE.

1. How many degrees in 600 min.?
2. How many minutes in 780 sec.? In 60°?

WRITTEN PROBLEMS.

1. What part of a circumference is 40°? 60°? 120°?
3. How many degrees in 7200' ? *Ans.* 2°.
3. How many degrees in 890' ? *Ans.* 14° 50'.

NOTE.—Degrees of latitude vary but little, but increase slightly as we approach the poles. A degree of longitude at the equator equals 69.16 common miles; at 30 degrees north latitude, 59.81 common miles; at 60 degrees north latitude, 34.53 miles; and at the poles, nothing.

COUNTING.

The following Table is used in counting such objects as buttons, screws, etc.:

12 things	= 1 dozen,	doz.
12 dozen	= 1 gross,	gro.
12 gross	= 1 great-gross,	gr. gro.
20 units	= 1 score.	

PAPER AND BOOKS.

24 sheets = 1 quire.
 20 quires = 1 ream.
 2 reams = 1 bundle.
 5 bundles = 1 bale.

In printing books, large sheets are folded and named as follows: When the sheet is folded into

	The book is called	1 sheet makes
2 leaves	a folio.	4 pp.
4 "	a quarto, or 4to.	8 "
8 "	an octavo, or 8vo.	16 "
12 "	a duodecimo, or 12mo.	24 "
16 "	a 16mo.	32 "
18 "	an 18mo.	36 "

ORAL EXERCISE.

1. How many years are three score and ten?
2. How many eggs in $6\frac{3}{4}$ doz.?
3. How many screws in $1\frac{1}{2}$ gro.? In 1 gr. gro.?
4. How many sheets of paper in 6 quires.? In $\frac{1}{2}$ ream?
5. How many scores in 160 yr.?
6. How many sheets of paper will be required to make a quarto volume of 160 pp.?
7. How many sheets of paper will be required to make an 8vo of 240 pp.?
8. How many sheets of paper will be required to make a 12mo volume of 252 pp.?
9. How many pages of a 16mo book will a quarto of 80 pp. make?

SECTION III.

REDUCTION.

129. Reduction of denominate numbers is the process by which their denomination is changed without altering their value.

130. Reduction Descending is the process of changing from a higher to a lower denomination; as from yards to inches.

131. Reduction Ascending is the process of changing from a lower to a higher denomination; as from inches to yards.

1. REDUCTION DESCENDING.

ORAL EXERCISE.

1. How many shillings in 6 £? Pence in 5 £?
2. How many ounces in 4 lb.? In 3 lb. 5 oz.?
3. How many pounds in $\frac{3}{4}$ of a ton? In $\frac{1}{2}$ ton?
4. How many pennyweights in 3 oz.? Ounces in 3 lb. 3 oz. gold.
5. How many ounces in 4 lb. avoirdupois? In 4 lb. Troy?
6. How many drams in 8 $\frac{3}{4}$ apothecaries' weight?
7. How many yards in 20 rd.? In $\frac{1}{2}$ m.?
8. How many inches in 6 ft. 3 in.? Rods in $\frac{1}{2}$ m.?
9. How many square feet in 3 sq. yd.?
10. How many cubic feet in 4 cu. yd.?
11. How many acres in 3 sq. m.? In 3 miles square?
12. How many cubic feet in 5 cd.?
13. How many pints in 6 gal.? In 5 gal. 3 qt.?
14. How many pints in 3 pk.? In 1 bu. 1 pk.?
15. How many quarts in 10 bu. 2 pk.?
16. How many days in 16 wk. 5 d.? In 12 wk. 3 d.?
17. How many degrees in 3 signs? In 3 quadrants?
18. How many in $6\frac{1}{4}$ doz.? In 4 score and 5?
19. How many minutes in $4\frac{1}{4}$ hr.?
20. How many pennyweights in 3 lb.?

WRITTEN PROBLEMS.

1. Reduce 24 bu. 3 pk. 2 qt. to quarts. *Ans.* 794 qt.

SOLUTION.

24 bu. 3 pk. 2 qt.

$$\begin{array}{r}
 4 \\
 96 \text{ pk.} \\
 \underline{3} \\
 99 \text{ pk.} \\
 \underline{8} \\
 792 \text{ qt.} \\
 \underline{2} \\
 794 \text{ qt.}
 \end{array}$$

Explanation.—1 bu. = 4 pk.; 24 bu. = 24×4 pk. = 96 pk.; 96 pk. + 3 pk. = 99 pk.; 1 pk. = 8 qt.; 99 pk. = 99×8 qt. = 792 qt.; 792 qt. + 2 qt. = 794 qt. Hence, 24 bu. 3 pk. 2 qt. = 794 qt.

2. Reduce 248 £ to pence. *Ans.* 59520 d.

3. Reduce 32 £ 10 d. to farthings. *Ans.* 30760 far.

4. Reduce 56 £ 12 s. to farthings. *Ans.* 54336 far.

5. Reduce 6 mi. 30 rd. 3 yd. to feet. *Ans.* 32184 ft.

6. Reduce 6 A. 10 sq. yd. to sq. ft. *Ans.* 261450 sq. ft.

7. Reduce 10 sq. rd. $3\frac{1}{2}$ sq. ft. to sq. in. *Ans.* 392544 sq. in.

8. Reduce 12 T. 3 cwt. 10 lb. to pounds. *Ans.* 24310 lbs.

9. Reduce the year 1876 to minutes. *Ans.* 527040 min.

10. Reduce 16 cd. 3 cu. ft. to cu. ft. *Ans.* 2051 cu. ft.

11. Reduce $12\frac{1}{2}$ gal. to gills. *Ans.* 400 gills.

12. Reduce 12 lb. 6 oz. 3 dwt. to pennyweights. *Ans.* 3003 dwt.

13. Reduce 3 wk. 5 d. 4 hr. to minutes. *Ans.* 37680 min.

14. Reduce 8 bu. 2 pk. 2 qt. 1 pt. to pints. *Ans.* 549 pt.

15. What is the value of 3 lb. 6 oz. of gold at 80 cents a pennyweight? *Ans.* \$672.

16. A farmer put 1001 bu. of wheat in sacks containing 2 bu. 3 pk. each: how many sacks were there? *Ans.* 364.

17. A grocer put a lot of sugar, 6 bbl. of 220 lb. each, in packages of 8 lb. each: how many packages were there? *Ans.* 165.

18. Bought 60 gal. of molasses at 80 cents a gallon, and sold it at 25 cents a quart: what was the gain? *Ans.* \$12.

19. How many pint bottles can be filled from a hogshead containing 63 gal. of wine? *Ans.* 504.
20. How many pounds in $14\frac{1}{2}$ bbl. of flour? *Ans.* 2842.
21. What is the value in U. S. money of 12 sovereigns? *Ans.* \$58.398.
22. If a clock tick seconds, how often did it tick in January and February, 1876? *Ans.* 5184000 times.
23. How many times will a wheel 15 ft. in circumference revolve in going 3 mi.? *Ans.* 1056.
24. How many pages in a 12mo book containing 20 sheets? *Ans.* 480.
25. Bought 12 bu. of chestnuts at \$3.50 a bushel, and sold them at 8 cents a pint: what was the gain? *Ans.* \$19.44.

2. REDUCTION ASCENDING.

ORAL EXERCISE.

1. How many £ in 80 shillings? In 480 pence?
2. How many yards in 108 in.? In 112 in.?
3. How many bushels in 60 pk.? In 120 pt.?
4. How many cubic yards in 108 cu. ft.?
5. How many hands in 81 in.? Feet in 81 in.?
6. In 1200 dwt. how many ounces? How many pounds?
7. In 2600 lb. how many tons?
8. In 5760 gr. Troy how many ounces? How many pounds?
9. In 1820 lb. of wheat how many bushels?
10. In 3920 lb. of flour how many barrels?
11. In 18000 sec. how many minutes? How many hours?
12. In 1800 how many dozen? How many gross?
13. In 256 gi. how many pints? Quarts? Gallons?
14. In 160 O. how many gallons?
15. In 224 pt. how many pecks? Bushels?
16. In 120 quires how many reams?
17. In 256 cu. ft. how many cords?
18. In 60 sq. ch. how many acres?

19. Change 48 oz. Troy to pounds; 48 oz. avoirdupois to pounds.

20. Change 1800 hr. to days; to weeks.

WRITTEN PROBLEMS.

1. Reduce 40325 oz. to tons. *Ans.* 1 T. 5 cwt. 20 lb. 5 oz.

SOLUTION. *Explanation.*—Since 1 lb. = 16 oz., $\frac{1}{16}$ of the number of ounces equals the number of pounds; $\frac{1}{16}$ of 40325 is 2520 + 5 oz. 40325 oz. is 2520, and 5 oz. remaining.
 4) 2520 + 5 oz. Since 25 lb. equal 1 quarter, $\frac{1}{4}$ of 2520 lb. = 630. 2520 lb. is 630 quarters, which is 100, and 20 lb. remaining.
 20) 100 + 20 lb. 1 T. 5 cwt. 20 lb. 5 oz.

Since 4 qr. = 1 cwt., $\frac{1}{4}$ of 100, or 25, equals the number of cwt.; and since 20 cwt. = 1 T., $\frac{1}{20}$ of 25 equals the number of tons, which is 1, with 5 cwt. remaining. Hence, 40325 oz. = 1 T. 5 cwt. 20 lb. 5 oz.

2. Reduce 1000 far. to pounds. *Ans.* 1 £ 10 d.

3. Reduce 12740 min. to weeks.

Ans. 1 wk. 1 d. 20 hr. 20 min.

4. Reduce 187620 min. to weeks. *Ans.* 18 wk. 4 d. 7 hr.

5. Reduce 26480 pch. to acres. *Ans.* 165 A. 80 P.

6. Reduce 126784 sq. in. to sq. yds.

Ans. 97 sq. yd. 7 sq. ft. 64 sq. in.

7. Reduce 186482 cu. ft. to cords.

Ans. 1456 cd. 114 cu. ft.

8. Reduce 200720" to degrees. *Ans.* 55° 45' 20".

9. Reduce 240600 gr. to pounds Troy.

Ans. 41 lb. 9 oz. 5 dwt.

10. Reduce 1200 Ⓓ to pounds. *Ans.* 4 lb 2 3.

11. Reduce 796863 lb. to tons. *Ans.* 398 T. 8 cwt. 63 lb.

12. Reduce 8000 gr. to pounds. *Ans.* 1 lb 4 3 5 3 1 Ⓓ.

13. Reduce 24717 cu. in. to wine gallons. *Ans.* 107.

14. If a cubic foot of stone weigh 160 lb., what will a cubic yard weigh? *Ans.* 4320 lb.

15. What will a ton of coal cost at $\frac{1}{2}$ ct. a pound?

Ans. \$10.

16. A ton of feed at \$1.15 per cwt. will cost how much?

Ans. \$23.

17. What will a ton of hay cost at $1\frac{1}{2}$ cts. a pound?

Ans. \$24.

18. What is the cost of a load of wheat weighing 7920 lb., at \$1.30 per bushel?

Ans. \$171.60.

19. If a man save 5 cents per hour, how much would he save in the first four months of 1876?

Ans. \$145.20.

20. At 11 cents a pound, how much sugar can be bought for \$5390?

Ans. $24\frac{1}{2}$ tons.

21. How many cubic feet in a box 27 in. long, 18 in. deep and 20 in. wide?

Ans. 5 cu. ft. 1080 cu. in.

22. What is the cost of 12 bbl. of flour at 4 cts. a pound?

Ans. \$94.08.

23. A man retails some oil at 12 cts. a pint: how much does he make on 3 bbl. of 36 gal. each, costing him 60 cts. a gallon?

Ans. \$38.88.

SECTION IV.

COMPOUND NUMBERS.

1. ADDITION OF COMPOUND NUMBERS.

132. Addition of Compound Numbers is the process of finding the sum of two or more similar compound numbers.

WRITTEN PROBLEMS.

1. Find the sum of 16 £ 13 s. 7 d., 125 £ 12 s. 6 d., 116 £ 9 s. 5 d., 12 £ 10 s. 8 d.

Ans. 271 £ 6 s. 2 d.

PROCESS.

16 £	13 s.	7 d.
125	12	6
116	9	5
12	10	8
271 £	6 s.	2 d.

Explanation.—We write the numbers in a vertical column, so that units of the same denomination shall be found in the same column. The sum of the first right-hand column is 26 d. = 2 s. 2 d.; writing the 2 d. in the column of pence and adding the 2 s. to the column of shillings, the sum of the second column is 46 s. = 2 £ 6 s.; writing the 6 s. in the column of shillings and adding the 2 £ to the column of pounds, the sum of the third column is 271 £. Hence, the sum of the numbers is 271 £ 6 s. 2 d.

From the solution of this and similar problems we derive the following rules for the addition of compound numbers:

RULES.

1. *Write the numbers so that units of the same denomination shall stand in the same column.*

2. *Beginning with the lowest denomination, add as in simple numbers, and reduce the sum to the next higher denomination, writing the remainder, if any, under the column added, and adding the quotient, obtained by the reduction, to the next column.*

3. *Proceed in the same manner with all the columns to the last.*

NOTE.—If any places are wanting, supply ciphers.

What is the sum of the following:

(1.)

£	s.	d.
13	10	3
16	18	10
10	10	11

(2.)

£	s.	d.
17	17	2
116	17	5
149	18	9

(3.)

£	s.	d.
18	11	9
941	10	11
7	18	9

(4.)

£	s.	d.
0	17	6
3	4	7
16	11	12
75	3	3

(5.)

bu.	pk.	qt.	pt.
37	1	3	1
34	2	5	0
16	2	7	1
41	1	3	1

(6.)

pk.	qt.	pt.
3	6	0
2	4	1
3	4	1
2	6	0

(7.)

yd.	ft.	in.
3	2	9
7	1	11
6	0	10
8	2	5

(8.)

d.	hr.	min.
7	9	13
16	10	20
11	12	30
15	18	53

(9.)

lb.	oz.	dwt.
16	3	8
19	10	12
14	8	7
16	12	12

(10.)				(11.)				
3	3	9	gr.	cwt.	qr.	lb.	oz.	dr.
10	7	2	12	19	2	12	3	6
3	4	1	19	25	3	19	14	10
1	11	2	12	75	2	24	15	11
8	9	1	13	106	1	20	12	9

12. Add 20 T. 15 cwt. 8 oz.; 5 T. 8 cwt. 50 lb.; 8 cwt. 15 oz.; 1 ton 17 cwt. 40 lb. 5 oz.; 10 T. 27 lb. 12 oz.

Ans. 38 T. 9 cwt. 19 lb. 8 oz.

13. Add 16 bu. 2 pk. 6 qt.; 7 bu. 3 qt. 1 pt.; 12 bu. 7 qt. 2 pt.; 7 bu. 3 pk. 4 qt. 1 pt.; 12 bu. 3 pk. 4 qt.

Ans. 56 bu. 3 pk. 2 qt.

14. What is the sum of 17 lb. 8 oz. 12 pwt. 13 gr. Troy; 12 lb. 2 oz. 9 pwt. 15 gr.; 1 lb. 11 oz. 19 pwt. 16 gr.; and 12 lb. 3 oz. 18 pwt. 10 gr.?

Ans. 44 lb. 3 oz. 6 gr.

15. Add 13 mi. 3 fur. 39 rd. 1 yd. 7 in.; 7 mi. 7 fur. 32 rd. 2 ft. 8 in.; 15 mi. 6 fur. 23 rd. 1 yd. 1 ft. 6 in.; 23 mi. 7 fur. 28 rd. 1 ft. 9 in.; 30 mi. 5 fur. 27 rd. 2 ft. 4 in.

Ans. 91 mi. 7 fur. 29 rd. 4 yd. 2 ft. 10 in.

16. Add 71 T. 13 cwt. 1 qr. 16 lb. 11 oz.; 14 T. 13 cwt. 3 qr. 13 lb. 14 oz.; 14 T. 17 cwt. 2 qr. 15 oz.; 13 T. 17 cwt. 1 qr. 15 lb. 13 oz.; 1 T. 6 cwt. 2 qr. 13 oz.

Ans. 116 T. 8 cwt. 2 qr. 23 lb. 2 oz.

17. Add 12 mi. 5 fur. 39 rd. 12 ft. 7 in.; 16 mi. 6 fur. 18 rd. 5 ft. 7 in.; 10 mi. 5 fur. 32 rd. 4 ft. 6 in.

Ans. 40 mi. 2 fur. 10 rd. 2 yd. 2 in.

18. A farmer raised in four fields the following quantities of wheat: 75 bu. 2 pk. 3 qt.; 108 bu. 1 pk. 1 pt.; 600 bu. 1 pt.; and 106 bu. 3 pk. 7 qt.: how much did he raise in the four fields?

Ans. 890 bu. 3 pk. 3 qt.

19. How far is it from A, which is east of a given point 3 mi. 3 fur. 9 rd. 3 yd., to B, which is west of the same point 6 mi. 1 fur. 2 yd. 6 in.?

Ans. 9 mi. 4 fur. 9 rd. 5 yd. 6 in.

20. How long is it from 3 hr. 16 min. and 6 sec. before 12 o'clock to 15 min. past 9 o'clock in the evening?

Ans. 12 hr. 31 min. 6 sec.

21. The four sides of a field measure as follows: 1st, 16 rd. 3 yd. 2 ft. 9 in.; 2d, 18 rd. 3 in.; 3d, 12 rd. 4 yd. 1 ft.; 4th, 15 rd. 2 yd. 2 ft. 11 in.: how far is it around the field?

Ans. 63 rd. 11 in.

2. SUBTRACTION OF COMPOUND NUMBERS.

133. Subtraction of Compound Numbers is the process of finding the difference between two similar compound numbers.

EXAMPLE.—From 120 lb. 6 oz. 13 dwt. 20 gr. take 75 lb. 9 oz. 8 dwt. 22 gr.

SOLUTION.

120 lb. 6 oz. 13 dwt. 20 gr.
 75 " 9 " 8 " 22 "
 44 lb. 9 oz. 4 dwt. 22 gr.

Explanation.—We write the numbers as in addition of compound numbers, and begin to subtract at the lowest denomination. We cannot take 22 gr. from 20 gr.; we therefore take 1 dwt., equal to 24 gr., which added to 20 gr. equals 44 gr.; 22 gr. from 44 gr. leave 22 gr.; 8 dwt. from 12 dwt. leave 4 dwt. We cannot take 9 oz. from 6 oz.; we therefore convert 1 lb. to ounces; 12 oz. and 6 oz. are 18 oz.; 9 oz. from 18 oz. leave 9 oz.; 75 lb. from 119 lb. leave 44 lb.

NOTE.—Let the pupil derive the **RULE**.

(1.)

From 35 £ 6 s. 10 d. 3 far.
 Take 13 " 18 " 3 " 1 "

(2.)

23 £ 14 s. 3 d. 1 far.
 15 " 14 " 8 " 3 "

(3.)

From 18 lb. 11 oz. 16 dwt. 18 gr.
 Take 12 " 10 " 18 " 15 "

(4.)

116 lb. 8 oz. 19 dwt. 16 gr.
 80 " 6 " 14 " 18 "

(5.)

From 117 yr. 6 mo. 3 wk. 2 d.
 Take 109 " 8 " 7 " 5 "

(6.)

6 m. 110 rd. 13 ft.
 2 " 60 " 15 "

(7.)

From 5 bbl. 16 gal. 1 qt. 1 pt.
 Take 2 " 20 " 3 " 0 "

(8.)

3 bu. 2 pk. 5 qt. 1 pt.
 1 " 3 " 7 " 1 "

9. From 16 cu. yd. 3 cu. ft. 1200 cu. in. take 3 cu. yd. 12 cu. ft. 1500 cu. in. *Ans.* 12 cu. yd. 17 cu. ft. 1428 cu. in.

10. From 16 bu. take 3 bu. 3 pk. 3 qt.

Ans. 12 bu. 5 qt.

11. From 20 T. 17 cwt. 40 lb. 5 oz. take 5 T. 15 cwt. 7 lb. 10 oz.

Ans. 15 T. 2 cwt. 32 lb. 11 oz.

12. A barrel contained 36 gal. of vinegar; from it was sold 13 gal. 2 qt. 1 pt. : how much remained?

Ans. 22 gal. 1 qt. 1 pt.

13. A miller had 600 bu. 2 pk. 3 qt. of wheat; he ground into flour 276 bu. 3 pk. 1 pt. : how much had he remaining?

Ans. 323 bu. 3 pk. 2 qt. 1 pt.

14. From 6 yd. take 2 ft.

Ans. 5 yd. 1 ft.

15. A man had 107 bu. 3 pk. of wheat; he bought 190 bu. 1 pt., and sold 250 bu. 2 pk. 5 qt. : how much had he remaining?

Ans. 47 bu. 3 qt. 1 pt.

16. A grocer bought 10 bbl. of sugar of 206 lb. each, and sold 1615 lb. 6 oz. : how much had he remaining?

Ans. 444 lb. 10 oz.

17. Three piles of wood contain, respectively, 16 cd. 3 cu. ft.; 19 cd. 3 cu. ft.; 27 cd. 18 cu. ft.; there were sold from them 50 cd. 3 cu. ft.; how much remained? *Ans.* 12 cd. 21 cu. ft.

18. A farm contains 250 A. 16 pch., of which 121 A. 3 pch. 9 sq. yd. are cleared : how much woodland is there yet on the farm?

Ans. 129 A. 12 pch. 21¼ sq. yd.

TO FIND THE DIFFERENCE OF DATES.

A man was born Feb. 8, 1825, and died Oct. 6, 1870 : how old was he?

SOLUTION.

yr.	mo.	d.
1870	10	6
1825	2	8
45	7	28

Remark.—Let the pupil give the explanation of this solution.

NOTE.—In finding the difference of time, 30 d. equal 1 mo., and 12 mo. equal 1 yr.

WRITTEN PROBLEMS.

1. What is the difference in time between Mar. 28, 1840, and Feb. 2, 1876? *Ans.* 35 yr. 10 mo. 4 d.
2. Franklin was born Jan. 6, 1706, and died Apr. 17, 1790: how old was he? *Ans.* 84 yr. 3 mo. 11 d.
3. Milton was born Dec. 9, 1608: how long was that before Franklin's birth? *Ans.* 97 yr. 27 d.
4. How long was it from Sept. 17, 1839, to Mar. 28, 1871? *Ans.* 31 yr. 6 mo. 11 d.
5. How long does a note run if dated Jan. 16, 1873, and made payable June 10, 1876? *Ans.* 3 yr. 4 mo. 24 d.
6. A gentleman went to Europe May 16, 1874, leaving the station at 9 o'clock 45 min. in the forenoon, and returned to the same station at 10 o'clock 20 min. in the evening, Feb. 18, 1875: how long was he gone? *Ans.* 9 mo. 2 d. 12 hr. 35 min.
7. Find the exact number of days from Oct. 16, 1875, to Jan. 12, 1876. *Ans.* 88 d.
8. Find the exact number of days from Oct. 16, 1875, to Mar. 3, 1876. *Ans.* 139 d.
9. How many years, months and days are you old?
10. What is the time from 15 min. past 2 o'clock in the afternoon to 10 min. before 12 at night? *Ans.* 9 hr. 35 min.

3. MULTIPLICATION OF COMPOUND NUMBERS.

134. Multiplication of Compound Numbers is the process of finding the product of two numbers, one of which is compound.

Multiply 7 bu. 3 pk. 5 qt. by 6.

SOLUTION.		Explanation.—	
7 bu. 3 pk. 5 qt.		6 times 5 qt. are	30 qt., or 3 pk.
		and 6 qt.; 6 times 3 pk. are	18 pk., which added
	6	to 3 pk. are	21 pk., or 5 bu. and 1 pk.; 6 times 7
47 bu. 1 pk. 6 qt.		bu. are	42 bu., which added to 5 bu. are 47 bu.

Remark.—Let the pupil derive a rule from the analysis of the foregoing and similar problems.

WRITTEN PROBLEMS.

(1.)	(2.)
19 cwt. 3 qr. 10 lb. 8 oz.	600 mo. 10 d. 3 hr. 14 min. 16 sec.
Multiplied by 6	7

(3.)	(4.)
3 mi. 2 fur. 16 rd. 5 yd. 2 ft.	16 £ 8 s. 3 d. 2 far.
12	16

5. A man has seven loads of wheat, each containing 73 bu. 2 pk. : how much wheat has he? *Ans.* 514 bu. 2 pk.

6. Multiply 15 gal. 3 qt. 1 pt. 2 gi. by 15.
Ans. 239 gal. 2 gi.

7. What will 24 spoons weigh if each weigh 2 oz. 9 dwt. 16 gr.? *Ans.* 59 oz. 12 dwt.

8. A hogshead of sugar weighs 6 cwt. 3 qr. 10 lb. 2 oz. : how much will 8 hhd. weigh? *Ans.* 54 cwt. 3 qr. 6 lb.

9. If it take 3 d. 6 hr. 15 min. 10 sec. to build a machine, how long will it require to build 9 machines at the same rate?
Ans. 29 d. 8 hr. 16 min. 30 sec.

10. A merchant bought goods in England at 3 £ 9 s. 6½ d. a box : how much was paid at the same rate for 25 boxes?
Ans. 86 £ 18 s. 6½ d.

11. If wine cost \$1.23 a gallon, how much will 10 casks cost, each containing 29 gal. 3 qt. 1 pt. *Ans.* \$367.46.

12. If a man's expenses are 1 £ 10 s. 6 d. a day, what are his expenses in a year of 365 days? *Ans.* 556 £ 12 s. 6 d.

13. If an acre of ground produce 45 bu. 3 pk. 2 qt. of corn, how much will 65 acres produce?
Ans. 2977 bu. 3 pk. 2 qt.

4. DIVISION OF COMPOUND NUMBERS.

135. Division of Compound Numbers is the process of dividing one number by another, the dividend always being compound.

Divide 35 £ 9 s. 7 d. by 6.

SOLUTION. *Explanation.*—Write the divisor on the left of the
 $6 \overline{) 35 \text{ £ } 9 \text{ s. } 7 \text{ d.}}$ dividend, and proceed in this problem to find $\frac{1}{6}$ of
 5 £ 18 s. $3\frac{1}{2}$ d. the number. $\frac{1}{6}$ of 35 £ is 5 £ and a remainder of
 5 £, or 100 s.; 100 s. plus 9 s. are 109 s.; $\frac{1}{6}$ of 109 s. is 18 s., and a re-
 mainder of 1 s., or 12 d.; 12 d. plus 7 d. are 19 d.; $\frac{1}{6}$ of 19 d. is $3\frac{1}{2}$ d.

Perform the following divisions :

$\begin{array}{r} (1.) \\ 7 \overline{) 40 \text{ bu. } 3 \text{ pk. } 6 \text{ qt.}} \\ \underline{5 \text{ bu. } 3 \text{ pk. } 3\frac{1}{2} \text{ qt.}} \end{array}$	$\begin{array}{r} (2.) \\ 8 \overline{) 107 \text{ bu. } 2 \text{ pk. } 6 \text{ qt.}} \\ \underline{13 \text{ bu. } 1 \text{ pk. } 6\frac{1}{2} \text{ qt.}} \end{array}$
$\begin{array}{r} (3.) \\ 7 \overline{) 32 \text{ T. } 14 \text{ cwt. } 0 \text{ qr. } 15 \text{ lb.}} \end{array}$	$\begin{array}{r} (4.) \\ 12 \overline{) 118 \text{ mi. } 1 \text{ fur. } 0 \text{ rd. } 12 \text{ ft.}} \end{array}$
$\begin{array}{r} (5.) \\ 5 \overline{) 62 \text{ £ } 7 \text{ s. } 9 \text{ d. } 3 \text{ far.}} \end{array}$	$\begin{array}{r} (6.) \\ 8 \overline{) 79 \text{ cwt. } 2 \text{ qr. } 18 \text{ lb.}} \end{array}$

7. If 9 men earn 116 £ 10 s. 10 d., how much can one man earn?
Ans. 12 £ 18 s. $11\frac{2}{3}$ d.

8. If 15 casks hold 400 gal. 3 qt. 1 pt., how much will one of these casks hold?
Ans. 26 gal. 2 qt. $1\frac{1}{3}$ pt.

9. If 19 turkeys weigh 3 cwt. 32 lb. 8 oz., how much will one turkey weigh?
Ans. 17 lb. 8 oz.

10. A man bought 8 T. 5 cwt. 3 qr. 17 lb. of hay, and hauled it in 6 loads: what was the average weight of the loads?
Ans. 1 T. 7 cwt. 2 qr. $15\frac{1}{2}$ lb.

NOTE.—When both divisor and dividend are concrete numbers, reduce both to the lowest denomination in either, and divide as in simple numbers.

Ex. Divide 24 £ 3 s. 8 d. by 6 £ 11 d.

$$\text{SOLUTION.} \quad \frac{24 \text{ £ } 3 \text{ s. } 8 \text{ d.}}{6 \text{ £ } 11 \text{ d.}} = \frac{5804 \text{ d.}}{1451 \text{ d.}} = 4.$$

11. Divide 80 £ 8 s. by 2 £ 10 s. 3 d. *Ans.* 32.

12. A merchant has 181 cwt. 1 qr. 19 lb. of coal: how many loads of 12 cwt. 3 qr. 21 lb. each has he? *Ans.* 14.

13. A farmer has 35 A. 1 R. 16 P. of land divided into fields of 4 A. 1 R. 27 P. each: how many fields are there? *Ans.* 8.

14. How many bales of hay, each weighing 5 cwt. 3 qr. 16 lb., in 4 T. 14 cwt. 2 qr. 6 lb.? *Ans.* 16.

NOTE.—When the divisor is composite the work may be shortened by dividing by its factors.

PRACTICAL PROBLEMS FOR REVIEW.

1. A farmer has four fields containing respectively the following areas: 16 A. 17 P.; 9 A. 57 P.; 5 A. 25 P.; 11 A. 102 P.: how much land has he? *Ans.* 42 A. 41 P.

2. A man had 640 acres of land; he gave his son 163 A. 9 P., and his daughter 172 A. 10 P.: how much has he remaining? *Ans.* 304 A. 141 P.

3. A man travels 127 mi. 120 rd. a day for 15 days: how much does he lack of having travelled 2500 miles? *Ans.* 589 mi. 120 rd.

4. If 16 men gather 150 bu. 3 pk. of oysters, how many can one man gather? *Ans.* 9 bu. 1 pk. 5 qt. 1 pt.

5. If 4 yd. of cloth cost 5 £ 4s. 8d., how much will 6 yd. cost? *Ans.* 7 £ 17 s.

SECTION V.

LONGITUDE AND TIME.

136. Longitude is the distance east or west from a given meridian, measured on the equator.

The **Meridian** from which the distance is reckoned is marked 0°; and from this, either east or west, longitude is reckoned by degrees from 0 up to 180.

The English and the Americans reckon from the meridian passing through Greenwich, England; the French, from the meridian of Paris.

Since the earth revolves from west to east in 24 hours, the sun appears to pass around the earth, or through 360 degrees of longitude, in the same space of time, or through 15 degrees of longitude in *one hour* of time. In 1 minute of time it

passes through $\frac{1}{60}$ of 15° , or $\frac{1}{4}$ of a degree, equal to $15'$ of longitude, and in 1 second of time through $15''$ of longitude.

For a difference of	There is a difference of
15° in longitude	1 hour in time.
$15'$ " "	1 minute "
$15''$ " "	1 second "

Or,

1° in longitude is equivalent to 4 minutes in time.

$1'$ " " " 4 seconds "

$1''$ " " " $\frac{1}{6}$ second "

Places east of a given meridian are said to have faster time, because the sun appears to them earlier or passes over their meridian earlier; and, for a similar reason, places west are said to have slower time.

When one of two places is in east and the other in west longitude, the difference is found by adding the longitude of the two places; but if both are east or both west, the difference is found by subtracting.

ORAL EXERCISE.

1. What part of a revolution does the earth make in 6 hr.? In 12 hr.?
2. How many degrees of longitude are equivalent to 1 hr.? To 6 hr.? To 10 hr.?
3. When it is noon at Philadelphia, what is the time 15° east? 15° west? 30° west?
4. If the difference in the longitude of two places is 60° , what is the difference of their time?
5. If the difference in time between two places is 3 hr., what is their difference in longitude?
6. What difference of longitude causes a difference of 4 min. in time? Of 1 min.? Of 1 hr.?
7. A man leaves his home at noon, and travelling 5 days finds his watch 1 hr. slow: what direction and how many degrees of longitude is he from home?

8. If a man travel a month, and find his watch $1\frac{1}{2}$ hr. fast, how many degrees of longitude and in what direction is he from home?

9. Two men see an eclipse at the same moment; one's watch marks 10 o'clock, the other's 12 o'clock 15 min.: how far apart are they, and which is farthest east?

10. What is the difference in the time of sunrise between two places 110° apart?

WRITTEN PROBLEMS.

11. The longitude of Cincinnati is $84^\circ 26'$ W., and that of Philadelphia is $75^\circ 10'$ W.: what is the difference in time?

Ans. 37 min. 4 sec.

12. Berlin is in $13^\circ 23' 53''$ E. longitude: what is the difference in longitude between Berlin and Philadelphia?

Ans. $88^\circ 33' 53''$.

13. What is the difference in time between Berlin and Philadelphia?

Ans. 5 hr. 54 min. $15\frac{8}{15}$ sec.

14. When it is noon at Berlin, what is the time at Philadelphia?

Ans. 6 hr. 5 min. $44\frac{7}{15}$ sec. A. M.

15. The longitude of San Francisco is $122^\circ 26' 15''$ W. When it is noon at San Francisco, what is the time at Berlin?

Ans. 9 hr. 3 min. $20\frac{8}{15}$ sec. P. M.

16. When it is noon at Philadelphia, what is the time at San Francisco?

Ans. 8 hr. 50 min. 55 sec. A. M.

17. The difference in the time between two places is 2 hr. 3 min. 5 sec.: what is the difference in longitude?

Ans. $30^\circ 46' 15''$.

18. Two men starting at different points travel till they meet; one finds his watch 1 hr. 10 min. fast, and the other 2 hr. 6 min. slow: how far and in what direction did each travel?

Ans. 1st, $17^\circ 30'$ W.; 2d, $31^\circ 30'$ E.

19. The difference in the time between two points is 50 min. 30 sec.; the longitude of the one having the slower time is $75^\circ 10'$ W.: what is the longitude of the other point?

Ans. $62^\circ 32' 30''$ W.

20. How much earlier does the sun rise at Philadelphia than at Cincinnati, which is in $84^{\circ} 26' W.$ lon.?

Ans. 37 min. 4 sec.

SECTION VI.

REDUCTION OF DENOMINATE FRACTIONS.

137. A **Denominate Fraction** is one whose unit is a denominate number; as $\frac{1}{2}$ lb., $\$ \frac{3}{4}$, etc.

CASE I.

To Reduce a Common Denominate Fraction to a Lower Denomination.

Ex.—Reduce $\frac{1}{10}$ yd. to a lower denomination.

PROCESS.— $\frac{1}{10}$ in. $\times \frac{1}{4} \times \frac{1}{2} = \frac{1}{80}$ in. $= 3 \frac{3}{4}$ in.

Explanation.—Since there are 3 ft. in a yard, in $\frac{1}{10}$ yd. there are *three* times as many feet as yards; and since in 1 ft. there are 12 in., there are 12 times as many inches as feet: hence, in $\frac{1}{10}$ yd. there are $12 \times 3 \times \frac{1}{10}$ in., or $3 \frac{3}{4}$ in.

PROBLEMS.

- | | |
|---|-----------------------------------|
| 1. Reduce $\frac{3}{4}$ gal. to pints. | <i>Ans.</i> $\frac{1}{2}$ pt. |
| 2. Reduce $\frac{1}{2000}$ T. to pounds. | <i>Ans.</i> $\frac{1}{2}$ lb. |
| 3. Reduce $\frac{3}{8}$ bu. to pints. | <i>Ans.</i> $\frac{3}{4}$ pt. |
| 4. Reduce $\frac{1}{8}$ mi. to yards. | <i>Ans.</i> $29 \frac{1}{2}$ yd. |
| 5. Reduce $\frac{5}{8400}$ A. to square feet. | <i>Ans.</i> $\frac{5}{8}$ sq. ft. |
| 6. Reduce $\frac{1}{360}$ wk. to hours. | <i>Ans.</i> $4 \frac{2}{3}$ hr. |
| 7. Reduce $\frac{1}{8}$ bu. to quarts. | <i>Ans.</i> $4 \frac{1}{2}$ qt. |
| 8. Reduce $\frac{1}{80}$ T. to pounds. | <i>Ans.</i> $87 \frac{1}{2}$ lb. |
| 9. Reduce $\frac{1}{16}$ oz. Troy to grains. | <i>Ans.</i> $\frac{3}{4}$ gr. |

CASE II.

To Reduce a Common Denominate Fraction to Integers.

Ex.—What is the value of $\frac{3}{4}$ of a mile?

PROCESS.— $\frac{3}{4} \times 8$ fur. $= \frac{3}{1} \times 8$ fur. $= 5 \frac{1}{2}$ fur.

$\frac{1}{4} \times 40$ rd. $= \frac{1}{1} \times 40$ rd. $= 13 \frac{1}{2}$ rd.

$\frac{1}{2} \times \frac{1}{2}$ yd. $= \frac{1}{1} \times \frac{1}{2}$ yd. $= 1 \frac{1}{2}$ yd.

$\frac{3}{4} \times 3$ ft. $= \frac{3}{1} \times 3$ ft. $= 2 \frac{3}{4}$ ft.

$\frac{3}{4} \times 12$ in. $= \frac{3}{1} \times 12$ in. $= 6$ in.

Explanation.—Since there are 8 fur. in a mile, in $\frac{3}{4}$ mi. there are $\frac{3}{4}$ of

8 fur., or $5\frac{1}{2}$ fur.; since there are 40 rd. in a furlong, in $\frac{1}{2}$ of a furlong there is $\frac{1}{2}$ of 40 rd., or $13\frac{1}{2}$ rd.; since there are $5\frac{1}{2}$ or $1\frac{1}{2}$ yd. in a rod, in $\frac{1}{2}$ of a rod there is $\frac{1}{2}$ of $1\frac{1}{2}$ yd., or $1\frac{1}{4}$ yd.; since there are 3 ft. in a yard, in $\frac{1}{4}$ of a yard there are $\frac{1}{4}$ of 3 ft., or $2\frac{3}{4}$ ft.; since there are 12 in. in a foot, in $\frac{3}{4}$ of a foot there are $\frac{3}{4}$ of 12 in., or 6 in.: hence, $\frac{3}{4}$ mi. is equal to 5 fur. 13 rd. 1 yd. 2 ft. 6 in.

PROBLEMS.

1. What is the value of $\frac{3}{4}$ A.? *Ans.* 96 sq. rd.
2. What is the value of $\frac{3}{4}$ gal.? *Ans.* 2 qt. $\frac{1}{2}$ pt.
3. What is the value of $\frac{1}{2}$ T.? *Ans.* 3 cwt. 1 qr. 8 lb. $5\frac{1}{2}$ oz.
4. What is the value of $\frac{1}{2}$ lb. Troy? *Ans.* 2 oz. 8 dwt.
5. What is the value of $\frac{3}{4}$ bu.? *Ans.* 3 pk. 2 qt. $1\frac{1}{2}$ pt.
6. What is the value of $\frac{1}{2}$ cd.? *Ans.* 2 cd. ft. 10 cu. ft. 1152 cu. in.
7. What is the value of $\frac{1}{2}$ mi.? *Ans.* 6 fur. 8 rd. 4 yd. 2 ft. 8 in.

CASE III.

To Reduce a Denominate Decimal Fraction to a Lower Denomination.

Ex.—Reduce .648 T. to a lower denomination.

PROCESS.	Explanation.
.648	Since there are 20 cwt. in a ton, in .648 T.
20 cwt.	there are $.648 \times 20$ cwt., or 12.960 cwt.; since there are 4
12) .960	qrs. in a cwt., in .960 cwt. there are .960 times 4 qr., or
4 qr.	3.840 qr.; since there are 25 lb. in a quarter, in .840 qr.
3) .840	there are .840 times 25 lb., or 21 lb.: hence, .648 T. equal
25 lb.	12 cwt. 3 qr. 21 lb.
21.	

PROBLEMS.

Find the value—

1. Of .9375 gal. *Ans.* 3 qt. 1 pt. 2 gi.
2. Of .965625 mi. *Ans.* 7 fur. 29 rd.
3. Of .323 T. *Ans.* 6 cwt. 1 qr. 21 lb.
4. Of .8 bu. *Ans.* 3 pk. 1 qt. 1.2 pt.
5. Of .875 rd. *Ans.* 4 yd. 2 ft. 5.25 in.
6. Of .976 A. *Ans.* 156 sq. rd. 4 sq. yd. 7 sq. ft. 80.64 sq. in.

CASE IV.

To Reduce a Common Denominate Fraction to a Higher Denomination.

Ex. Reduce $\frac{3}{8}$ gi. to the fraction of a gallon.

PROCESS.— $\frac{3}{8} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{3}{128}$, or $\frac{3}{128}$ gal.

Explanation.—Since there are 4 gills in a pint, $\frac{1}{4}$ of the number of gills equals the number of pints; for a similar reason, $\frac{1}{4}$ of the number of pints equals the number of quarts, and $\frac{1}{4}$ of the number of quarts equals the number of gallons.

PROBLEMS.

1. Reduce $\frac{3}{4}$ far. to the fraction of a pound. *Ans.* $\frac{1}{1280}$ £.
2. Reduce $\frac{3}{4}$ pt. to the fraction of a bushel. *Ans.* $\frac{3}{896}$ bu.
3. Reduce $\frac{5}{8}$ of a dram avoirdupois to the fraction of a pound. *Ans.* $\frac{5}{2048}$ lb.
4. Reduce $\frac{3}{8}$ gi. to the fraction of a gallon. *Ans.* $\frac{3}{128}$ gal.
5. Reduce $\frac{3}{4}$ lb. to the fraction of a ton. *Ans.* $\frac{3}{8000}$ T.
6. Reduce .625 lb. to the fraction of a ton. *Ans.* $\frac{1}{1600}$ T.
7. Reduce 3.16 gi. to the fraction of a gallon. *Ans.* $\frac{79}{800}$ gal.

NOTE.—Write decimals as common fractions, and solve as the others.

CASE V.

To Reduce Denominate Numbers to Common Fractions.

Ex. Reduce 3 pk. 6 qt. to the fraction of a bushel.

PROCESS.— $\frac{3 \text{ pk. } 6 \text{ qt.} = 30 \text{ qt.}}{1 \text{ bu.} = 32 \text{ qt.}} = \frac{15}{16}$ bu.

Remark.—Let the pupil give the explanation.

PROBLEMS.

1. Reduce 10 oz. 6 dr. to the fraction of a pound. *Ans.* $\frac{83}{128}$ lb.
2. Reduce 3 pk. 4 qt. 1 pt. to the fraction of a bushel. *Ans.* $\frac{57}{64}$ bu.
3. Reduce 9 cwt. 1 qr. 6 lb. to the fraction of a ton. *Ans.* $\frac{231}{2000}$ T.

4. Reduce 9 gal. 3 qt. 1 pt. of wine to the fraction of a barrel.
Ans. $\frac{79}{57}$ bbl.
5. What part of a mile is 3 fur. 10 rd. 2 yd.?
Ans. $\frac{717}{1780}$ mi.
6. What part of 5 bu. is 3 bu. 2 pk. 1 pt.?
Ans. $\frac{41}{4}$.

CASE VI.

To Reduce Denominate Numbers to Denominate Decimal Fractions.

Ex. Reduce 6 cwt. 1 qr. 20 lb. to the decimal of a ton.

$$\begin{aligned}\text{SOLUTION 1.}—6 \text{ cwt. 1 qr. 20 lb.} &= 645 \text{ lb.} \\ 1 \text{ T.} &= 2000 \text{ lb.} \\ \frac{645}{2000} \text{ T.} &= .3225 \text{ T.}\end{aligned}$$

Explanation.—6 cwt. 1 qr. 20 lb. = 645 lb.; 1 T. = 2000 lb.: hence, 6 cwt. 1 qr. 20 lb. = $\frac{645}{2000}$ of a ton, or .3225 ton.

SOLUTION 2. *Explanation.*—Since there are 25 lb. in a quarter, $\frac{1}{4}$ of the number of pounds equals the number of quarters;
 25)20.
 4)1.8
 20)6.45
 .3225
 $\frac{1}{4}$ of 20 is .8, which with 1 qr. equals 1.8 qr. Since there are 4 qr. in 1 cwt., $\frac{1}{4}$ of the number of quarters equals the number of hundredweight; $\frac{1}{4}$ of 1.9 = .45; .45 cwt. plus 6 cwt. = 6.45 cwt. For a similar reason as before, $\frac{1}{20}$ of the number of hundredweight equals the number of tons; $\frac{1}{20}$ of 6.45 = .3225.

NOTE.—The first of these solutions is similar to that in *Case V.*, with the common fractions changed to decimals.

PROBLEMS.

1. Reduce 6 qt. 1 pt. to the decimal of a peck.
Ans. .8125 pk.
2. Reduce 1 gal. 1 qt. 1 pt. to the decimal of a gallon.
Ans. 1.375 gal.
3. Reduce 90 rd. to the decimal of a mile. *Ans.* .28125 mi.
4. What part of a ton is 3 cwt. 6 lb. 3 oz.?
Ans. .15309375 T.
5. What part of 6 bu. 3 pk. 4 qt. is 2 bu. 1 pk. 2 qt.?
Ans. .33636+.
6. What part of 19 lb. 3 oz. 10 dr. is 10 lb. 1 oz. 8 dr.?
Ans. .52499—.

7. What part of 2 lb. 2 oz. avoirdupois is 2 lb. 2 oz. Troy?

Ans. .83899+.

8. What part of 6 ft. square is 6 sq. ft? *Ans.* .166+.

PRACTICAL PROBLEMS.

1. What cost 1 T. 2 cwt. 3 qr. and 5 lb. of hay at \$22 a ton?

Ans. \$25.08.

2. What will 12 horses cost if 5 horses cost 200 £ 10 s. 6 d.?

Ans. 481 £ 5 s. 2½ d.

3. How much will 3 loads of wheat, each containing 70 bu. 3 pk., cost, at \$1.50 a bushel? *Ans.* \$318.37½.

4. A man bought 19 T. 3 cwt. 10 lb. of coal at \$5.75 a ton: how much did it cost? *Ans.* \$110.14+.

5. What cost 6 bbl. of wine, each containing 32 gal. 2 qt. 1 pt., at \$2.10 a gallon? *Ans.* \$411.075.

6. A farmer had 1263 bu. 3 pk. of wheat; he used for flour 116 bu. 2 pk., and sold the remainder at \$1.30 a bushel: how much did he get for what he sold? *Ans.* \$1491.425.

7. If 12 bu. of wheat weigh 730 lb. 8 oz., how much will 10 bu. weigh? *Ans.* 608 lb. 12 oz.

8. A man has 310 A. 16 P. of land; he sells ¼ of it, and divides the remainder among his four sons: how much did he sell, and how much does each son get?

Ans. Sold, 77 A. 84 P.; sons, each 58 A. 23 P.

9. A marketman sells 18 lb. 6 oz. of butter at 25 cents a pound, and takes muslin at 12 cts. a yard in exchange: how much muslin does he get? *Ans.* 38.28125 yd.

10. Bought 106 T. 2 cwt. 3 qr. 10 lb. of iron at \$30 a ton: what did it cost me? *Ans.* \$3184.275.

11. A druggist bought 20 lb. 10 oz. avoirdupois of drugs at \$8.25 a pound: what was the cost? *Ans.* \$170.156+.

12. What cost 16.723 bu. of corn at 1½ cts. a pint?

Ans. \$16.05+.

13. A farmer had 160 A. 32 P. of land; he sold 112 A. 10 P. at \$112 an acre, and the remainder at \$120 an acre: how much did he get for all? *Ans.* \$18327.50.

14. If 3 T. 12 cwt. 10 lb. of coal cost \$17.304, how much will 10 T. 3 cwt. cost at the same rate? *Ans.* \$48.72.

15. A newspaper has a circulation of 1200 copies a week: how many reams of paper will be used in a year of 52 weeks? *Ans.* 130.

16. A book contains 312 pages 12mo: how much paper will it take to furnish 1000 copies?

Ans. 27 reams 1 quire 16 sheets.

17. A grocer's scales weigh one-fourth of an ounce short on every pound; he sells a barrel of sugar, 216 lb., at 15 cents a pound: how much does he gain by short weight?

Ans. \$.506+.

18. Bought 6 bu. 3 pk. 1 pt. of beans at \$2 a bushel, and gave sugar in exchange at 13 cents a pound: how much sugar did it take? *Ans.* 104.086+lb.

19. Bought 3 loads of hay; the first weighed 1 T. 3 cwt. 20 lb.; the second, 1.45 T.; the third, 18 cwt. 6 lb.: how much did it cost me at \$25 a ton? *Ans.* \$87.825.

20. A farmer received \$1.25 a bushel for wheat; he sold 5 loads as follows: 62.75 bu., 3672 lb., 68 bu. 3 pks., 73 bu. 18 lb., $52\frac{6}{7}$ bu.: how much did he get for all? *Ans.* \$398.

CHAPTER VIII.

RATIO.—PROPORTION.

SECTION I.

RATIO.

138. Ratio is the relation between two similar numbers. It is usually expressed by dividing the first by the second.

139. The Sign of Ratio is the colon (:), supposed to be the sign of division with the line omitted.

Thus, the ratio of 12 to 3 is expressed 12 : 3, or $12 \div 3$, or $\frac{12}{3}$,

and is read, "the ratio of 12 to 3," or "twelve divided by three."

140. The **Terms** of a ratio are the numbers compared.

141. The **Antecedent** is the first term, or the dividend.

142. The **Consequent** is the second term, or the divisor.

143. The **Value** of a ratio is the quotient obtained by dividing the *antecedent* by the *consequent*, and is always an abstract number.

144. The terms of a ratio form a **Couplet**.

ORAL EXERCISE.

1. What is the relation of 12 to 6? Of 18 to 3?
2. What is the relation of \$15 to \$5? 60 horses to 12 horses?
3. What is the ratio of 64 to 8? Of 64 to 16? Of 120 to 80?
4. What is the ratio of 100 to 25? Of 100 to 16? Of 100 to 8?
5. What is the ratio of 11 to 4? Of 12 to 5? Of 19 to 6?
6. What is the ratio of 4 to 8? Of 3 to 9? Of 9 to 45?
7. What is the ratio of 3 to 7? Of 7 to 50? Of 16 to 75?
8. Is there any ratio between \$9 and 3 books?

WRITTEN PROBLEMS.

1. What is the ratio of 5 to 17? *Ans.* $\frac{5}{17}$.
2. What is the ratio of 106 to 7? *Ans.* $15\frac{1}{7}$.
3. What is the ratio of 12 lb. 9 oz. to 4 lb. 3 oz. *Ans.* 3.
4. What is the ratio of 326 bu. 2 pk. to 40 bu. 3 pk. 2 qt.? *Ans.* 8.
5. What is the ratio of 191 lb. 10 oz. to 27 lb. 6 oz.? *Ans.* 7.
6. What is the ratio of 3 yd. 2 ft. 5 in. to 30 yd. 1 ft. 4 in.? *Ans.* $\frac{1}{3}$.
7. What is the ratio of $\frac{2}{3}$ to $\frac{1}{14}$? *Ans.* 12.
8. What is the ratio of $\frac{1}{3}$ to $4\frac{3}{16}$? *Ans.* $\frac{2}{3}$.
9. What is the ratio of $6\frac{1}{2}$ to $7\frac{1}{4}$? *Ans.* $\frac{2}{3}$.
10. What is the ratio of $6\frac{1}{2} \times \frac{2}{3}$ to $4\frac{1}{3} \times 1\frac{1}{5}$? *Ans.* $\frac{1}{5}$.

11. The antecedent is 60, and the ratio 5: what is the consequent? *Ans.* 12.

12. The consequent is 9, the antecedent 81: what is the ratio? *Ans.* 9.

13. The consequent is 12, the ratio $5\frac{1}{2}$: what is the antecedent? *Ans.* 66.

14. What is the ratio of $\frac{3\frac{1}{2}}{6\frac{1}{4}}$ to $\frac{12\frac{1}{4}}{3\frac{1}{8}}$? *Ans.* $\frac{7\frac{1}{2}}{5\frac{1}{2}}$.

15. What is the ratio of $\frac{1}{2}$ of $\frac{7\frac{1}{2}}{\frac{1}{2}}$ to $\frac{1}{4}$ of $\frac{16\frac{1}{2}}{\frac{2}{3}}$? *Ans.* $16\frac{1}{2}$.

SECTION II.

PROPORTION.

145. Proportion is an expression of equality between ratios.

The equality of ratios is expressed in two ways—either by the sign of equality or by the double colon ($::$). Thus, to show that the ratio of 9 to 3 equals the ratio of 12 to 4, we may write it—

$$9 : 3 = 12 : 4,$$

or, $9 : 3 :: 12 : 4$, and it is usually read 9 is to 3 as 12 is to 4.

146. Since a proportion represents an equality of ratios, there must be at least *two couplets* and at least *four terms*.

147. The first term of each couplet is called, as in Ratio, an *antecedent*, and the second in each couplet a *consequent*.

148. The first and fourth terms are called the *extremes*, and the second and third the *means*.

149. The first and second terms are the *first couplet*, and the third and fourth terms the *second couplet*.

PRINCIPLES.—Since in every proportion the ratio of the first couplet equals the ratio of the second couplet, the proportion $12 : 3 :: 8 : 2$ becomes $\frac{12}{3} = \frac{8}{2}$. Multiplying each member by 2 and 3, the denominators, the equation becomes $2 \times 12 = 3 \times 8$; hence, the following Principles:

1. In every proportion the product of the means equals the product of the extremes.

2. The product of the means divided by either extreme will give the other extreme.

3. The product of the extremes divided by either mean will give the other mean.

EXERCISE.

Find the value of the unknown term (x) in the following proportion: $3 : 2 :: 18 : x$.

PROCESS. $\frac{2 \times 18}{3} = 12.$ See Principle 2.

Find the unknown term in the following:

- | | |
|---|--|
| 1. $16 : 8 :: 36 : x.$ | 6. $4 \text{ yd.} : 10 \text{ yd.} :: 7 : x.$ |
| 2. $21 : 3 :: 60 : x.$ | 7. $4\frac{1}{2} : 1\frac{1}{2} :: 20 : x.$ |
| 3. $25 : 2 :: 60 : x.$ | 8. $7.5 : 18 :: 2.25 : x.$ |
| 4. $9 \text{ bu.} : 3 \text{ bu.} :: \$10 : x.$ | 9. $\frac{3}{4} : \frac{1}{2} :: \frac{5}{8} : x.$ |
| 5. $125 : 15 :: 140 : x.$ | 10. $36 : 19 :: 90 \text{ gal.} : x.$ |

11. $17 \text{ bu.} : 20 \text{ bu.} :: \$20 : x.$
12. $25 \text{ gal.} : 10 \text{ gal.} :: \$10 : x.$
13. $20 \text{ bu.} : 3 \text{ bu.} 2 \text{ pk.} :: \$18 : x.$
14. $3 \text{ gal.} 2 \text{ qt.} : 1 \text{ gal.} :: \$5 : x.$
15. $25 \text{ bbl.} : 1\frac{1}{2} \text{ bbl.} :: 45 : x.$

150. SIMPLE PROPORTION, sometimes called **Rule of Three**, is employed in the solution of problems where three of four terms are given, so related that from them may be found the fourth by a comparison of ratios.

ORAL EXERCISE.

1. If 6 pairs of shoes cost \$18, what will 8 pairs cost?

SOLUTION.—If 6 pairs cost \$18, 8 pairs will cost as many times \$18 as the ratio of 8 to 6, or $\frac{4}{3}$ of \$18, which is \$24.

2. If 3 oranges cost 10 cts., how much will 15 oranges cost?
3. If 6 boys can earn \$10 in a day, how much can 9 boys earn in the same time?

4. Five bbl. of sugar cost \$120: how much will 2 bbl. cost?
5. If 4 bu. of potatoes cost \$5, how much will 20 bu. cost?
6. If 7 bbl. of apples cost \$14, how much will 20 bbl. cost?
7. If a man can earn \$24 in 3 mo., how much can he earn in 10 mo.?

8. If 8 horses eat a quantity of hay in 6 wk., how long would the same quantity last 10 horses?

Remark.—It would last 10 horses a shorter time than 8 horses; it would therefore last as many times 6 wk. as the ratio of 8 to 10, or $\frac{8}{10} \times 6$ wk. = $4\frac{3}{5}$ wk., or $4\frac{6}{10}$ wk.

9. I paid 90 cts. for 3 bu. of apples: how much would 25 bu. cost at the same rate?

10. I paid 90 cts. for 3 bu. of apples: how many bushels could I buy for \$4.50?

11. If 10 hens cost \$3, how much will 60 hens cost?

12. If 10 hens cost \$3, how many hens can I buy for \$12?

13. If a tree 20 ft. high cast a shadow 50 ft., how long will be the shadow of a tree 50 ft. high?

14. If a tree 18 ft. high cast a shadow 45 ft., how high will be a tree that casts a shadow 135 ft.?

15. A family of 6 pay \$21 a week for boarding: what will be the cost for the boarding of a family of 8 at the same rate?

WRITTEN PROBLEMS.

1. If 12 lb. of sugar cost \$2, what will 30 lb. cost at the same rate?

Ans. \$5.

SOLUTION.

12 lb. : 30 lb. :: \$2 : x.

$$\begin{array}{r} 30 \\ 2 \\ \hline 12 \overline{)60} \\ 5, \text{ or } \$5. \end{array}$$

Explanation.—Since 12 lb. and 30 lb. have the same unit (*lb.*) we can compare them, and place them for the first couplet. For the same reason \$2, the cost of 12 lb., and the cost of 30 lb., will form the second couplet; then by Principle 2 we have $\$x = \frac{30 \times \$2}{12} = \$5$.

The following is the rule for *Simple Proportion*:

RULE.

Place for the third term that number which is of the same kind as the required answer.

When, from the nature of the question, the answer is to be greater than the third term, place the larger of the two numbers in the first couplet for the second term; and when the answer is to be smaller than the third term, place the smaller of the two numbers in the first couplet for the second term.

Multiply together the means and divide by the given extreme; the result will be the required answer.

2. If 20 bbl. of beef cost \$220, what will 12 bbl. cost?

Ans. \$132.

Remark.—Let the teacher ask questions somewhat like the following: What two numbers have the same unit in this problem? What is the unit of the third term? What is the unit of the required term? Should the answer be larger or smaller than the third term? If larger, which of the first couplet should be the larger, the antecedent or the consequent? If smaller, which? How then do we find the fourth term?

3. If 6 horses cost \$1250, what will 10 horses cost?

Ans. \$2083 $\frac{1}{3}$.

4. If 4 pairs of stockings cost \$1.20, what will 6 pairs cost?

Ans. \$1.80.

5. If 12 bu. of wheat cost \$15.60, what will 25 bu. cost?

Ans. \$32.50.

6. If 12 T. of hay cost \$264, what will 13 T. cost?

Ans. \$286.

7. What will 25 lb. of coffee cost if 19 lb. cost \$7.03?

Ans. \$9.25.

8. If 12 bu. of wheat weigh 726 lb., how much will a load of 73 bu. weigh?

Ans. 4416 $\frac{1}{2}$ lb.

9. A man can walk 125 mi. in 4 days: how far can he walk in 12 days?

Ans. 375 mi.

10. A clerk's expenses for 22 days are \$37.50: what will be his expenses for a year of 365 days?

Ans. \$622.16—.

11. If 6 $\frac{1}{2}$ cd. of wood cost \$14 $\frac{1}{2}$, how many can be bought for \$21?

Ans. 9 $\frac{1}{2}$ cd.

12. If 3 yd. of velvet cost $\$5\frac{1}{4}$, how much will $7\frac{1}{4}$ yd. cost?

Ans. $\$13.12\frac{1}{2}$.

13. When the tax on a property valued at $\$600$ is $\$53.25$, how much is the tax on a property valued at $\$1450$?

Ans. $\$128.69 -$.

14. If 3 cows eat $6\frac{1}{2}$ T. of hay in a season, how much will 7 cows eat?

Ans. $15\frac{1}{2}$ T.

15. A railroad train travels 220 mi. in 8 hr.: how far will it travel in 5 hr.?

Ans. $137\frac{1}{4}$ mi.

16. If a train travel 220 mi. in 8 hr., how long will it be in travelling 446 mi. at the same rate?

Ans. $16\frac{1}{2}$ hr.

17. If a train travel 300 mi. in 9 hr. 40 min., how long will it be in travelling 223 mi.?

Ans. 7 hr. 11 min. 8 sec.

18. If a train travel 223 mi. in 9 hr. 15 min., how far will it travel in 5 hr. 20 min.?

Ans. $128\frac{1}{11}$ mi.

19. If 6 T. of coal cost $\$28.50$, what cost 20 carloads of 4 T. 3 cwt. each?

Ans. $\$394.25$.

20. If a carload of coal weighing 4 T. 4 cwt. cost $\$22.05$, how much will 12 carloads of 4 T. 13 cwt. each cost?

Ans. $\$292.95$.

21. A field of 12 A. 30 P. yields 255 bu. 2 qt. of wheat: how much will a field of 16 A. 10 P. yield at the same rate?

Ans. $336.159 +$ bu.

22. When 180 qt. of milk produce 24 lb. of butter, how many pounds will 500 qt. of the same milk produce?

Ans. $66\frac{2}{3}$ lb.

23. If 360 qt. of milk produce 48 lb. of butter, how much milk will be required to produce 50 lb. of butter?

Ans. 375 qt.

24. If 16 men can build a house in 20 days, how long would it take 11 men to build it?

Ans. $29\frac{1}{11}$ d.

25. If 12 men can build a house in 30 days, how many men can build it in 24 days?

Ans. 15 men.

26. A merchant failing paid 33 cents on the dollar. He owed B $\$6742.50$: how much did B receive?

Ans. $\$2225.02\frac{1}{2}$.

27. If an 8-cent loaf of bread weigh 12 oz. when flour is worth \$8.50 a barrel, how much should it weigh when flour is worth \$6.80 a barrel? *Ans.* 15 oz.

28. If $\frac{3}{4}$ of a store is worth \$5200, how much is $\frac{1}{4}$ of it worth? *Ans.* \$8190.

29. If $12\frac{1}{2}$ tons of hay cost \$246, how much can be bought for \$82? *Ans.* $4\frac{1}{8}$ T.

30. A pole 50 ft. high casts a shadow 27 ft. long: how long at the same time of day will be the shadow of a pole 70 ft. high? *Ans.* $37\frac{3}{4}$ ft.

31. A pole 25 ft. high casts a shadow 18 ft. long: how high a pole will cast a shadow 39 ft. long? *Ans.* 54 ft. 2 in.

32. If 20 bbl. of flour supply 180 men 6 weeks, how many barrels will be required for 260 men for the same time? *Ans.* 28 $\frac{2}{3}$.

33. If it require 36 yd. of carpeting $\frac{3}{4}$ yd. wide to cover a floor, how many yards $\frac{1}{2}$ yd. wide would be required to cover the same floor? *Ans.* 30 $\frac{3}{4}$.

34. If a train of cars moving at the rate of 22 miles an hour run from one city to another in 12 hr. and 15 min., how long will it take for a train running 25 mi. an hour to perform the same journey? *Ans.* 10 hr. 46 min. 48 sec.

35. If a tax of 7 m. on the property of a town produce a revenue of \$16000, what will be the revenue at 9 m. on the same property? *Ans.* \$20571.43—.

36. A has corn at \$.75 a bu. to exchange for wheat with B at \$1.20. If B puts his wheat at \$1.40, how much a bushel should A have for his corn? *Ans.* 87 $\frac{1}{2}$ cts.

37. If $\frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{1}{2}$ bbl. of beef cost \$78, how much will $\frac{7}{8}$ of $\frac{5}{6}$ of $3\frac{1}{2}$ bbl. cost? *Ans.* \$74.66 $\frac{2}{3}$.

38. A milkman sells in 1 d. 175 qt. of milk at 8 cts. a quart; his measure lacks $\frac{1}{4}$ of a gill of holding a full quart: what is the actual worth of the milk sold? *Ans.* \$13.56 $\frac{1}{4}$.

39. A merchant became insolvent, owing \$6850, and had only \$4932 with which to pay his creditors: how much should a creditor whose claim is \$1540 receive? *Ans.* \$1108.80.

SECTION III.

COMPOUND PROPORTION.

151. A Compound Proportion is one in which one or both of the ratios is compound,

$$\text{as, } \left\{ \begin{array}{l} 4 : 6 \\ 3 : 4 \end{array} \right\} :: 6 : 12; \quad \text{or, } \left\{ \begin{array}{l} 4 : 6 \\ 3 : 4 \end{array} \right\} :: \left\{ \begin{array}{l} 3 : 8 \\ 4 : 3 \end{array} \right\}.$$

NOTE.—All problems in Compound Proportion may be solved by analysis, and it would probably be best for classes to solve them in this way in passing through the book the first time. Analysis may then be combined with the statement, as in the following example:

Ex. If a man earn \$192 in 8 d. by working 6 hr. a day, how much can he earn in 20 d. by working 10 hr. a day?

Ans. \$800.

SOLUTION.

$$\left. \begin{array}{l} 8 \text{ d.} : 20 \text{ d.} \\ 6 \text{ hr.} : 10 \text{ hr.} \end{array} \right\} :: \$192 : x$$

$$\frac{\$192 \times 20 \times 10}{8 \times 6} = \$800.$$

Analysis.—The required answer is dollars; hence we place for the third term \$192. Since the means are known, and but one of the extremes, the members of the means are all multipliers, and the members of the extreme all divisors.

If a man earn \$192 in 8 days, in one day he will earn but $\frac{1}{8}$ as much; 8 days is therefore a divisor, or member of the first term, and its corresponding number, 20 days, is therefore a multiplier, or member of the second term. If a man earn a sum of money by working 6 hours a day, by working 1 hour a day he will earn $\frac{1}{6}$ as much; 6 hours is therefore a divisor, or member of the first term, and its corresponding number, 10 hours, a multiplier, or member of the second term.

Taking the product of the means and dividing by the given extreme, we have for the other extreme, or unknown term, \$800.

The following is the rule for Compound Proportion :

RULE.

Place for the third term that number which is of the same kind as the answer.

Arrange the couplets as in Simple Proportion, making all divisors antecedents, and all multipliers consequents in these couplets.

Find the product of the members of the second and third terms, and divide by the product of the members of the first term. The result is the fourth term.

WRITTEN PROBLEMS.

1. If the wages of 16 men for 8 d. be \$256, what will be the wages of 22 men for 12 d.? *Ans.* \$528.

2. If 3 men in 10 d. of 8 hr. each can earn \$96, how much can 5 men earn in 12 d. of 10 hr. each? *Ans.* \$240.

3. If a piece of marble 8 ft. long, 60 in. wide and 6 in. thick weigh 1400 lb., how much will a slab 5 ft. long, 20 in. wide and 7 in. thick weigh? *Ans.* $340\frac{5}{8}$ lb.

4. If 6 men can build a wall 60 ft. long and 8 ft. high in 10 d., how long will it take 3 men to build a wall of the same thickness 100 ft. long and 6 ft. high? *Ans.* 25 d.

5. If 2 horses eat 3 bu. of oats in 4 d., how much will 6 horses eat in 12 d.? *Ans.* 27 bu.

6. If it require 1 bbl. of flour to supply a family of 6 persons for 20 d., how much will be needed to supply a family of 8 persons 30 d.? *Ans.* 2 bbl.

7. It costs a family of five persons \$135 for 6 wk.'s board: how much will it cost a family of 7 persons at the same rate for 3 wk.? *Ans.* \$94.50.

8. If \$500 gain \$60 in 2 yr. at 6%, how much will \$800 gain in 3 yr. at 8%? *Ans.* \$192.

9. If \$600 gain \$96 in 2 yr. at 8%, how long will it require \$700 on interest at 5% to gain \$105? *Ans.* 3 yr.

10. Three men in 6 d. of 12 hr. each plough a field 80 rd. long and 40 rd. wide: how long will it require 6 men by working 10 hr. a day to plough a field 60 rd. long and 60 rd. wide? *Ans.* 4 d. 30 min.

11. If a 10-cent loaf weigh 15 oz. when flour is \$8 a barrel, how much will a 6-cent loaf weigh when flour is worth \$6 a barrel? *Ans.* 12 oz.

12. If it require 90000 bricks, 8 in. long, 4 in. wide, $2\frac{3}{8}$ in. thick, to erect a building, how many will it require if the bricks are $8\frac{1}{2}$ in. long, $4\frac{1}{8}$ in. wide and $2\frac{3}{8}$ in. thick? *Ans.* 82139+.

13. If 6 compositors in 12 d. of 10 hr. each can compose 24 sheets of 24 pages to the sheet, 40 lines to the page and 40 letters to the line, in how many days of 12 hr. each can 10 compositors compose an 8vo volume of 20 sheets, with 50 lines in a page and 48 letters to the line? *Ans.* 5.

14. If 9 persons eat \$24 worth of bread in 4 mo. when flour is worth \$6 a barrel, how much bread will a family of 8 persons consume in 3 mo. when flour is worth \$8 a barrel?

Ans. \$21 $\frac{1}{2}$ worth.

15. A farmer owning 25 horses, traded them for sheep. If 3 horses are worth 12 cows, 6 cows are worth 42 pigs, and 25 pigs are worth 30 sheep, how many sheep did he get for his horses?

Ans. 840.

16. If it cost \$45 to carpet a room with ingrain carpet 1 yd. wide at \$1.20 a yard, how much will it cost to carpet the same room with Brussels carpet $\frac{3}{4}$ yd. wide at \$1.75 a yard?

Ans. \$87.50.

17. If it cost \$36 to carpet a room 18 ft. long and 15 ft. wide, how much will it cost to carpet a room 15 ft. long and 9 ft. wide?

Ans. \$18.

18. A commission merchant sells 3000 lb. of butter at 40 cts. a pound, and receives \$120 commission: how much commission would he receive on 6550 lb. at 35 cts. a pound?

Ans. \$229.25.

19. If it cost \$150 to dig a cellar 40 ft. long, 30 ft. wide and 6 ft. deep, how much will it cost to dig a cellar 30 ft. square and 5 $\frac{1}{2}$ ft. deep?

Ans. \$103.12 $\frac{1}{2}$.

20. A liveryman finds that 20 horses consume 105 bu. of oats in 14 d.; he had 180 bu. When it was half eaten he bought 4 more horses: how long did it last?

Ans. 22 d.

21. If 16 men can dig a ditch 40 rd. long, 3 ft. wide, 2 $\frac{1}{2}$ ft. deep in 10 d. of 12 hr. each, in how many days of 10 hr. each can a ditch 60 rd. long, 3 $\frac{1}{2}$ ft. wide, 3 ft. deep be dug by 20 men, if 8 of the men work till but $\frac{3}{4}$ of the work is done?

Ans. 23 d. 5 hr. 12 min.

SECTION IV.

DISTRIBUTIVE PROPORTION.

152. Distributive Proportion is the process of dividing a number into parts which are proportional to two or more given numbers.

ORAL EXERCISE.

1. A and B hired a horse and buggy for \$40; A used it 6 wk., and B 4 wk.: how much should each pay?

SOLUTION.—If A used it 6 wk. and B 4 wk., they together used it 6 wk. and 4 wk., or 10 wk. If the use of it for 10 wk. cost 40 dollars, the use of it for 1 wk. cost $\frac{1}{10}$ of 40 dollars, or 4 dollars. A therefore paid 6 times 4 dollars, or 24 dollars; and B, 4 times 4 dollars, or 16 dollars.

2. Three men agree to pay 60 dollars' rent for a pasture-lot; the first pastures 3 cows; the second, 5 cows; and the third, 4 cows: how much should each pay?

3. Two men pay \$108 for pasturage; the first pastures 4 cows for 6 wk.; the second, 2 cows for 15 wk.: how much does each pay?

4. A, B and C gain in trade \$600; A has invested \$200 for 3 mo.; B, \$300 for 3 mo.; C, \$150 for 6 mo.: how much does each gain?

5. The feed of 6 horses for 3 wk., 4 cows for 2 wk. and 10 sheep for 3 wk. costs 60 dollars: how much does the feed of each cost per week if a cow eat 5 times as much as a sheep, and a horse twice as much as a cow?

6. Divide 60 into three parts which are to one another as 2, 3 and 5.

SOLUTION.—Since the parts are to each other as 2, 3 and 5, we may divide 60 into $2+3+5$, or 10 equal parts: 1 of these equal parts is $\frac{1}{10}$ of 60, or 6; 2 parts equal 2×6 , or 12; 3 parts equal 3×6 , or 18; and 5 parts equal 5×6 , or 30.

7. The sum of two numbers is 60; the first is to the second as 5 is to 7: what are the numbers?

8. Divide 80 into three parts which are to one another as 2, 5 and 9.

9. The sum of three numbers is 120; they are to one another as $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$: what are the numbers?

10. If \$1600 be divided in the proportion of $1\frac{1}{2}$, $\frac{2}{3}$ and $1\frac{3}{4}$, it will give respectively $\frac{1}{4}$ A's money, $\frac{1}{5}$ B's, and $\frac{1}{6}$ C's: how much money has each?

11. John and George have 25 apples; John has 3 more than George: how many has each?

SOLUTION.—George's number plus 3 equals John's, which, added to George's, equals twice George's plus 3, or 25 apples; hence, 25 apples minus 3 apples, or 22 apples, equal twice George's number. Once George's number equals one-half of 22 apples, or 11 apples; and John's equals 11 apples plus 3 apples, or 14 apples.

12. Divide 48 into two parts, so that the first part shall be 10 more than the second.

13. A has \$2500 more than B, and they together have \$7000: how much has each?

14. A hat cost \$1.50 more than a vest, and both cost \$11.50: how much did each cost?

15. A coat, a pair of boots and a hat cost 31 dollars; the boots cost 10 dollars less than the coat, and the hat 4 dollars less than the boots: how much did each cost?

16. A and B have 50 cents; A has 4 times as many as B: how many has each?

SOLUTION.—If A has 4 times as many as B, 4 times B's number equals A's, which added to B's is 5 times B's number, or both, which is 50 cents. If 5 times B's number is 50 cents, B's number is $\frac{1}{5}$ of 50 cents, or 10 cents; and A's is 4 times 10 cents, or 40 cents.

17. Two men have \$6000; the first has twice as much as the second: how much has each?

18. The ages of two men, C and D, equal 120 years; D is $\frac{2}{3}$ as old as C: how old is each?

19. A, B and C buy a farm; B pays twice as much as A, and C three times as much as B; the farm cost 18000 dollars: how much did each pay?

20. A farm, a store and a hotel cost \$40000; the farm cost twice as much as the hotel, and the store \$4000 more than three times as much as the farm: how much did each cost?

21. Three-fourths of the price of a piano equals $1\frac{1}{2}$ times the price of a cabinet organ; both cost 780 dollars: how much did each cost?

WRITTEN PROBLEMS.

1. Three men perform a piece of work; the first works 27 days; the second, 30 days; the third, 22 days; they receive \$165.90: how much does each get, and what is the daily wages per man?

Ans. 1st, \$56.70; 2d, \$63; 3d, \$46.20; daily wages, \$2.10.

2. A, B and C enter into partnership to perform some work for \$767.25. A sends 6 men 20 days; B, 5 men 25 days; and C, 4 men 24 days: how much did each receive, and what was the average daily wages?

Ans. A, \$270; B, \$281.25; C, \$216; daily wages, \$2.25.

3. I paid \$900 for 6 horses, 3 cows and 12 sheep; the cows each cost three times as much as a sheep, and the horses each three times as much as a cow: how much did I pay for each?

Ans. Sheep, \$12 each; cows, \$36 each; horses, \$108 each.

4. The sum of two numbers is 1260; they are to each other as 57 and 48: what are the numbers?

Ans. 684 and 576.

5. Divide the number 950 into three parts, which are to each other as 6, 18 and 26. *Ans.* 114, 342 and 494.

6. If \$72500 be divided in the proportion of $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$, it will give respectively $1\frac{1}{2}$ A's, $1\frac{1}{2}$ B's, and $3\frac{1}{2}$ C's money: how much has each? *Ans.* A, \$15000; B, \$15000; C, \$9375.

7. Divide the number $3\frac{1}{2}$ into two such parts that the first shall equal the second plus $\frac{1}{15}$. *Ans.* $1\frac{2}{3}$ and $1\frac{1}{6}$.

8. Mr. G is worth \$27050 more than Mr. E, and they together are worth \$81000: how much is each worth?

Ans. G, \$54025; E, \$26975.

9. Bought three farms; the first cost \$700 more than the second, and the third \$800 less than the second. They all cost \$20900: how much did each cost?

Ans. 1st, \$7700; 2d, \$7000; 3d, \$6200.

10. Bought three houses: the first cost \$600 more than the second, and the third \$1200 less than the first. They all cost \$10800: what was the cost of each?

Ans. 1st, \$4200; 2d, \$3600; 3d, \$3000.

11. A, B and C earned \$3456; B earned three times as much as C, and A four times as much as B: how much did each earn?

Ans. A, \$2592; B, \$648; C, \$216.

12. Two men lay 6750 bricks: the first lays $\frac{4}{5}$ as many as the other: how many does each lay?

Ans. 1st, 3000; 2d, 3750.

13. D and E have 2255 bu. of wheat; $\frac{3}{4}$ of D's equals $\frac{5}{6}$ E's amount: how many bushels has each?

Ans. D, 1100; E, 1155.

14. The difference between two numbers is 120, and $\frac{7}{8}$ of the first equals $\frac{5}{6}$ of the second: what are the numbers?

Ans. 1296 and 1176.

15. The sum of two fractions is $1\frac{7}{12}$, and $\frac{4}{5}$ of the first equals $\frac{2}{3}$ of the second: what are the fractions?

Ans. $\frac{5}{6}$ and $\frac{2}{3}$.

16. John has \$637 less than William, and $\frac{4}{5}$ of William's money equals $1\frac{1}{2}$ of John's: how much has each?

Ans. John, \$728; William, \$1365.

17. A and B have \$13650; 5 times A's money equals 8 times B's: how much has each?

Ans. A, \$8400; B, \$5250.

18. A farmer raised 6720 bu. of wheat and corn; 3 times the number of bushels of corn, plus 120 bu., equals the number of bushels of wheat: how many bushels of each were there?

Ans. 1650 bu. corn; 5070 bu. wheat.

19. A man bought a farm, a store and a house for \$27700; the farm cost 3 times as much as the house, plus \$450; and the store cost twice as much as the farm, minus \$650: what was the cost of each?

Ans. Store, \$16450; farm, \$8550; house, \$2700.

CHAPTER IX.

P E R C E N T A G E.

SECTION I.

DEFINITIONS AND PRINCIPLES.

153. Percentage is the name applied to computations in which 100 is the unit or measure.

154. Per cent. is an abbreviation of the Latin phrase *per centum*, meaning *by the hundred*.

ORAL EXERCISE.

1. What is $\frac{1}{100}$ of 100? $\frac{2}{100}$? $\frac{7}{100}$? $\frac{20}{100}$? $\frac{50}{100}$?
2. What is $\frac{1}{100}$ of 600? $\frac{5}{100}$ of 600? $\frac{5}{100}$ of 1200?
3. How many hundredths of \$100 are \$7? \$20? \$25?

NOTE.— $\frac{1}{100}$ of any number is 1 per cent. of it; $\frac{2}{100}$ is 2 per cent., and, in general, any number of one-hundredths of a number is so many per cent. of it.

4. How many hundredths of a number is 6 per cent. of it? 10 per cent. of it? 25 per cent. of it? 60 per cent. of it?

5. How many hundredths of a number is $5\frac{1}{2}$ per cent. of it? 120 per cent. of it? 400 per cent. of it? $\frac{2}{3}$ of 1 per cent. of it?

6. What per cent. of a number is $\frac{5}{100}$ of it? $\frac{12}{100}$ of it? .16 of it? .37 of it? .005 of it? 1.125 of it?

Per cent. is usually written %; thus, 20% is read "twenty per cent."

7. How many hundredths is 40%? 60%? $62\frac{1}{2}\%$? 125%? $8\frac{1}{2}\%$? $18\frac{3}{4}\%$? $56\frac{1}{4}\%$?

8. What per cent. of a number is $\frac{1}{4}$ of it?

NOTE.— $\frac{1}{4}$ is $\frac{25}{100}$, or 25%.

9. What per cent. of a number is $\frac{1}{2}$ of it? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{8}$? $\frac{9}{10}$? $\frac{8}{9}$? $\frac{1}{10}$? $1\frac{1}{2}$?

10. What fractional part of a number is 20% of it?

ANALYSIS.—20% = $\frac{20}{100}$, or $\frac{1}{5}$; hence, 20% of a number is $\frac{1}{5}$ of it.

11. What fractional part of a number is 10%? 30%? 60%? 75%? $12\frac{1}{2}\%$? 12%? $31\frac{1}{4}\%$? $33\frac{1}{3}\%$? $66\frac{2}{3}\%$? $87\frac{1}{2}\%$? $55\frac{1}{2}\%$?

12. What part of a number is $\frac{1}{2}\%$ of it?

ANALYSIS.— $\frac{1}{2}\% = \frac{1}{2}$ of $\frac{1}{100} = \frac{1}{200}$.

13. What part of a number is $\frac{1}{3}\%$ of it? $\frac{1}{4}\%$? $\frac{2}{3}\%$? $\frac{5}{8}\%$?

WRITTEN EXERCISE.

Express decimally the following:

1. 1%; 12%; 70%; 95%; 60%; 50%; 87%.

2. 12%; 37%; $56\frac{1}{2}\%$; $5\frac{1}{2}\%$; $9\frac{1}{4}\%$; 220%.

Change to expressions having the per cent. (%) sign the following:

3. .16; .125; .875; $.33\frac{1}{3}$; $.62\frac{1}{2}$; $.00\frac{1}{2}$; $.18\frac{2}{3}$.

4. $.31\frac{1}{4}$; $.87\frac{5}{8}$; $.00\frac{1}{4}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$.

PRINCIPLES AND DEFINITIONS.

155. The **Rate** is the number of hundredths. Thus, in 3%, or $\frac{3}{100}$, 3 is the rate, and $\frac{3}{100}$ itself the *rate per cent.*

156. The **Rate per cent.** is the fraction which denotes how many hundredths are taken.

157. The **Base** is the number of which the per cent. is taken.

158. The **Percentage** is the result of taking a per cent. of the base.

159. The **Amount** is the base added to the percentage.

160. The **Difference** is the remainder found by subtracting the percentage from the base.

In computation, any per cent. less than 100 may be expressed as a decimal; as 16% = .16; 45% = .45; 7% = .07, etc.

Any per cent. greater than 100 may be expressed as a whole number and a decimal; as 125% = 1.25; $187\frac{1}{2}\%$ = 1.87 $\frac{1}{2}$, etc.

SECTION II.

GENERAL CASES IN PERCENTAGE.

CASE I.

The Base and the Rate given, to find the Percentage.

ORAL EXERCISE.

1. What is 15% of 160?

SOLUTION.—15% is $\frac{15}{100}$, or $\frac{3}{20}$; $\frac{3}{20}$ of 160 is 24.

2. What is 5% of 60?

5. What is 6% of 300?

3. What is 7% of 400?

6. What is 25% of 24?

4. What is 12% of 75?

7. What is $12\frac{1}{2}\%$ of 60?

8. What is $33\frac{1}{3}\%$ of 15 sheep?

9. What is $56\frac{1}{4}\%$ of 120 cents?

10. What is $87\frac{1}{2}\%$ of 96 yd. of cloth?

11. What is $3\frac{1}{5}\%$ of 90 bu. of corn?

12. A farmer had 50 sheep, and bought 20% of his number more: how many had he then?

13. A laborer who received \$60 a month, had his salary diminished 10%: how much did he get then?

14. Bought a knife for \$2, and sold it at a gain of 25%: what did I get for it?

15. A man had a flock of chickens consisting of 75; they increased during the summer 120%: how many had he at the close of the summer?

16. Bought goods at \$36, and gained on them $16\frac{2}{3}\%$: what was the selling price?

17. Having \$80, I expended 15% of it for bread, 30% for clothes and 25% in travelling: how much did I spend for each, and how much have I remaining?

18. A has \$40, and B has 4 times as much; B gives 25% of his money to A: how much has each then?

19. 15% of \$700 is $\frac{7}{10}$ of what I paid for a horse; I sold him at a gain of 12%: what did I get for him?

20. Two men, C and D, owned a store worth \$16500; C owned $\frac{2}{5}$ as much as D; D bought 20% of C's: how much did each then own?

WRITTEN PROBLEMS.

1. What is 27% of \$6723?

SOLUTION.

$$\begin{array}{r}
 \$6723 \\
 \underline{.27} \\
 47061 \\
 \underline{13446} \\
 \$1815.21
 \end{array}$$

Explanation.—Since 27% equals .27, the required percentage is .27 of \$6723, or \$1815.21.

From the foregoing solution and explanation we derive the following

RULE.

Multiply the base by the rate per cent. ; the result is the percentage.

2. What is 63% of 127 A.?
3. What is 19% of \$27?
4. What is $16\frac{1}{2}\%$ of \$127?
5. What is $66\frac{1}{2}\%$ of \$186.50?
6. What is $\frac{1}{8}\%$ of \$100?
7. What is $\frac{1}{2}\%$ of $7\frac{1}{2}\%$ of \$647?
8. What is 6% of \$140 + 9% of \$160 + 23% of \$175?
Ans. \$63.05.
9. Find the value of 11% of \$180 + 22% of \$160 + 92% of \$63.
Ans. \$112.96.
10. Find the value of 7% of \$6.50 + 12% of \$12.50 + 25% of \$19.64.
Ans. \$6.865.
11. Find the value of 12% of 25 £ 6 s. 6 d.
Ans. 3 £ 9.36 d.
12. Find the value of 25% of 6 yd. 2 ft. 6 in.
Ans. 1 yd. 2 ft. $1\frac{1}{2}$ in.
13. What is the value of $\frac{1}{8}$ of $4\frac{1}{2}\%$ of $\frac{5}{8}$ T.? *Ans.* 9 lb. 6 oz.
14. What is the value of $2\frac{1}{4}$ times $\frac{1}{8}\%$ of $66\frac{1}{2}$ gal.? *Ans.* 1 qt. 1 pt. $3\frac{3}{4}$ gi.
15. If iron ore yields 63% of pure metal to the ton, how much iron is there in 16 T. of ore? *Ans.* 10 T. 160 lb.
16. How much silver in 26 T. of ore that yields $2\frac{1}{2}\%$ of metal to the ton? *Ans.* 1300 lb.
17. An operator mined 12600 T. of coal; he paid 20% for mining and 25% for transportation: how much did he have remaining? *Ans.* 6930 T.

18. A and B each invest \$3300 in business; at the end of a year A has gained 12%, and B has lost $5\frac{1}{4}\%$: how much has each? *Ans.* A, \$3696; B, \$3126.75.

19. A man has an annual income of \$2550; he spends 15% in travelling, 20% for board and $6\frac{1}{2}\%$ for clothing: how much does he save? *Ans.* \$1491.75.

20. If a man have a yearly income of \$2500, and spend the first year 25% of it, the second 35% of it, and the third $42\frac{1}{4}\%$ of it, how much does he save in three years?

Ans. \$4937.50.

CASE II.

The Base and the Percentage being given, to find the Rate.

1. What per cent. of 40 is 10?

ANALYSIS.—10 is $\frac{1}{4}$ of 40; it is therefore $\frac{1}{4}$ of 100% of 40, or 25% of 40.

2. What per cent. of \$25 is \$5? Of 40 cows is 4 cows?

3. What per cent. of 75 yd. is 15 yd.? Of 72 sheep is 9 sheep?

What per cent.—

4. Of 28 is 7?

5. Of 42 mills is 7 mills?

6. Of 88 is 11?

7. Of \$90 is \$15?

8. Of \$120 is \$40?

9. Of \$800 is \$160?

10. Of 18 is 6? Is 3?

11. Of $31\frac{1}{4}$ is $6\frac{1}{4}$?

12. Of $12\frac{1}{2}$ is $2\frac{1}{2}$?

13. Of $16\frac{2}{3}$ is $1\frac{2}{3}$?

14. From a barrel containing 36 gal. of vinegar a grocer draws 20 gal: what per cent. of the vinegar remains?

15. A boy having 90 cents, spent 60 cents: what per cent. of his money did he spend, and what per cent. has he remaining?

16. Bought 4 doz. of eggs, but found that 6 of the eggs were not good: what per cent. were good?

17. $\frac{2}{3}$ of \$25 is what per cent. of $\frac{3}{4}$ of \$96?

18. $\frac{2}{3}$ of 20% is what per cent. of $2\frac{1}{2}$ times 12%?

19. A man has \$325, and spends $\frac{1}{4}$ of it in one week, and $\frac{1}{4}$ the next week : what per cent. remains ?

20. A drover after selling $\frac{1}{3}$ of his cattle found that 25 was $\frac{5}{12}$ of the remainder : had he sold 40, what per cent. would he have had left ?

WRITTEN PROBLEMS.

1. What per cent. of 125 is 15 ?

SOLUTION.

125)15 (.12

125

250

250

Explanation.—15 is $\frac{1}{8}$ of 125, or $\frac{1}{8}$ of 100 per cent. of 125, which is $\frac{1}{8} \times 100$ per cent., equal to 12 per cent.

RULE.

To find the rate, divide the percentage by the base.

2. What per cent. of 300 is 180 ?
3. What per cent. of 160 is 56 ?
4. What per cent. of \$12 is 30 cts. ? *Ans.* $2\frac{1}{2}$.
5. What per cent. of 890 lb. is 120.15 lb. ? *Ans.* $13\frac{1}{2}$.
6. What per cent. of \$18 is \$19.20 ? *Ans.* $106\frac{2}{3}$.
7. What per cent. of 640 mi. is 90.88 mi. ? *Ans.* $14\frac{1}{2}$.
8. What per cent. of 24 A. is 15 A. ? *Ans.* $62\frac{1}{2}$.
9. What per cent. of 60 lb. is $1\frac{1}{2}$ lb. ? *Ans.* $2\frac{1}{2}$.
10. What per cent. of 1 £ is 1 s. ? *Ans.* 5.
11. What per cent. of 6 bu. is 3 pk. ? *Ans.* $12\frac{1}{2}$.
12. What per cent. of 10 gal. is 3 qt. ? *Ans.* $7\frac{1}{2}$.
13. What per cent. of 2 gal. 2 qt. is 9 gal. ? *Ans.* 360.
14. What per cent. of a pound avoirdupois is a pound Troy ? *Ans.* $82\frac{2}{3}$.
15. A man failing in business had only \$2100 ; he owed \$6000 : what per cent. would his creditors receive ? *Ans.* 35%.
16. Bought goods for \$1200, and sold them for \$1500 ; my expenses were \$100 : what per cent. did I gain ? *Ans.* $16\frac{2}{3}$.
17. A farmer had his money invested in three farms, as follows : in the first, \$6000 ; in the second, \$8960 ; in the third, \$9600 : what per cent. of his money was invested in each farm ? *Ans.* 1st, $24\frac{1}{3}\frac{2}{3}$; 2d, $36\frac{1}{3}\frac{2}{3}$; 3d, $39\frac{2}{3}\frac{2}{3}$.

18. A merchant invested \$6000 in business; he gained 23% the first year; the second year he lost 20% of what he had at the end of the first year: what per cent. of his money has he?

Ans. 98 $\frac{1}{2}$ %.

CASE III.

The Rate and the Percentage being given, to find the Base.

ORAL EXERCISE.

1. 16 is 4 per cent. of what number?

ANALYSIS.—4% is $\frac{4}{100}$, or $\frac{1}{25}$; since $\frac{1}{25}$ of a certain number is 16, $\frac{11}{11}$, or the number, is 25 times 16, or 400.

Or, If 4% or $\frac{4}{100}$ of a number is 16, $\frac{100}{4}$ is $\frac{1}{4}$ of 16, which is 4; and $\frac{100}{4}$, or the number, is 100 times 4, or 400.

Remark.—Pupils may be taught to give either of the foregoing analyses.

2. 15 is 12% of what number?

3. 50 is 10% of what number?

4. 96 is 8% of what number?

5. $1\frac{1}{2}$ is 14% of what number?

6. 24 is 20% of what number?

7. 5 cwt. is 40% of what?

8. 3 qt. is 15% of how many gallons?

9. \$60 is 37 $\frac{1}{2}$ % of what my horse cost: how much did he cost?

10. I receive \$630 rent for my house, which is 14 per cent. of the value: what is the value?

11. A store rents for \$600 per annum, which is 12 $\frac{1}{2}$ % of its value: what is the value?

12. \$40 is 8% of what a watch and a chain cost; the watch cost 9 times as much as the chain: what was the cost of each?

13. A coal-dealer sold 480 tons of coal, which is 12 $\frac{1}{2}$ % of all he had: how much had he?

14. A farmer sold 2500 bu. of corn, and had 37 $\frac{1}{2}$ % of his crop left: how much was the crop?

15. 75% of \$8400 is what a store cost, and 60% of the cost of the store is 72% of the cost of a farm: what was the cost of the farm?

WRITTEN PROBLEMS.

1. 150 is 60% of what number?

SOLUTION 1. $60\% = 150$

$$1\% = \frac{1}{60} \text{ of } 150 = \frac{150}{60}$$

$$100\% = 100 \times \frac{150}{60} = 250.$$

Let the pupil give the explanation.

Explanation to Solution 2.—

SOLUTION 2. $150 \div .60 = \frac{150.00}{.60} = 250.$

If .60 of a number is 150, the number is as many as .60 is contained times in 150, or 250.

RULE.

To find the base when the rate per cent. and the percentage are given, divide the percentage by the rate per cent.

2. \$6840 is 30% of a man's money: how much money has he? *Ans.* \$22800.

3. If a man rent a store property at \$650 per annum, which is 13% of the value of the property, what is the value of it? *Ans.* \$5000.

4. 1240 bu. of corn is 62% of a farmer's crop: how much does he raise? *Ans.* 2000 bu.

5. I sold a horse for \$225 dollars, which is 112½% of what I paid for him: what did he cost me? *Ans.* \$200.

6. A merchant bought a store for \$11562.50, which is 62½% of what he sold it for: what did he receive for it? *Ans.* \$18500.

7. If \$6.51 is 31% of the fare from Philadelphia to Chicago, what is the fare? *Ans.* \$21.

8. A man's expenses are \$36 a month, and this amount is 54% of his salary: how much does he save? *Ans.* \$30.66½.

9. The number of pupils in attendance at school in a certain town is 996, which is 83% of the whole number of pupils: what is the number? *Ans.* 1200.

10. The number of children in a city attending school is 13650, which is 21% of the population: what is the population? *Ans.* 65000.

11. A clerk spent 25% of his salary for board, 15% for books, 12½% for clothing, and he saves \$712.50: how much was his salary? *Ans.* \$1500.

12. A merchant owning $\frac{3}{4}$ of a ship, sold 25% of his share for \$24000: how much was the ship worth? *Ans.* \$128000.

13. A farmer sold 13 A. 1 R. 38 P. of land, which was 13% of his farm: how large was the farm? *Ans.* 103 A. 3 R.

14. A miller had a certain quantity of flour; a flood damaged 20% of it, and he sold 20% of the remainder for \$2560; what was the value of all? *Ans.* \$16000.

15. Mr. A bought 175 sheep, which is 7% of what he already had. He then sold 24% of his number: what per cent. of his original number remained? *Ans.* 81½%.

16. A man's income in 3 years was \$4560; his income the second year was 35% more than the first, and his income the third year was 45% more than the first: what was his income each year? *Ans.* 1st, \$1200; 2d, \$1620; 3d, \$1740.

CASE IV.

The Amount or the Difference, and the Rate per cent. being given, to find the Base.

ORAL EXERCISE.

1. What number being increased by 25% of itself will equal 40?

SOLUTION.—25% equals $\frac{1}{4}$; $\frac{1}{4}$ plus $\frac{1}{4}$ equals $\frac{1}{2}$. If $\frac{1}{2}$ of a number equals 40, $\frac{1}{2}$ of that number is $\frac{1}{2}$ of 40, or 8; and $\frac{1}{2}$, or the number, is 4 times 8, or 32.

2. What number increased by 20% of itself equals 60?

3. What number diminished by 40% of itself equals 30?

4. 165 is 10% more than what number?

5. \$7 is 16⅔% more than what?

6. A merchant sold coffee at 36 cents a pound, and thereby made 33⅓%: what did it cost?

7. A carter sold his horse for \$127, which was 27% more than the cost: what was the cost of the horse?

8. If cloth is sold for \$3.75 a yard at a gain of 25%, what is the cost?

9. A miller sold flour at \$7.28 per barrel, which is 12% more than cost: what did it cost?

10. If a merchant sell goods at 15% less than cost, and get \$2.55 a yard, what was the cost?

11. After selling 120 yd. of carpet from a roll, there remained 25%: how much was there at first?

12. What number diminished by 17% of itself equals 747?

13. What number increased by 17% of itself equals 819?

14. What number increased by the difference between 40% and 16% of itself equals 372?

15. Two boys have 63 apples; one has $33\frac{1}{3}\%$ more than the other: how many has each?

16. Divide \$2300 between C and D so that D shall have 30% more than C.

17. What is the length of a log which being increased by 40%, and 12 ft. more, equals twice its length?

18. A house and a store cost \$6000; the store cost 40% more than the house: what was the cost of each?

WRITTEN PROBLEMS.

1. What number increased by 25% of itself equals 525?

SOLUTION 1.

$$125\% = 525$$

$$1\% = \frac{525}{125}$$

$$100\% = 100 \times \frac{525}{125} = 420.$$

SOLUTION 2.

$$125\% = \frac{5}{4} = 525$$

$$\frac{5}{4} = \frac{5}{4} \times 525 = 105$$

$$\frac{5}{4} = 4 \times 105 = 420.$$

NOTE.—Let the pupil give the explanation of both solutions.

RULES.

1. When the amount and the rate per cent. are given, to find the base, divide the amount by 1 plus the rate per cent.

2. When the difference and the rate per cent. are given, to find the base, divide the difference by 1 minus the rate per cent.

2. What number increased by 20% of itself equals 768?

Ans. 640.

3. What number increased by 32% of itself equals 198?
Ans. 150.
4. What number increased by 15% of itself equals 1748?
Ans. 1520.
5. What number diminished by 70% of itself equals 432?
Ans. 1440.
6. What fraction increased by 20% of itself equals $\frac{4}{5}$?
Ans. $\frac{4}{5}$.
7. What fraction diminished by 40% of itself equals $\frac{4}{5}$?
Ans. $\frac{4}{5}$.
8. A has \$306, which is 66% less than B has : how much has B?
Ans. \$900.
9. A merchant cut 34 yd. from a piece of calico, and 15% remained : how much was there in the piece? *Ans.* 40 yd.
10. A man failing in business had \$3350 cash, which paid 67% of his debts : how much were his debts?
Ans. \$5000.
11. A merchant failing paid 27% of his debts with \$4546.80 : what was his indebtedness?
Ans. \$16840.
12. The number of pupils enrolled in the schools of a certain town is 1161, which is 35% more than the number in attendance : what is the number in attendance? *Ans.* 860.
13. The population of a town increased in 10 yr. 72% ; in 1870 its population was 11180 : what was it in 1860?
Ans. 6500.
14. The profits on a business in two years were \$3105 ; the profits the second year were 7% more than the first year : what were the profits each year? *Ans.* 1st, \$1500 ; 2d, \$1605.
15. A man sold two farms for \$7000 each ; for the first he received $12\frac{1}{2}\%$ less than its value, and for the second $16\frac{2}{3}\%$ more than its value : what was the value of each?
Ans. 1st, \$8000 ; 2d, \$6000.
16. When gold is at a premium of $12\frac{1}{2}\%$, what is the value in gold of \$1 currency? *Ans.* 88 $\frac{3}{4}$ cents.
17. When gold was at a premium of 115, what was the value in gold of \$1 currency? *Ans.* 46 $\frac{2}{3}$ cents.

18. Bought a house for \$3200 when property was depreciated 40% in value: what was the value of the house?

Ans. \$5333 $\frac{1}{3}$.

19. \$2460 is 59% less than A's and B's money; A's is 60% of B's: how much has each? *Ans.* A, \$2250; B, \$3750.

20. Sold gold at a premium of 15 $\frac{1}{2}$ %, and received \$1386 currency: how much gold did I sell? *Ans.* \$1200.

APPLICATIONS OF PERCENTAGE.

161. The principal applications of percentage are *Profit and Loss, Commission and Brokerage, Capital and Stocks, Taxes, Customs, Insurance and Bankruptcy*, in all of which time is not an element.

Also, *Interest, Discount, Exchange, Equation of Payments and Equation of Accounts*, in which time enters as an element or factor.

All the problems in these applications are solved by the application of one or more of the four cases of Percentage.

SECTION III.

PROFIT AND LOSS.

162. **Profit and Loss** are the terms used to indicate gain or loss in business transactions. The difference gained between the *cost* and the *selling* price is called *Profit*, and the difference between the *cost* and the *selling* price when an article is sold at less than cost is called *Loss*.

The *cost* may be regarded as the *base*, and the profit or the loss as a percentage of the cost.

CASE I.

When the Cost and Rate are given, to find the Gain or Loss.

ORAL EXERCISE.

1. If a man have a horse worth \$100, for how much must he sell him to gain 25%?

SOLUTION.—If he gains 25%, he gains $\frac{25}{100}$, or $\frac{1}{4}$; $\frac{1}{4}$ the cost, plus $\frac{1}{4}$ the gain, equals $\frac{5}{4}$ of the cost, or the selling price; $\frac{5}{4}$ of \$100 equal \$125.

2. Bought muslin at 10 cents a yard, and sold it at a gain of 20% : what did I sell it at?

3. A house cost me \$1200, and I sold it at a gain of 40% : what did I receive for it?

4. A merchant bought 40 bbl. of flour for \$240, and sold them at a gain of $16\frac{2}{3}\%$: what was the selling price per barrel?

5. If eggs cost me 20 cents a dozen, for how much must I sell them to make 20%?

6. Paid \$12 for 20 hats : for how much must they be sold to gain 25%?

7. Bought a house for \$2500, but property depreciating in value I sold it at a loss of 30% : how much did I get for it?

8. A bought a horse for \$120 ; he sold it to B at a gain of 25% ; B sold it to C at a gain of 10% : how much did C pay for the horse?

9. A miller bought \$1200 worth of wheat, and lost on it 10% ; he bought a second lot for \$1500, and made on this 12% : how much did he gain or lose?

10. A drover bought 12 cows at an average of \$60 each, and sold them at a gain of 35% : what did he receive for the lot?

WRITTEN PROBLEMS.

1. Paid \$1620 for a house and lot, and I sell it at a gain of 22% : what is the gain?

SOLUTION.

\$1620

.22

3240

3240

\$356.40

Let the pupil explain ; also derive the rule from the solution.

2. A merchant invests \$6575 in spring goods, and sells them at a gain of 18% : what is his gain? *Ans.* \$1183.50.

3. Bought some hay for \$560 : for how much must I sell it to gain 15%? *Ans.* \$644.

4. Bought 500 tons of coal at \$4.20 a ton, and sold it at 27% advance : what was the total gain, and the gain per ton?

Ans. \$567 ; per ton, \$1.13 $\frac{1}{3}$.

5. A miller bought 700 bu. of wheat at \$1.20 per bushel, and sold it at a gain of $15\frac{1}{2}\%$, after paying \$28 for hauling: what was his gain? *Ans.* \$102.20.

6. The cost of coal is \$2.60, and the freight \$1.30 per ton: at what must I sell it to gain 30%? *Ans.* \$5.07 per ton.

7. Bought 60 bbl. of flour at \$6 a bbl., paid for freight \$36: at what must I sell it per bbl. to gain 30%? *Ans.* \$8.58.

8. If 16 cows cost \$464.80, how much will be the profit on each cow at 18% gain? *Ans.* \$5.23—.

9. A man has \$6740 invested at 6%, but his taxes are $1\frac{1}{2}\%$: how much does he make? *Ans.* \$283.08.

10. Bought a farm for \$16500; sold it to B at a gain of 12%; he sold it to C at a loss of 3%: what did B and C each pay for the farm? *Ans.* B, \$18480; C, \$17925.60.

CASE II.

The Base and the Gain or Loss given, to find the Rate.

ORAL EXERCISE.

1. If a boy buy a knife at 12 cts., and sell it at 15 cts., what per cent. does he gain?

SOLUTION.—If he pay 12 cts. for the knife, and sell it at 15 cts., he gains the difference between 15 cts. and 12 cts., which is 3 cts.; 3 cts. is $\frac{1}{4}$ of 12 cts., or $\frac{1}{4}$ of 100% of 12 cts., which is 25%.

2. Bought a hat at \$4, and sold it at \$6: what is the gain per cent.?

3. If I buy \$300 worth of grain, and sell it for \$360, what is the gain per cent.?

4. What per cent. is lost on delaine worth 60 cts. a yard when it is sold at 55 cts. a yard?

5. A farmer refused to sell his potatoes at 50 cts. a bushel, but afterward was compelled to sell them at 35 cts. a bushel: what was the loss per cent.?

6. A huckster bought chickens at 7 cts. a pound; he paid 1 ct. a pound freight on them, and sold them in the city at 11 cts. a pound: what per cent. did he gain?

7. A fruit-dealer bought lemons at 15 cts. a dozen, and sold them at 2 for 5 cts.: what was his gain per cent.?

8. If \$140 is $\frac{3}{4}$ of the cost of a horse, what per cent. will be gained by selling him at \$273?

9. If $\frac{3}{4}$ of a farm is sold for the value of $\frac{3}{4}$ of it, what is the gain per cent.?

10. Bought chestnuts at \$4 a bushel, and retailed them at 10 cts. a pint: what was the gain per cent.?

WRITTEN PROBLEMS.

1. If I buy an acre of land for \$80, and sell it for \$90, what is the gain per cent.?

SOLUTION. Let the pupil give the explanation and derive the rule.
 $90 - 80 = 10$
 $10 \div 80 = \frac{1}{8} = 12\frac{1}{2}\%$

2. A knife cost \$1.20, and is sold for \$1.40: what is the gain per cent.?

Ans. $16\frac{2}{3}\%$.

3. If tea costs 75 cents, and I sell it for 95 cents, what is the gain?

Ans. $26\frac{2}{3}\%$.

4. Bought a house for \$1250, and sold it for \$1400: what was the gain per cent.?

Ans. 12.

5. Bought a buggy for \$220, and sold it for \$240: what was the gain per cent.?

Ans. $9\frac{1}{11}\%$.

6. A merchant having \$5000 worth of goods, lost 25% by fire, and sold the remainder at a gain of 40%: what was the gain or loss per cent.?

Ans. 5% gain.

7. Sold a bookcase for \$125, and cleared $\frac{1}{6}$ of this money: what would I have gained per cent. by selling at \$120?

Ans. $6\frac{2}{3}\%$.

8. \$147 is $\frac{7}{8}$ of the cost of a piano: what per cent. is gained by selling it at \$300?

Ans. $19\frac{1}{2}\%$.

9. A carload of grain worth \$620 is found to be damaged, and is sold at \$500: what is the loss per cent.?

Ans. $19\frac{1}{11}\%$.

10. $\frac{3}{4}$ of a farm is sold for the value of $\frac{3}{4}$ of it: what is the loss per cent.?

Ans. $23\frac{1}{3}\%$.

CASE III.

The Gain or Loss and the Rate being given, to find the Base.

ORAL EXERCISE.

1. A man sold wheat at 30 cents a bushel more than it cost, and gained 20% : what did it cost ?

SOLUTION.—20%, or the gain, is $\frac{1}{5}$ of the cost; if $\frac{1}{5}$ of the cost is 30 cents, $\frac{4}{5}$, or the cost, is 5 times 30 cents, or \$1.50.

2. A boy gains 25% by selling his ball for 10 cents more than it cost : what was its cost ?

3. A house was sold for \$400 more than it cost, and the owner gained $16\frac{2}{3}\%$: what was the cost ?

4. A merchant sold goods at 20% more than cost, and gained 7 cents a yard : what was the cost per yard ?

5. By selling a book at 27 cents more than cost I gain 18% : what was the cost ?

WRITTEN PROBLEMS.

1. A drover by selling cattle at \$19.50 less than they cost lost 20% : what did they cost ?

SOLUTION. Let the pupil give the explanation and derive the rule.
 $20\% = \frac{20}{100} = \frac{1}{5} = \$19.50$
 $\frac{4}{5} = 5 \times \$19.50 = \$97.50.$

2. A house and lot was sold for \$830 more than it cost, which was at a gain of $12\frac{1}{2}\%$: what was the cost of the property ?
Ans. \$6640.

3. John lost $16\frac{2}{3}\%$ by selling his watch at \$10 less than cost : what did he get for it ?
Ans. \$50.

4. If I lose 13% by selling goods for 39 cents a yard less than cost, how should I sell them to gain 8% ?
Ans. \$3.24 a yard.

5. I gain 23% by selling 24 bbls. of apples for \$16.56 more than cost : what was the cost per barrel ?
Ans. \$3.

6. By selling 12 bu. of chestnuts at \$23.76 less than cost I lose 44% : at what shall I sell them per bushel to gain 16% ?
Ans. \$5.22.

CASE IV.

The Selling Price and the Rate of Gain or Loss being given,
to find the Cost.

ORAL EXERCISE.

1. A merchant sold a coat for \$25, on which he gained 25% :
what was the cost?

SOLUTION.—Since he gained 25%, or $\frac{1}{4}$ of the cost, he sold the coat for $\frac{1}{4} + \frac{1}{4}$, or $\frac{1}{2}$, of the cost. If $\frac{1}{2}$ of the cost equals \$25, $\frac{1}{4}$ of the cost is $\frac{1}{2}$ of \$25, or \$5; and $\frac{1}{4}$, or the cost, is 4 times \$5, or \$20.

2. A grocer sold potatoes at 60 cents a bushel, and thereby gained 20% : what was the cost?

3. A farmer sold a cow for \$38, and lost 5% of what she was worth : what was she worth?

4. A jeweller sold a watch for \$214, and thereby gained 7% :
what was the cost?

5. Flour was sold at \$8 a barrel, and the owner gained 33 $\frac{1}{3}$ % : what did it cost him?

6. If by selling coffee at 15 cents a pound I lose 16 $\frac{2}{3}$ %, what is the coffee worth?

7. By selling corn at 70 cents a bushel I lose 12 $\frac{1}{2}$ % : how much would I lose by selling at 75 cents a bushel?

8. By selling corn at 90 cents a bushel I gain 12 $\frac{1}{2}$ % : what per cent. would I lose by selling at 65 cents?

9. I sell two books at \$3 each ; on the first I gain 25%, and on the second I lose 20% : how much do I gain or lose?

WRITTEN PROBLEMS.

1. A horse was sold at \$144, which was at a gain of 20% :
what was the cost?

SOLUTION 1.
 $.20 + 1.00 = 1.20$

1.20)144.00(120
 $\frac{120}{240}$
 $\frac{240}{0}$

SOLUTION 2.
 $20\% = \frac{1}{5}$

$\frac{1}{5} + \frac{1}{5} = \frac{2}{5} = \$144$
 $\frac{1}{5} - \frac{1}{5} \times \$144 = \$24$
 $\frac{2}{5} - 5 \times \$24 = \$120$

Remark.—Let the pupil give the explanation and derive the rule in each solution.

RULE.

Divide the amount by 1 + the rate expressed decimally; or, the difference by 1 - the rate expressed decimally.

2. A horse was sold for \$252, which was at a gain of 12% : what was the cost? *Ans. \$225.*

3. I sold land at \$96.05 an acre, which was at a gain of 13% : what was my gain on 65 acres? *Ans. \$718.25.*

4. A dealer sold \$1543.50 worth of clover-seed, which was at a gain of 22½% : what did he gain? *Ans. \$283.50.*

5. Sold a farm at \$8128, and made 27% on the cost : what was the cost of the farm? *Ans. \$6400.*

6. A merchant sold carpet at 92 cents a yard, and cleared 15% : at what should he have sold it per yard to clear 18½%? *Ans. 95 cents.*

7. If by selling a house at \$1425 the owner gains \$175, how much would he have gained per cent. by selling it for \$1600? *Ans. 28.*

8. If an investment pays 13% by selling for \$7345, what would it realize at \$7500? *Ans. 15⅔%.*

9. Sold two houses at \$2484 each ; I gained on one 8%, and lost 8% on the other : what did I gain or lose? *Ans. Lost \$32.*

10. A piano was sold for \$297, at a gain of 35% : what would have been the gain per cent. if it had been sold at \$300? *Ans. 36⅔.*

SECTION IV.

COMMISSION AND BROKERAGE.

163. Commission is an allowance made to an agent for selling goods for another.

164. An Agent is one who transacts business for another.

165. A Factor is an agent who buys and sells or transacts mercantile business for another.

166. A Commission Merchant is one who buys and sells merchandise in his own name for another.

167. A Broker is one who buys and sells stocks, real estate, bonds and the like for another person.

168. A Consignee is the person to whom goods are sent for sale; the sender is called the **Consignor**, and the goods themselves a **Consignment**.

169. The commission allowed to a broker is called **Brokerage**.

Commission and brokerage are computed at a certain rate per cent. on the sale, purchase, collection or amount of business transacted.

170. The Base is the sum expended or received.

171. The Amount is the base plus the commission.

172. The Net Proceeds is the remainder after deducting the commission and other charges.

The four cases of Commission correspond to the four cases of Percentage.

CASE I.

Given the Base and the Rate, to find the Commission.

ORAL EXERCISE.

1. If I collect a debt of \$220, and charge 5% commission, what commission do I receive?

SOLUTION.— $5\% = \frac{5}{100}$, or $\frac{1}{20}$. If I receive 5% commission, therefore, I receive $\frac{1}{20}$ of the amount collected, or $\frac{1}{20}$ of \$220, which is \$11.

2. A lawyer collects a note of \$650, and charges 5% commission: what amount does he receive?

3. A tax-collector collects \$12500 at a commission of 4%: what is his commission?

4. An auctioneer sells a farm for \$6400 at $2\frac{1}{2}\%$ commission: what is his commission, and what does the owner receive?

5. I sold 600 bbl. of flour at \$7.50 per barrel on a commission of $1\frac{1}{2}\%$: what is my commission, and what does the owner of the flour receive?

WRITTEN PROBLEMS.

1. A commission merchant sold \$2350 worth of produce at a commission of 3% : what did he make?

SOLUTION.

\$2350	Let the pupil give the explanation and derive the rule.
.03	
\$70.50	

2. What commission must I pay an agent to sell a farm for me, worth \$12350, at $\frac{1}{2}\%$? *Ans.* \$61.75.
3. A commission merchant receives and sells 12600 bu. of wheat at \$1.37 per bushel on a commission of $3\frac{1}{2}\%$: what was his commission ? *Ans.* \$604.17.
4. An agent sold 60 bales of cotton, each bale weighing 380 lb., at $11\frac{1}{2}$ cts. a pound, on a commission of $2\frac{1}{4}\%$: what was his commission ? *Ans.* \$72.10 $\frac{1}{2}$.
5. A commission merchant sold the following consignment on $3\frac{1}{4}\%$ commission : 640 lb. poultry at \$.17 ; 850 lb. butter at \$.33 ; 1260 doz. eggs at \$.27 ; 2750 lb. dressed pork at \$.08 $\frac{1}{2}$: how much is his commission, and what does he pay the consignor ? *Ans.* Com., \$36.12+ ; consignor, \$927.13-.
6. An attorney collects a claim of \$650.50 ; he pays \$23.75 costs, and charges 5% for collection : what does he pay the owner of the claim ? *Ans.* \$594.225.

CASE II.

Percentage and Base given, to find the Rate.

ORAL EXERCISE.

1. An attorney collects a debt of \$300, and charges \$15 fees : what per cent. does he charge?

SOLUTION.—If he charges \$15 on \$300, on 1 he charges $\frac{1}{20}$ of 15, or $\frac{15}{20}$, which equals $\frac{3}{4}$, or 5%.

2. An attorney charges \$16 for collecting a note of \$200 : what per cent. does he charge?
3. A land-agent charges \$35 for selling a property for \$700 : what per cent. does he charge?

4. A commission merchant sells \$1875 worth of goods, and charges \$75 commission: what is his per cent.?

5. An auctioneer sells goods to the amount of \$480, and charges \$12: what per cent. does he receive as commission?

WRITTEN PROBLEMS.

1. My agent sells for me a town-lot at \$1080, and charges me \$81 commission: what per cent. does he charge? *Ans.* $7\frac{1}{2}$.

SOLUTION.

$\frac{1080}{81.00}$ Let the pupil explain and derive a rule.
 $.07\frac{1}{2}$

2. I paid a broker \$27 brokerage to purchase for me \$5400 bank stock: what rate per cent. did he charge me? *Ans.* $\frac{1}{2}$.

3. A commission merchant charged \$17.28 for selling 640 bu. of potatoes at 60 cents a bushel: what was the rate per cent.? *Ans.* $4\frac{1}{2}$.

4. A commission merchant sold 127 bbl. of flour for me at \$7.50 per bbl; he paid \$12.85 freight, and this with his commission amounted to \$70: what was the rate per cent. of his commission? *Ans.* 6.

5. Paid an agent \$6.70 cartage and \$27 freight on \$6300 worth of goods. His entire bill was \$6680.20: what was the rate per cent. of his commission? *Ans.* $5\frac{1}{2}$.

CASE III.

The Commission or Brokerage and the Rate being given, to find the Base.

ORAL EXERCISE.

1. At 6% commission a merchant charged \$75: what was the amount of goods sold?

SOLUTION.— $6\% = \frac{6}{100}$ or $\frac{3}{50}$; if $\frac{3}{50}$ of the amount expended is \$75, $\frac{1}{50}$ is $\frac{1}{3}$ of \$75, or \$25; and $\frac{3}{50}$, or the amount invested, is 50 times \$25, or \$1250.

2. I paid \$26 commission at 4% to an agent in purchasing silks: what amount did he buy for me?

3. A tax-collector received \$160 for collecting taxes at 5%: what amount did he collect?

4. A commission merchant sold produce for me, and charged \$54 at 8% commission: what amount did he sell?

WRITTEN PROBLEMS.

1. An attorney's fees for collecting a debt at $2\frac{1}{2}\%$ commission were \$65: what was the debt? *Ans.* \$2600.

SOLUTION. $2\frac{1}{2}\% = .025$
 $\$65 \div .025 = \$2600$

Or, $2\frac{1}{2}\% = \frac{2\frac{1}{2}}{100}$ or $\frac{1}{40} = \$65$

$\frac{1}{40} = 40 \times \$65 = \$2600.$

Let the pupil explain and derive a rule.

2. When \$56.25 brokerage is paid at 9%, what is the amount of the sale? *Ans.* \$625.

3. When \$171.78 is paid as commission at $3\frac{1}{2}\%$, what is the amount of sales? *Ans.* \$4908.

4. Paid an attorney \$57.82 $\frac{1}{2}$ for collecting a claim at $4\frac{1}{2}\%$: what was the claim? *Ans.* \$1285.

5. A commission merchant charged \$25.50 commission at $2\frac{1}{2}\%$ for selling 120 bbl. of flour: what did he sell it at per barrel, and how much did he pay over to the owner?

Ans. \$8.50 per bbl.; paid over \$994.50.

CASE IV.

When the Rate and the entire Cost are given, to find the Commission.

ORAL EXERCISE.

1. A man sends his agent \$260 to invest in goods, after retaining his commission of 4% on the amount expended: what was the agent's commission?

SOLUTION.— $4\% = \frac{4}{100}$; $\frac{4}{100}$, the amount expended, $+\frac{4}{100}$ of the amount expended $= \frac{8}{100}$ of the amount expended, which is equal to \$260; $\frac{4}{100}$ of the amount expended $= \frac{4}{100}$ of \$260, or \$10; and $\frac{4}{100}$, or the amount expended, equals 25 times \$10, or \$250.

2. What amount of goods can be purchased for \$318 if the agent retain 6% on the amount expended?

3. A real-estate agent received \$3000 to invest in land, after deducting his commission of $6\frac{1}{4}\%$: what amount did he invest?

4. How many barrels of flour at \$7 a barrel can an agent buy for \$441, after taking out his commission of 5%?

5. A laborer agreed to husk corn for 8% of the farmer's share. The whole amount husked was 540 bu.: how much did each get?

WRITTEN PROBLEMS.

1. An agent is paid 6% for buying goods: what amount can he buy with \$1325, after deducting his commission?

Ans. \$1250.

SOLUTION.

$$\begin{array}{r} 1.06 \overline{)1325} \\ 1250 \end{array}$$

Let the pupil give the explanation.

RULE.

Divide the amount by 1, plus the rate expressed decimally.

2. If I pay an agent $3\frac{1}{4}\%$ commission to purchase goods, what amount can he purchase for \$2846.25? *Ans.* \$2750.

3. A merchant sent an agent \$1610 with which to buy goods; the agent charged \$11 cartage, and $2\frac{1}{2}\%$ commission on the amount purchased: what amount was purchased?

Ans. \$1560.

4. A broker sold 125 shares of railroad stock at \$48 a share, and invested the proceeds in bank stock at \$39.80 per share, after deducting his commission of $\frac{1}{2}\%$ on the sale: how many shares did he buy? *Ans.* 150.

5. A country merchant forwarded 160 bbl. of flour to be sold at \$6.25 a barrel, the agent receiving a commission of 3% for selling. After paying \$5.45 for cartage and deducting his commission of $1\frac{1}{2}\%$, he invested the proceeds in plaster at \$19 a ton: how many tons did he buy? *Ans.* 50.

6. An agent is paid $1\frac{1}{2}\%$ for purchasing goods: what amount does he purchase for \$629.30? *Ans.* \$620.

SECTION V.

INSURANCE.

173. Insurance is a contract by which a company agrees to pay the party insured a certain amount of money in case of loss or damage.

174. There are two kinds of insurance—*Property* insurance and *Life* insurance.

175. The written agreement between the company and the party insured is called a *Policy of Insurance*.

176. The **Insurer**, or **Underwriter**, is the one who agrees to make good the loss, and the person insured is called the **Policy-holder**.

177. The **Premium** is the sum paid for the insurance. It is a certain percentage of the amount insured.

Insurance is generally conducted by companies: these are either joint-stock companies or mutual companies, or a combination of the two, called mixed companies.

178. A **Joint-stock** company is one in which all the profits or the losses are divided among the stockholders.

179. A **Mutual** company is one in which the profits or the losses are shared alike by all the policy-holders.

180. Property insurance embraces *Fire Insurance* and *Marine and Inland Insurance*.

181. Fire Insurance is indemnity for loss of property by fire.

182. Marine and Inland Insurance is indemnity for loss of vessels or cargoes, whether at sea or on inland waters.

183. Accident Insurance, in which a company agrees to indemnify one against accident, is usually combined with life insurance.

184. Stock Insurance is that in which a company agrees to insure the lives of horses, cattle, etc.

ORAL EXERCISE.

1. A house worth \$1500 is insured at 2% : what is the premium?

SOLUTION.—2%, or $\frac{2}{100}$ of the value, equals the premium; $\frac{2}{100}$ of \$1500 is \$30, the premium.

2. I insure \$1200 worth of furniture at 1% : what is the premium?

3. A man's house worth \$5000 is insured for 3 years at 3% : what is the premium?

4. What is the premium at $\frac{3}{4}$ % on a building worth \$2000?

5. If I pay \$40 premium on an insurance of \$2000, what is the rate?

6. What is the rate of \$3000 worth of insurance to require \$15 premium?

7. A insures his store, worth \$6000, and pays \$90 premium : what is the rate?

8. My premium at 4% on property insured is \$60 : what is the value of the property?

9. I insure my furniture at $1\frac{1}{2}$ %, and pay \$21 : what is the furniture worth?

10. A piano is insured at $\frac{3}{4}$ %, and the premium is \$4.50 : what is the value?

WRITTEN PROBLEMS.

1. A property valued at \$1250 is insured at $1\frac{1}{2}$ % : what is the premium?

Ans. \$18.75.

SOLUTION.

$$\begin{array}{r} \$1250 \\ .01\frac{1}{2} \\ \hline \$18.75 \end{array}$$

Let the pupil explain and derive the rule.

2. A store valued at \$7500 is insured at $2\frac{1}{4}$ % : what is the premium?

Ans. \$168.75.

3. A planing-mill, worth \$3200, is insured at $4\frac{1}{2}$ % : what is the premium?

Ans. \$144.

4. A merchant insures his stock for \$3500 at $1\frac{3}{4}$ %, and his house, worth \$3000, at $\frac{3}{4}$ % : what is the premium?

Ans. \$83.75.

5. A man insures his house, worth \$5500, at $1\frac{1}{4}\%$; his barn, worth \$3300, at 1% ; and his furniture, worth \$1500, at $1\frac{1}{4}\%$: what was his total premium? *Ans.* \$118.62 $\frac{1}{2}$.

6. A dealer buys 1500 bu. of wheat at \$1.10 per bushel, and insures it at \$1.20 per bushel at $2\frac{1}{2}\%$: if destroyed by fire, what is his gain? *Ans.* \$105.

7. A stock of goods worth \$6000, and insured at $2\frac{1}{4}\%$, was damaged $65\frac{1}{4}\%$: what did the company have to pay above the premium? *Ans.* \$3810.

8. A block of buildings worth \$150000 is insured for $\frac{2}{3}$ of its value in three companies; the first takes a $\frac{1}{3}$ risk at $\frac{3}{8}\%$; the second a $\frac{1}{3}$ risk at $\frac{3}{8}\%$; and the third, the remainder at $\frac{3}{8}\%$: what was the premium? *Ans.* \$717.19-.

9. If the block named in the last problem should be damaged by fire to the extent of \$75000, how much insurance money would each company have to pay?

Ans. 1st, \$15000; 2d, \$18750; 3d, \$41250.

10. An insurance company took a risk on a vessel for \$100000 at $\frac{3}{4}\%$, but deeming the risk too great reinsured \$60000 at $\frac{7}{8}\%$: what was the amount received more than paid? *Ans.* \$225.

11. Paid \$54 to insure property valued at \$3600: what was the rate? *Ans.* $1\frac{1}{2}\%$.

12. Paid \$78.75 to insure my store for \$4500: what was the rate? *Ans.* $1\frac{3}{4}\%$.

13. If I pay \$150 for insuring \$8000 worth of goods, what is the rate? *Ans.* $1\frac{7}{8}\%$.

14. If I pay \$234 to insure a house worth \$5200, what is the rate? *Ans.* $4\frac{1}{2}\%$.

15. If a person 33 yr. of age pay \$123.90 annual premium on a life-insurance policy of \$5000, what is the rate?

Ans. $2\frac{3}{8}\frac{3}{8}\%$.

16. If the cost of insuring a property at $2\frac{1}{4}\%$ is \$60.75, what is the value of the property? *Ans.* \$2700.

17. If at $2\frac{1}{2}\%$ my premium is \$88.75, what is the value of the property insured? *Ans.* \$3550.

18. The premium at 3% is \$378: what is the value of the property insured? *Ans.* \$12600.

19. At what must a property worth \$1950 be insured at 2½% so as to include premium in case of loss? *Ans.* \$2000.

SUGGESTION.— $1950 \div .97\frac{1}{2} = \$2000$.

20. A dealer stores 9700 bu. of wheat worth \$1.20 a bushel; he insures it at 3%, so as to cover premium and loss: what was the amount insured for? *Ans.* \$12000.

SECTION VI.

TAXES.

185. A Tax is a sum of money assessed on the property or income of an individual for the support of the government or for other public purposes.

186. A Poll Tax, called also a *capitation tax*, is a tax on every male citizen 21 years of age or upward, without regard to his property.

187. A Property Tax is assessed on property according to its estimated value.

188. Property is of two kinds—*Real Estate* and *Personal Property*.

189. Real Estate is fixed property—such as houses, lands, etc.

190. Personal Property is movable property, and consists of stocks, watches, furniture, cattle, etc.

191. An Assessor is an officer appointed to estimate the value of property, prepare the assessment rolls and apportion the taxes.

192. An Assessment Roll is a list containing the names of all persons in the district liable to taxation, together with the valuation of each one's property.

193. A Tax-Rate is the rate per cent. of the valuation of the property of a district.

WRITTEN PROBLEMS.

1. The tax-rate in a district is $1\frac{1}{2}\%$: what is B's tax if his property is valued at \$2500?
Ans. \$37.50.

2. Mr. Smith's property is valued at \$1600; he pays a poll-tax of \$1, and the tax-rate is 23 mills on the dollar: what is his tax?
Ans. \$37.80.

3. My house is assessed at \$1500, a vacant lot at \$300, and my watch at \$50; my occupation is valued at \$200, and the tax-rate for school purposes is $1\frac{3}{10}\%$: what is my tax?
Ans. \$26.65.

4. The property of a town is valued at \$2500000; a tax of \$28000 is necessary to support the public schools: what is the rate?
Ans. $1\frac{2}{5}\%$.

5. A school board find it necessary to raise a tax of \$2750; the assessed value of property is \$210000. If a tax-rate of $1\frac{4}{10}\%$ is to be adopted, what will be the surplus tax raised?
Ans. \$190.

6. It is necessary to raise a tax of \$3200 on an assessed valuation of \$180000: what is the rate if \$140 is the amount of poll-tax?
Ans. .017.

7. In a district in which the property is valued at \$950000 a tax of \$33250 is raised: what is the rate, and how much tax does A pay, whose property is assessed at \$15370?
Ans. Rate, $3\frac{1}{2}\%$; A's tax, \$537.95.

8. In a district whose valuation was fixed at \$640000 a tax of \$8960 is raised: what is B's property worth if he pay a tax of \$102.90?
Ans. \$7350.

9. In a city whose valuation is \$6748950, the tax-rate is fixed at $3\frac{4}{10}\%$; the collector is paid $1\frac{1}{2}\%$ for collection: what is the net amount collected, and what are the collector's fees?
Ans. Amt., \$239317.77 -; fees, \$3644.43 +.

10. A tax of \$2910 is to be raised in a town whose valuation is \$175600: what is the rate if there are 276 polls at \$1 each, and what is B's tax, who owns a house valued at \$1200, a watch worth \$50, and pays 1 poll?
Ans. Rate, $1\frac{1}{2}\%$; B's tax, \$19.75.

SECTION VII.

CUSTOMS, OR DUTIES.

194. Customs, or Duties, is the name given to taxes which are levied on goods exported or imported.

In the United States this tax is levied on imported goods only.

195. Certain towns are designated by the government as places for exportation and importation. These are called *Ports of Entry*, and in each of these ports there is established a *Custom-House*, where these duties are assessed and collected. The officer appointed to take charge of the custom-house is called the *Collector of Customs* or *Collector of the Port*.

196. Duties are of two kinds—*Specific* and *Ad Valorem*.

197. A Specific Duty is one laid on an article without reference to its value. An **Ad-Valorem Duty** is a percentage on the cost of an article where purchased.

198. An Invoice, or Manifest, is an inventory or list of goods, setting forth their cost at the place from which they are imported.

199. A Tonnage Duty or Tax is a tax laid on the vessel with regard to its cargo for the privilege of entering port.

200. In specific duties certain allowances are made, called *Draft, Tare, Leakage, Breakage*, etc.

201. Draft is an allowance made for waste, so that goods may be of full measure when retailed.

202. Tare is an allowance made for the weight of the box, cask, etc., containing the goods.

203. Leakage is an allowance made for the waste of liquids imported in casks or barrels.

204. Breakage is an allowance for the loss of liquids imported in bottles.

205. Gross weight or quantity is the weight or quantity before allowance is made.

206. Net weight or quantity is the weight or quantity after the allowances have been deducted.

WRITTEN PROBLEMS.

1. What is the duty on 644 yd. of silk, worth \$1.20 a yard, at 40% ? *Ans.* \$309.12.

SOLUTION.

$644 \times \$1.20 = \$772.80$ Let the pupil explain, and
40% of \$772.80 = $.40 \times \$772.80 = \$309.12$ form the rule.

2. What is the duty at 30% on 60 pieces of satin, each piece containing $48\frac{1}{2}$ yd., at \$1.80 a yard? *Ans.* \$1571.40.

3. A merchant imported 25 cases of cloth containing 24 pieces each, with 45 yd. in a piece, and worth \$1.13 a yard: what was the duty at 28% ? *Ans.* \$8542.80.

4. What is the duty on 80 hhd. and 20 gal. of syrup, worth 60 cts. a gallon, duty at $12\frac{1}{4}\%$? *Ans.* \$379.50.

5. A merchant imported 1250 yd. of silk worth \$2.75 a yard, 1640 yd. of cloth at \$2.05 a yard, and 1080 yd. of cassimere at \$1.85 a yard: what is the total duty if he pays 32% on woollen and 45% on silk goods? *Ans.* \$3262.07+.

6. What is the duty on 12 casks of wine containing 42 gal. each, invoiced at \$1.30 per gallon, at $31\frac{1}{2}\%$, leakage 5% ? *Ans.* \$196.07-.

7. A dealer imported 120 doz. champagne, invoiced at \$23 a dozen, breakage $12\frac{1}{2}\%$: what was the duty at 22% ? *Ans.* \$531.30.

8. A grocer imports 540 bags of coffee, gross weight 160 lb. each, tare $3\frac{1}{4}\%$, invoiced at 9 cents a pound: what was the duty at 15% ? *Ans.* \$1125.576.

9. A paid \$184.80 duty on 50 bbl. of sugar weighing 220 lb. each, gross, tare 4%, invoiced at 5 cents a pound: what was the rate? *Ans.* 35%.

10. A wine-merchant imported 60 casks of wine, 54 gal. each, invoiced at \$1.80 a gallon; transportation, 25 cents a gallon; cartage, \$8.50; leakage, 5%; and duty 20%: what was the entire cost? *Ans.* \$7758.58.

11. A merchant imported 40 rolls carpet, 160 yd. each, and paid \$1024 duty at 20% : at what price per yard was the carpet invoiced ?
Ans. 80 cents.

SECTION VIII.

CAPITAL AND STOCK.

207. A **Company** is a number of individuals associated to transact business.

208. A **Corporation** is an association of individuals authorized by law to transact business as one person.

209. A **Charter** is the legal act of incorporation defining the powers and obligations of the corporation.

210. The **Capital Stock** of a corporation is the money employed to carry on the business of the company.

211. A **Certificate of Stock** is a paper showing the number of shares of stock to which one is entitled in the corporation.

212. A **Share** is one of the equal parts into which the stock is divided. The value of a share varies in different corporations. In railway and similar companies it is usually \$100.

213. The **Par Value** is the sum for which the certificate was issued.

214. The **Market Value** is the sum for which it can be sold.

215. Stock is said to be *above par* or *at a premium* when it will sell at more than its face or par value, and *below par* or *at a discount* when it sells for less than its face value. When shares sell at their face value, the stock is said to be *at par*.

216. **Instalments** are portions of the capital stock subscribed by each stockholder, and are paid at different times as demanded by the company.

217. The **Gross Earnings** of a corporation are the total receipts from the regular business of the company.

218. The **Net Earnings** are the receipts after the entire expenses of the company are deducted.

219. Dividends are certain portions of the net earnings distributed among the stockholders according to the amount of stock owned by each.

220. Assessments are sums required of the stockholders, according to the amount of stock held by each, to meet losses or expenses of the company.

221. Dealing in stocks is called *stock-jobbing*, and the persons engaged in the business are called *stock-jobbers* or *stock-brokers*.

CASE I.

To find the Total Cost, the Value of a Share and the Number of Shares being given.

ORAL EXERCISE.

NOTE.—When the par value of shares is not mentioned, it is understood to be \$100.

1. When stock is at 5% premium, what is the market value of \$1 in stock? Of \$20? Of \$100?
2. When stock is at 10% discount, what is the value of \$1 in stock? Of \$50? Of \$100? Of \$125?
3. What cost 20 shares of railroad stock at 15% premium?
4. What is the cost of 40 shares of railroad stock at 20% discount?

WRITTEN PROBLEMS.

1. What is the cost of 45 shares of bank stock at $5\frac{1}{8}\%$ premium, brokerage $\frac{1}{8}\%$?

$$\begin{aligned}\text{SOLUTION.} & -100\% + 5\frac{1}{8}\% + \frac{1}{8}\% = 105\frac{1}{8}\% \\ & 105\frac{1}{8}\% \times \$100 = \$105.50 \\ & 45 \times \$105.50 = \$4747.50\end{aligned}$$

2. What cost 62 shares Erie Railroad stock at 27% discount, brokerage $\frac{1}{8}\%$? Ans. \$4533.75.
3. What is the cost of 65 shares Erie Railroad stock when quoted at $85\frac{1}{4}$, brokerage $\frac{1}{8}\%$? Ans. \$5549 $\frac{3}{8}$.

4. Find the cost of 75 shares bank stock when the market value is $103\frac{1}{2}$, brokerage $\frac{1}{8}\%$? *Ans.* \$7790 $\frac{1}{2}$.

5. Bought railroad stock at $63\frac{1}{2}$, and sold at 75, brokerage $\frac{1}{8}\%$ each way: what did I make on 100 shares? *Ans.* \$1150.

CASE II.

To find the Number of Shares, the Loss or Gain and the Loss or Gain per cent. being given.

ORAL EXERCISE.

1. I bought mining stock at 80, and selling at 65 I lost \$3000: how many shares did I buy?

SOLUTION.—If I bought stock at 80 and sold at 65, I lost the difference, or \$15 a share, which is 15% of the par value. \$3000 is 15% of \$20000, which is the value of as many shares as \$100 is contained times in \$20000, or 200 shares.

2. Bought canal stock at par and sold at 60%, thereby losing \$200: how many shares did I buy?

3. Bought telegraph stock at 95 and sold at 103, thereby making \$400: how many shares did I buy?

4. Bought railroad stock at 115, but sold at 80, and lost \$7000: how many shares were bought?

WRITTEN PROBLEMS.

1. If I bought stock at par and sold at $107\frac{1}{2}$, thereby gaining \$1740, how many shares did I buy? *Ans.* 240.

2. Bought stock at par and sold at $76\frac{1}{2}$, thereby losing \$3720: how many shares were there? *Ans.* 160.

3. Bought bank stock at $96\frac{1}{2}$ and sold at $112\frac{1}{2}$, thereby gaining \$3556: how many shares were there? *Ans.* 224.

CASE III.

To find the Amount of Dividend or Assessment, the Rate and the Number of Shares being given.

ORAL EXERCISE.

1. A bank declares a dividend of $5\frac{1}{2}\%$: what is that man's share who owns 20 shares of stock?

SOLUTION.—Since the dividend is $5\frac{1}{2}\%$, it is $\$5\frac{1}{2}$ on the \$100, or $\$5\frac{1}{2}$ on a share; on 20 shares it is therefore 20 times $\$5\frac{1}{2}$, or \$110.

2. If a bank declare a dividend of $4\frac{1}{2}\%$, what is that man's dividend who holds 60 shares of stock?

3. A railway company declares a semi-annual dividend of $3\frac{1}{2}\%$: what is that man's annual dividend who owns 24 shares of stock?

4. An insurance company meeting with a heavy loss by fire levies an assessment of 15% : what is that man's assessment who holds \$2500 worth of stock?

WRITTEN PROBLEMS.

1. A capitalist holds 650 shares of stock in a corporation that declares a dividend of $6\frac{3}{4}\%$: what is his share?

Ans. \$4387.50.

2. A banking company declares a dividend of $12\frac{1}{4}\%$: what is that man's share of the dividend who owns 250 shares of stock?

Ans. \$3187.50.

3. When certain railroad stock was selling at 115 a capitalist invested \$11040. The company afterward declared a dividend of 9% : what was his share of the dividend?

Ans. \$864.

4. When telegraph stock was selling at $72\frac{1}{4}$ I invested \$12731.25. The company declared a quarterly dividend of $2\frac{1}{2}\%$: what was my annual dividend at this rate, and what per cent. did I make on my investment?

Ans. Div., \$1750; profit, $13\frac{3}{11}\%$.

CASE IV.

To find the Amount of Stock, the Dividend and the Rate per cent. being given.

ORAL EXERCISE.

1. A bank declares a dividend of 5% : how much stock do I own if I receive \$300 as my dividend?

\$300 is 5% , or $\frac{1}{20}$, of my stock. If \$300 is $\frac{1}{20}$ the value of my stock, $\frac{1}{20}$, or the value, is 20 times \$300, or \$6000.

2. If a stockholder in a bank receive \$240 as his share of a 6% dividend, how much stock does he own?

3. If a banking company declare a dividend of 15%, how much stock does that man own whose dividend is \$375?

4. Mrs. B received \$450 as her share of a semi-annual dividend of 3% on certain railroad stocks: how much did she own?

WRITTEN PROBLEMS.

5. A manufacturing company, wishing to make some improvements, made an assessment of $13\frac{3}{4}\%$: how much stock does A own, whose assessment is \$428? *Ans.* \$3200.

6. A gentleman receives \$700 as his share of a $12\frac{1}{4}\%$ dividend: how many shares of stock does he own if the shares are worth \$50 each? *Ans.* 112.

7. Bought some railroad stock at 73, then received \$945 as a dividend at $2\frac{1}{4}\%$, when I sold my stock at $86\frac{1}{4}$: how much stock had I, and what did I make?

Ans. 420 shares; made \$6510.

8. A capitalist bought stock at 65, and after receiving a dividend of $5\frac{1}{2}\%$ sold it at 82, and made \$1125: how much stock had he, and what per cent. did he realize?

Ans. 50 shares; realized $22\frac{1}{2}\%$.

CASE V.

To find the Rate per cent., the Amount of Stock and the Dividend being given.

ORAL EXERCISE.

1. If the stock of a company is \$20000, what rate of dividend can be declared if the earnings are \$4000?

SOLUTION 1.—If \$20000 gain \$4000, \$1 will gain $\frac{4000}{20000}$ of \$4000, or $\frac{4000}{20000} = \frac{1}{5}$, or 20%.

SOLUTION 2.—As many per cent. dividend can be declared as 4000 is per cent. of 20000; 4000 is 20% of 20000; hence, 20% dividend can be declared.

2. If \$200 worth of stock gain \$9, what rate of dividend can be declared?

3. If \$5000 worth of stock gain \$150, what rate of dividend can be declared?

4. A gentleman owning 20 shares of bank stock was paid an annual dividend of \$160: what was the rate?

WRITTEN PROBLEMS.

5. Mr. C owns 250 shares of bank stock, and receives as his semi-annual dividend \$1125: what was the rate? *Ans.* $4\frac{1}{2}\%$.

6. A company whose capital is \$500000 earns \$26500: if they declare a dividend whose rate is expressed by a whole number, what is the rate, and what the surplus?

Ans. Rate, 5%; surplus, \$1500.

7. Suppose a company with a capital of \$2000000 earns \$265000, what rate of dividend can it declare if \$15000 is placed aside as surplus? *Ans.* $12\frac{1}{2}\%$.

8. The earnings of a company whose capital is \$400000 are \$86500, and the expenses \$12500: if a surplus of \$12000 be reserved, what dividend can be declared? *Ans.* $15\frac{1}{2}\%$.

CASE VI.

To find the Amount of Stock, the Premium and the Investment being given.

ORAL EXERCISE.

1. How much stock can be bought at 10% premium for \$880?

SOLUTION.—At 10% premium, stock is worth \$110 per share; hence, for \$880 as many shares can be bought as \$110 is contained times in \$880, or 8 shares.

2. What amount of stock can be bought at 5% discount for \$1900?

3. How many shares of canal stock at 20% discount can be bought for \$1640, par value being \$25 a share?

4. When Reading Railroad stock is 12% above par, how many shares can be purchased for \$8400, par value of shares being \$50 each?

WRITTEN PROBLEMS.

5. When bank stock is at a premium of $4\frac{3}{4}\%$, how many shares can be purchased for \$12570? *Ans.* 120.

6. I invested \$13125 in railroad stock at $87\frac{1}{2}\%$, and sold it at 91: how much stock did I own, and what was my gain?

Ans. Stock, \$15000; gain, \$525.

7. If stock is bought at $3\frac{1}{2}\%$ discount, and sold at a premium of $2\frac{1}{4}\%$, and the gain is \$258.75, what is the par value of the stock?
Ans. \$4500.

8. If a capitalist buy \$9040 worth of stock at 113, and after receiving a $5\frac{1}{2}\%$ dividend sells at $111\frac{1}{2}$, how much does he make, and how much stock did he buy?

Ans. Made, \$320; had \$8000 stock.

SECTION IX.

COMPARISON OF GOLD AND CURRENCY.

ORAL EXERCISE.

1. How much currency can be bought for \$200 gold when gold is at a premium of 13%?

SOLUTION.—When gold is at a premium of 13%, \$1 in gold is worth \$1.13 in currency, and \$200 in gold is worth 200 times \$1.13, or \$226 in currency.

2. When gold is at a premium of 10%, how much gold may be bought for \$132 currency?

SOLUTION.—When gold is at a premium of 10%, \$1 gold is worth \$1.10 in currency, and \$132 in currency is worth as much gold as \$1.10 is contained times in \$132, or \$120.

3. When gold is at a premium of 15%, how much currency can you buy for \$40 in gold?

4. When gold is at a premium of $12\frac{1}{2}\%$, how much currency can be bought for \$120 in gold?

5. How much gold at 12% premium can be bought for \$560 currency?

6. How much gold at 8% premium can be bought for \$1080 currency? For \$648 currency? For \$270 currency?

WRITTEN PROBLEMS.

7. What is the value in currency of \$2740 gold at a premium of $11\frac{3}{8}\%$?
Ans. \$3051.675.

8. What is the currency value of \$6500 gold at a premium of $13\frac{7}{8}\%$?
Ans. \$7401.87 $\frac{1}{2}$.

9. Bought goods for \$4850, payable in gold: what must be paid in currency when gold is at a premium of $16\frac{1}{4}\%$?

Ans. \$5638.12 $\frac{1}{2}$.

10. When gold is at a premium of 13%, what is the value in gold of a \$1 bill?

Ans. 88 $\frac{5}{11}$ cents.

11. Paid for a gold watch \$220 currency when gold was at a premium of 75%: what was the gold value of the watch?

Ans. \$125.71 $\frac{3}{4}$.

12. I imported some silks for which I paid in currency \$7695 when gold was at a premium of $12\frac{1}{2}\%$: what was the gold value?

Ans. \$6840.

SECTION X.

INTEREST.

222. Interest is a certain percentage paid for the use of money. It is reckoned at a certain rate per cent. for each year.

223. The Principal is the sum on which the interest is paid.

224. The Rate is the percentage paid per annum.

225. The Amount is the sum of the principal and the interest.

226. Legal Interest is the rate of interest fixed by law. When more than the legal rate is charged it is called **Usury**.

When in contracts, etc. no rate of interest is mentioned, the *legal rate* is always understood.

The following are the legal rates in the States named:

Delaware, Maryland, New Hampshire, Pennsylvania and the United States courts.....	6%
Connecticut, New Jersey, New York.....	7%
Alabama.....	8%
Idaho, Montana.....	10%
Wyoming.....	12%

In most of the other States the rate is not definitely fixed, but

is variable. In some also any rate can be fixed upon, if stipulated in writing.

227. Simple Interest is that which is reckoned upon the principal alone.

228. Compound Interest is that which is reckoned upon the *principal*, and also upon the interest as it comes due.

SIMPLE INTEREST.

1. GENERAL METHOD.

ORAL EXERCISE.

1. What is the interest of \$100 for 3 yr. at 6% ?

SOLUTION.—At 6%, $\frac{6}{100}$, or $\frac{3}{50}$ of the principal, equals the interest for 1 yr., and for 3 yr. 3 times $\frac{3}{50}$, or $\frac{9}{50}$ of the principal; $\frac{9}{50}$ of \$100 equal \$18; hence the interest of \$100 for 3 yr. at 6% is \$18.

2. What is the interest of \$200 for 1 yr. at 5% ?
3. What is the interest of \$600 for 3 yr. at 7% ?
4. What is the interest of \$450 for 2 yr. at 5% ?
5. What is the interest of \$320 for 2 yr. at 6% ?

SOLUTION.—If the interest for 1 yr. is 6% of the principal, for 2 yr. it is 2 times 6%, or 12% of the principal; and 12% of \$320 is \$38.40.

6. What is the interest of \$90 for 8 yr. at 5% ?
7. What is the interest of \$66 for 10 yr. at 6% ?
8. What is the interest of \$350 for 7 yr. at 6% ?
9. What is the interest of \$24 for 2 yr. at $16\frac{2}{3}\%$?
10. What is the interest of \$240 for 3 yr. at 15% ?
11. What is the interest of \$120 for 6 yr. at 7% ?
12. What is the interest of \$250 for 6 yr. 6 mo. at 6% ?

SOLUTION.—6 yr. 6 mo. equal $6\frac{1}{2}$ yr. If the interest for 1 yr. is 6%, for $6\frac{1}{2}$ yr. it is $6\frac{1}{2}$ times 6%, or 39% ; 39% of \$250 is \$97.50.

13. What is the interest of \$120 for 3 yr. 8 mo. at 6% ?
14. What is the interest of \$360 for 6 yr. 3 mo. at 8% ?
15. What is the interest of \$500 for 7 yr. 2 mo. at 6% ?
16. What is the interest of \$600 for 3 yr. 7 mo. at 5% ?
17. What is the interest of \$12 for 2 yr. 9 mo. at 10% ?
18. What is the interest of \$750 for 3 yr. 5 mo. at 7% ?

19. What is the interest of \$150 for 2 yr. 3mo. 18d., at 5% ?

SOLUTION.—18 d. equal $\frac{1}{8}$, or $\frac{1}{2}$ of a month; hence, 3 mo. and 18 d. equal $3\frac{1}{2}$, or $3\frac{1}{2}$ mo. $\frac{1}{2}$ of a month equals $\frac{1}{2}$ of $\frac{1}{12}$ of a year, which is $\frac{1}{24}$ of a year, and $3\frac{1}{2}$ mo. equals $3\frac{1}{2}$ of $\frac{1}{12}$, or $\frac{11}{8}$, or $1\frac{5}{8}$ of a year; hence, 2 yr. 3 mo. 18 d. equals $2\frac{5}{8}$ yr. If the interest is 5% for 1 yr., for $2\frac{5}{8}$ yr. it is $2\frac{5}{8}$ times 5%, or $11\frac{5}{8}\%$; $11\frac{5}{8}\%$ of \$150 is \$17.25.

20. What is the interest of \$300 for 6 yr. 2 mo. 12 d., at 6% ?
 21. What is the interest of \$270 for 3 yr. 7 mo. 6 d., at 5% ?
 22. What is the interest of \$300 for 3 yr. 24 d., at 15% ?
 23. What is the interest of \$600 for 1 yr. 6 mo. 20 d., at 9% ?
 24. What is the interest of \$250 for 1 yr. 1 mo. 6 d., at 10% ?

WRITTEN PROBLEMS.

1. What is the interest of \$620 for 6 yr. 3 mo., at 6% ?

Ans. \$232.50.

SOLUTION.

$$\begin{array}{r} \$620 \\ .06 \\ \hline 37.20 \\ 6\frac{1}{4} = 6 \text{ yr. } 3 \text{ mo.} \\ \hline 223.20 \\ 9.30 \\ \hline 232.50 \end{array}$$

Analysis.—6% of \$620 is \$37.20, the interest for 1 yr., and for 6 yr. 3mo., or $6\frac{1}{4}$ yr., the interest is $6\frac{1}{4}$ times \$37.20, or \$232.50.

RULE.

To find the simple interest, multiply the principal by the rate expressed decimally, and this result by the number of years.

2. What is the interest of \$225 for 6 yr. 8 mo., at 6% ?

Ans. \$90.

3. What is the interest of \$75 for 3 yr. 4 mo., at 5% ?

Ans. \$12.50.

4. What is the interest of \$630 for 6 yr. 4 mo., at 7% ?

Ans. \$279.30.

5. What is the amount of \$315 for 3 yr. 7 mo., at 8% ?

Ans. \$405.30.

NOTE.—To find the *amount*, add the principal to the interest.

6. What is the interest of \$625 for 2 yr. 3 mo. 12 d., at 6% ?

Ans. \$85.62 $\frac{1}{2}$.

7. What is the interest of \$126 for 5 yr. 6 mo. 18 d., at 5% ? *Ans.* \$34.96 $\frac{1}{4}$.

NOTE.—The years, months and days may be reduced to the decimal part of a year. Thus, 5 yr. 6 mo. 18 d. is 5.55 yr. The interest on \$126 for 1 yr. at 5% is \$6.30; and for 5.55 yr. it is 5.55 times \$6.30, or \$34.96 $\frac{1}{4}$. This method is often much the shortest.

8. What is the amount of \$1785 for 4 yr. 4 mo. 4 d., at 4% ? *Ans.* \$2095.19.

9. What is the interest of \$640.20 for 3 yr. 3 mo. 3d., at 6 $\frac{1}{2}$ % . *Ans.* \$135.59.

2. SIX PER CENT. METHOD.

In calculating interest 30 days are reckoned to the month, and 12 months, or 360 days, to the year.

The interest on \$1 for 1 year being 6 cents, for one month it is $\frac{1}{12}$ of 6 cents, or $\frac{1}{2}$ a cent; hence, the interest on a dollar is as many cents as half of the number of months.

The interest for 30 days being $\frac{1}{2}$ a cent, or 5 mills, for 1 day it is $\frac{1}{60}$ of 5 mills, or $\frac{1}{6}$ of a mill; hence, the interest on \$1 is as many mills as one-sixth the number of days.

From the foregoing we derive the following rule to find the interest at 6% per annum:

RULE.

Write one-half the number of months as cents, and one-sixth the number of days as mills, and multiply the sum of these by the number of dollars.

WRITTEN PROBLEMS.

1. What is the interest of \$955 for 5 yr. 3 mo. 12 d., at 6% ? *Ans.* \$302.735.

SOLUTION.
5 yrs. 3 mo. = 63 mo.

$$\begin{array}{r} \frac{1}{2} \text{ of } 63 = 31\frac{1}{2} = \$315 \\ \frac{1}{6} \text{ of } 12 = 2 = .002 \\ \hline .317 \end{array}$$

$$955 \times \$317 = \$302.735.$$

Analysis.—5 yr. 3 mo. equal 63 mo. Writing $\frac{1}{2}$ of 63 mo., or 31 $\frac{1}{2}$ mo., as cents, and $\frac{1}{6}$ of 12 days, or 2 days, as mills, we have for the interest of \$1 for the given time \$.317, and multiplying this by 955, the number of dollars, we have the required interest, \$302.735.

NOTE.—To find the interest at any other per cent., divide the interest at 6% by 6, and multiply by the given rate.

2. What is the interest of \$644 for 6 yr. 7 mo., at 6% ?
Ans. \$254.38.
3. What is the interest of \$725 for 9 yr. 5 mo., at 5% ?
Ans. \$341.35.
4. What is the interest of \$650.30 for 6 yr. 3 mo., at 8% ?
Ans. \$325.15.
5. What is the amount of \$720.50 for 3 yr. 9 mo., at 10% ?
Ans. \$990.69.
6. What are the interest and the amount of \$85 for 2 yr. 8 mo. 12 d., at 6% ?
Ans. Int., \$13.77; amt., \$98.77.
7. What are the interest and the amount of \$1250 for 3 yr. 7 mo. 15 d., at 8% ?
Ans. Int., \$362.50; amt., \$1612.50.
8. What is the interest on \$550 from May 8, 1876, to June 13, 1877, at 6% ?
Ans. \$36.208.
9. What is the interest on \$1116 from Aug. 16, 1873, to Mar. 28, 1875, at 7% ?
Ans. \$126.294.
10. What are the amount and the interest on \$125.40 from Sept. 25, 1876, to Mar. 16, 1878, at 6% ?
Ans. Amt., \$136.50; int., \$11.10.

3. METHOD BY ALIQUOT PARTS.

1. What is the interest of \$1250 for 3 yr. 5 mo. 18 d., at 8% ?
Ans. \$346.67.

SOLUTION.

\$1250 × .08 = \$100,	int. for 1 yr.	\$1250 for 1 yr., at 8% = \$100,
100 × 3 = 300,	int. for 3 yr.	and for 3 yr. 3 times \$100, or
25,	int. for 3 mo.	\$300; the interest for 3 mo. is $\frac{1}{4}$
16.67,	int. for 2 mo.	of \$100, or \$25; for 1 mo. it is
5,	int. for 18 d.	$\frac{1}{4}$ of \$25, or \$8.33 $\frac{1}{4}$, and for 2 mo.
Total interest = \$346.67.		2 times \$8.33 $\frac{1}{4}$, or \$16.67, and for

18 d., or $\frac{3}{4}$ of a month, $\frac{3}{4}$ of \$8.33 $\frac{1}{4}$, or \$5. Hence, the total interest for 3 yr. 5 mo. and 18 d. is \$300 + \$25 + \$16.67 + \$5, or \$346.67.

Explanation.—The interest of \$1250 for 1 yr., at 8% = \$100, and for 3 yr. 3 times \$100, or \$300; the interest for 3 mo. is $\frac{1}{4}$ of \$100, or \$25; for 1 mo. it is $\frac{1}{4}$ of \$25, or \$8.33 $\frac{1}{4}$, and for 2 mo. 2 times \$8.33 $\frac{1}{4}$, or \$16.67, and for

2. What is the interest of \$620 for 6 yr. 8 mo., at 6% ?
Ans. \$248.
3. What is the interest of \$800 for 3 yr. 3 mo. 15 d., at 7% ?
Ans. \$184.33.
4. What is the interest of \$660 for 2 yr. 2 mo. 2 d., at 9% ?
Ans. \$129.03.

5. What is the interest of \$300 for 1 yr. 1 mo. 1 d., at 10%?

Ans. \$32.58.

4. METHOD BY DAYS.

1. Find the interest of \$78 for 90 d., at 6%. *Ans.* \$1.17.

SOLUTION.

$$\frac{78 \times \overset{3}{90}}{\underset{2}{90}} = \frac{234}{2} = 117, \$1.17.$$

Analysis.—Since the interest on \$1 for 1 mo., or 30 d., at 6% is $\frac{1}{2}$ ct., for 60 d. it is 1 ct.; hence, $\frac{1}{6}$ of the number of days equals the number of cents interest. Multiplying this by 78, the number of dollars,

gives the total interest, 117 cts., or \$1.17.

From this we derive the following

RULES.

1. *Multiply the number of dollars by the number of days, and divide by 60; the result is the interest at 6% in cents.*

2. *When at any other per cent. than 6, multiply the number of dollars by the number of days, and this result by the given rate, and divide by 360.*

2. What is the interest of \$95 for 20 d., at 8%?

Ans. 42 cts.

$$\text{OPERATION. } \frac{95 \times \overset{4}{20} \times \underset{9}{8}}{\underset{360}{90}} = \frac{380}{9} = 42 = 42 \text{ cts.}$$

3. What is the interest of \$76 for 30 d., at 6%?

Ans. 38 cts.

4. What is the interest of \$1254 for 25 d., at 6%?

Ans. \$5.22 $\frac{1}{2}$.

5. What is the interest of \$96.50 for 36 d., at 6%?

Ans. \$.579.

6. What is the interest of \$117.35 for 48 d., at 6%?

Ans. \$.9388.

7. What is the interest of \$60 for 40 d., at 7%?

Ans. \$.46 $\frac{2}{3}$.

8. What is the interest of \$125 for 54 d., at 8%?

Ans. \$1.50.

9. What is the interest of \$1215 for 18 d., at 5 $\frac{1}{2}$ %?

Ans. \$3.34.

10. What is the interest of \$37.50 for 26 d., at 8%?

Ans. \$.21 $\frac{1}{2}$.

11. What is the interest of \$87.75 for 50 d., at 10%?

Ans. \$1.22.

12. What is the interest of \$118.63 for 45 d., at 7%?

Ans. \$1.038+.

13. What is the interest of \$78.45 for 1 mo. 6 d., at 6%?

Ans. \$.47+.

14. What is the interest of \$1260.60 for 6 mo. 14 d., at 7%?

Ans. \$47.55+.

229. Exact Interest may be found by reducing the time to days, dividing the interest for one year by 365, and multiplying by the given number of days.

Or, find the interest by any of the common methods, and take $\frac{7}{8}$ of the result for the common years, or $\frac{9}{11}$ for leap years.

SECTION XI.

GENERAL PROBLEMS IN INTEREST.

CASE I.

Principal, Interest and Time being given, to find the Rate.

ORAL EXERCISE.

1. At what per cent. will \$50 in 3 yr. give \$12 interest?

SOLUTION 1.—The interest of \$50 for 1 yr. at 1% is 50 cents, and for 3 yr. 3 times 50 cents, or \$1.50; to give \$12 interest will therefore require a rate of as many per cent. as \$1.50 is contained times in \$12, or 8%.

SOLUTION 2.—At 1% the interest on \$1 for 3 yr. is 3 cents, and on \$50 it is 50 times 3 cents, or \$1.50; it will therefore require as many per cent. as \$1.50 is contained times in \$12, or 8%.

2. At what per cent. will \$200 gain \$40 in 4 yr.?

3. At what per cent. will \$500 gain \$90 in 3 yr.?

4. At what per cent. will \$400 gain \$112 in $3\frac{1}{2}$ yr.?

5. At what per cent. will \$250 gain \$100 in 5 yr.?

6. At what per cent. will \$125 gain \$70 in 8 yr.?

7. At what per cent. will \$240 gain \$79.20 in $5\frac{1}{2}$ yr.?

8. At what per cent. will \$560 gain \$182 in 3 yr. 3 mo.?
9. At what per cent. will \$105 gain \$29.40 in 3 yr. 6 mo.?
10. At what per cent. will \$180 gain \$49.50 in 5 yr.?
11. At what per cent. will \$250 amount to \$287.50 in 3 yr.?
12. At what per cent. will \$325 amount to \$403 in 4 yr.?
13. At what per cent. will \$175 amount to \$224 in 3 yr. 6 mo.?
14. At what per cent. will \$940 amount to \$1236.10 in 5 yr. 3 mo.?
15. At what per cent. will a principal double itself in 2 yr.?
4 yr.? 5 yr.? 6 yr.? 8 yr.? 12 yr.? 20 yr.? 25 yr.?
16. At what per cent. will a principal treble itself in 10 yr.?
12 yr.? 16 yr.? 15 yr.? 30 yr.?
17. At what per cent. will a principal quadruple itself in
5 yr.? 10 yr.? 20 yr.? 25 yr.? 40 yr.?
18. At what rate per cent. will the interest of \$650 for 3 yr.
6 mo. amount to \$159.25?

WRITTEN PROBLEMS.

1. At what rate per cent. will \$630 in 3 yr. 8 mo. gain \$138.60 interest? Ans. 6.

SOLUTION. *Analysis.*—The interest of \$630 for 3 yr.
 3 yrs. 8 mo. = $3\frac{2}{3}$ yr. 8 mo., or $3\frac{2}{3}$ yr., at 1% is \$23.10. If \$23.10
 $\$630 \times .03\frac{2}{3} = \$23.10$ is the interest of \$630 at 1%, \$138.60 is the
 $\$138.60 \div \$23.10 = 6$ interest of the same sum at as many per cent.
 as \$23.10 is contained times in \$138.60, or 6%.

From the foregoing we derive the rule to find the rate per cent.:

RULE.

Divide the given interest by the interest of the given principal for the given time, at 1 per cent.

2. The interest on \$1255 for 6 yr. 3 mo. is \$313.75 : what is the rate per cent. ? Ans. 4.
3. The interest on \$125.50 for 2 yr. 4 mo. is \$17.57 : what is the rate per cent. ? Ans. 6.
4. The interest on \$640 from Mar. 12, 1873, to Jan. 27, 1875, was \$60 : what was the rate ? Ans. 5%.

5. \$150 was placed on interest for 12 yr. 6 mo.; at the end of that time it amounted to \$300: what was the rate per cent.?

Ans. 8.

6. A house worth \$4000 rents for \$275: what rate per cent. does this investment pay?

Ans. $6\frac{7}{8}$.

7. A house worth \$1800 rents for \$10 a month: what rate per cent. does it yield?

Ans. $6\frac{3}{4}$.

8. A house worth \$1600 rents for \$9 a month, and the owner pays \$36 a year taxes: what rate per cent. does it pay?

Ans. $4\frac{1}{2}$.

9. A merchant having \$15000 in business, yielding him a monthly income of \$150, has also some houses worth \$20,000, which after paying taxes yield him an annual income of \$2300: which is the better investment, and how much?

Ans. The business, by $\frac{1}{2}\%$.

10. The amount of a certain principal for 4 years at a certain per cent. is \$7200, and the amount of the same principal at the same rate per cent. for 7 years is \$8100: what is the principal, and what the rate per cent.?

Ans. Principal, \$6000; rate, 5%.

11. The amount of a certain principal for 6 yr. 6 mo., at a certain per cent., is \$7857, and the amount of the same principal at the same rate per cent. for 9 yr. 3 mo. is \$8896.50: what is the principal, and what the rate?

Ans. Principal, \$5400; rate, 7%.

CASE II.

Principal, Rate and Interest being given, to find the Time.

ORAL EXERCISE.

1. In what time will \$200, at 6%, give \$42 interest?

SOLUTION 1.—\$200 in 1 year, at 6%, will gain \$12 interest, and to gain \$42 will require as many years as \$12 is contained times in \$42, or $3\frac{1}{2}$ years.

SOLUTION 2.—The interest of \$200 for 1 year, at 6%, is \$12; hence, for \$200 to gain \$42 will require as many years as \$12 is contained times in \$42, or $3\frac{1}{2}$ years.

2. In what time will \$300, at 5%, gain \$45 interest?
3. In what time will \$500, at 6%, gain \$120 interest?
4. In what time will \$400, at 8%, gain \$80 interest?
5. In what time will \$150, at 6%, gain \$36 interest?
6. In what time will \$130, at 4%, gain \$26 interest?
7. In what time will \$125, at 10%, amount to \$150?
8. In what time will \$200, at 7%, amount to \$249?
9. In what time will \$105, at 5%, amount to \$141.75?
10. In what time will a principal double itself at 5%?
10%? 6%? 7%? 8%? 12%? 15%? $16\frac{2}{3}\%$?
11. In what time will a principal treble itself at 10%?
5%? 4%? 6%? 12%? 15%? $16\frac{2}{3}\%$? $12\frac{1}{2}\%$? 50%?
75%?
12. In what time will a principal quadruple itself at 5%?
10%? $12\frac{1}{2}\%$? $33\frac{1}{3}\%$? $66\frac{2}{3}\%$? 75%? 100%?
13. The amount of a certain principal for a given time, at 5%, is \$650, and at 7% for the same time is \$710: what is the time, and what the principal?
14. The amount of a certain principal for a given time, at 6%, is \$4840, and at 8% for the same time \$5720: what is the time, and what the principal?
15. The amount of my money invested in real estate, at 6%, in a certain time was \$20800, but by investing the same amount in bank stock, which earned a dividend of 9% per annum, it would have been worth to me \$23200 for the same time: what was the time, and what the sum invested?

WRITTEN PROBLEMS.

1. In what time will \$720 gain \$140.40 interest at 6%?

Ans. $3\frac{1}{4}$ yr.

SOLUTION.

$$\begin{array}{r}
 \$720 \\
 \underline{.06} \\
 \$43.20 \overline{) \$140.40} \quad 3\frac{1}{4} \\
 \underline{129.60} \\
 10.80 \\
 \underline{43.20} = \frac{1}{4}
 \end{array}$$

Let the pupil give the explanation and derive a rule.

2. In what time will \$1250 gain \$218.75 interest at 5 per cent. ? *Ans.* $3\frac{1}{2}$ yr.

3. In what time will \$127.30 gain \$31.82 $\frac{1}{2}$ interest at 6% ?
Ans. 4 yr. 2 mo.

4. If I pay \$99.06 interest on \$127, at 1 per cent. a month, how long have I the use of the money ? *Ans.* 6 yr. 6 mo.

5. I borrowed \$180, May 1, 1866, and when I returned it I paid \$196.47, interest being 6% per annum: when did I return the money ? *Ans.* Nov. 10, 1867.

6. I borrowed \$500, June 10, 1867, at 6% interest; I returned it when it amounted to \$600: when was the money returned ? *Ans.* Oct. 10, 1870.

7. In what time will \$140, at 6%, gain \$28.28 interest ?
Ans. 3 yr. 4 mo. 12 d.

8. In what time will \$126 amount to \$161.28, at 5% interest ?
Ans. 5 yr. 7 mo. 6 d.

9. In what time will \$260, at 4 $\frac{3}{4}$ %, gain \$39.52 interest ?
Ans. $3\frac{1}{2}$ yr.

10. In what time will the interest of any sum, at 6%, equal the principal? One-half the principal? One-fourth the principal? Twice the principal?

11. The amount of a certain principal, at 7%, for a given time is \$736.50, and at 10% for the same time it is \$795: what is the time, and what the principal?

Ans. Time, $3\frac{1}{4}$ yr.; principal, \$600.

CASE III.

Interest, Rate and Time being given, to find the Principal.

ORAL EXERCISE.

1. What sum will in 6 yr., at 5%, yield \$75 interest?

SOLUTION 1.—The interest of \$1 for 6 yr., at 5%, is 30 cts.; hence \$75 is the interest of as many dollars for the given time and rate as 30 cts. is contained times in \$75, which is \$250.

SOLUTION 2.—For 6 yr., at 5%, $\frac{3}{100}$, or $\frac{3}{100}$ of the principal, equals the interest. If \$75 is $\frac{3}{100}$ of the required principal, $\frac{1}{100}$ is $\frac{1}{3}$ of \$75, or \$25, and $\frac{1}{100}$, or the principal, is 10 times \$25, or \$250.

2. What sum will in 8 yr., at 5%, give \$40 interest? \$60? \$100?

3. What sum will in 7 yr., at 6%, give \$84 interest? \$105? \$210? \$252?

4. What principal in 5 yr. 4 mo., at 6%, will give \$48 interest? \$160?

5. What principal in $7\frac{1}{2}$ yr. will at 9% give \$207 interest? \$345?

6. If I pay an annual interest of \$420, at 7%, what is the amount borrowed?

7. The interest on a debt for 2 yr. 6 mo., at 6%, was \$240: what was the debt?

8. A man sold a farm and put the money received for it on interest, at 8%, for 2 yr. 9 mo.; when principal and interest, amounting to \$14640, were paid; what was the sum received for the farm?

9. How much money must be put on interest, at 10%, so that the amount in 5 yr. 6 mo. may be \$3875?

10. The interest of $\frac{2}{3}$ of A's fortune added to $\frac{1}{3}$ of B's, for 6 yr., at 6%, equals \$2700: how much has each, if A's is twice B's?

WRITTEN PROBLEMS.

1. What sum being placed on interest for $7\frac{1}{2}$ yr., at 8%, will yield \$278.40 interest? *Ans.* \$480.

SOLUTION 1.

$$\begin{array}{r} .08 \\ 7\frac{1}{2} \\ \hline .58 \overline{) \$278.40} \overline{) \$480} \\ \underline{232} \\ 464 \\ \underline{464} \end{array}$$

Let the pupil explain.

SOLUTION 2.

$$\begin{aligned} 7\frac{1}{2} \times .08 &= .58 \\ \frac{.58}{.08} &= \frac{58}{8} = \$278.40 \\ \frac{1}{50} &= \frac{1}{50} \times \$278.40 = \$9.60 \\ \frac{50}{50} &= 50 \times \$9.60 = \$480. \end{aligned}$$

RULE.

Divide the given interest by the product of the rate per cent. and the number of years; the result is the principal.

2. What sum of money will in 5 yr. 3 mo., at 7%, gain \$455.70 interest? *Ans.* \$1240.

3. A man placed a certain sum of money on interest, at 6%, when his son was born, and when the son became of age the amount was \$1130: what was the sum? *Ans.* \$500.

4. A man received a semi-annual interest of \$245 on a mortgage at 7%: what was the mortgage? *Ans.* \$7000.

5. What sum of money will yield \$2356 in 7 yr. 7 mo. 6 d., at 5%? *Ans.* \$6200.

6. What principal will in 5 yr. 4 mo., at 7%, amount to \$2101.20? *Ans.* \$1530.

7. The interest, at 6%, on $\frac{2}{3}$ of A's money added to $\frac{1}{3}$ of B's money, for 6 yr. 6 mo., is \$3198: how much money has each, if B's equals $\frac{2}{3}$ of A's? *Ans.* A, \$6000; B, \$4800.

8. The interest on the money paid on a farm, a store and a hotel for 3 yr. 4 mo. 15 d., at 8%, is \$7290: how much did each cost if the store cost twice as much as the hotel, and the farm three times as much as the store?

Ans. Hotel, \$3000; store, \$6000; farm, \$18000.

SECTION XII.

COMPOUND INTEREST.

230. Compound Interest is interest computed not only on the principal, but also on the interest as it falls due.

ORAL EXERCISE.

1. What is the compound interest of \$100 for 3 yr., at 10%?

SOLUTION.—The interest of \$100 for 1 yr. at 10% is \$10, which added to the principal, equals \$110, the new principal. The interest of \$110 for 1 yr. at 10% is \$11, which added to the principal, \$110, equals \$121, the new principal. The interest of \$121 for 1 yr. at 10% is \$12.10, which added to \$121, the principal, equals \$133.10, the amount. The amount, \$133.10, minus the principal, \$100, equals \$33.10, the compound interest of \$100 for 3 yr. at 10%.

2. What is the compound interest of \$500 for 2 yr., at 6%?

3. What is the compound interest of \$600 for 2 yr., at 5%?

4. What is the compound interest of \$1000 for 3 yr., at 5%?

5. What is the compound interest of \$400 for 2 yr., at 6% ?
6. What is the compound interest of \$200 for 2 yr., at 7% ?

WRITTEN PROBLEMS.

1. What is the compound interest of \$650 for 3 yr., at 6% ?

Ans. \$124.16.

OPERATION.

\$650	principal.
.06	
39.	int. of \$650 for 1 yr.
689	
.06	principal, 2d yr.
41.34	int. of \$689 for 1 yr.
689	
730.34	principal, 3d yr.
.06	
43.8204	int. of \$730.34 for 1 yr.
730.34	
774.1604	amount.
650.	principal deducted.
\$124.1604	compound interest.

Let the pupil explain.

RULE.

1. Find the amount of the principal for the first period of time ; this will be the principal for the second period. Find the amount of this principal for the second period, and proceed in this manner to the end of the given time ; the final result is the amount.

2. From the amount subtract the given principal, and the remainder is the compound interest.

2. What is the compound interest of \$620 for 3 yr., at 5% ?

Ans. \$97.73.

3. What is the compound interest of \$125 for 3 yr., at 6% ?

Ans. \$23.88.

4. What is the compound interest of \$75.40 for 4 yr., at 6% ?

Ans. \$19.79.

5. What is the compound interest of \$80.50 for 3 yr. 6 mo., at 8% ?

Ans. \$24.96.

6. What is the compound interest of \$190 for 2 yr., payable semi-annually, at 6% ?

Ans. \$23.484.

The computation of compound interest may be shortened by the use of the following table.

To find the compound interest by the given table, multiply the given principal by the number in the table corresponding to the given number of years and the given rate per cent., and from this deduct the principal.

TABLE.

Showing the Amount of \$1 at 3, 4, 5, 6, 7 and 8% Compound Interest, from 1 to 20 Years.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.03	1.04	1.05	1.06	1.07	1.08
2	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664
3	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597
4	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605
5	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693
6	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869
7	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138
8	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509
9	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990
10	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589
11	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316
12	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182
13	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196
14	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372
15	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722
16	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259
17	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000
18	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960
19	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157
20	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610

7. What is the interest of \$955 for 8 yr., at 8%, payable semi-annually? *Ans.* \$833.715.

SOLUTION.

8 yr., at 8%, is equivalent to
16 yr., at 4%.

\$1.8730 amt. of \$1 for 16 yr., at 4%.

955

93650

93650

168570

\$1788.715 amt. of \$955 for 16 yr., at 4%.

955.

\$833.715 int. of \$955 for 8 yr., at 8%, payable semi-annually.

Let the pupil explain.

8. What is the interest on \$1850 invested for 15 yr., at 4%, compound interest? *Ans.* \$1481.66½.

9. A guardian has in his possession \$3750: how much must he pay to his ward when coming of age if he hold the money from the time the child is three years old, money being worth 5%, compound interest? *Ans.* \$9024.75.

10. A father at his death left his daughter, who was then 10 yr. 3 mo. 6 d. old, \$5000, to be paid to her at the age of 18, interest compounding at 6%, payable semi-annually: what amount did she receive? *Ans.* \$7899.06.

SECTION XIII.

ANNUAL INTEREST.

231. Annual Interest is the interest due at the end of each year on the principal and on the interest remaining unpaid, so computed as not to increase the principal.

It is allowed in the case of notes and other contracts containing the words "*with interest, payable annually,*" or "*with annual interest.*"

232. Annual interest when not paid as it falls due draws simple interest at the legal rate until paid.

WRITTEN PROBLEMS.

1. What is the annual interest and the amount of \$500 for 6 yr., at 6% per annum? *Ans.* Int., \$207; amt., \$707.

SOLUTION 1.

Interest on \$500 for 1 yr. at 6% = \$30.

Interest on \$500 " 6 " " " = \$180.00

Interest on 1st annual interest (\$30) for 5 yr. at 6% = 9.00

Interest on 2d " " " " 4 yr. at 6% = 7.20

Interest on 3d " " " " 3 yr. at 6% = 5.40

Interest on 4th " " " " 2 yr. at 6% = 3.60

Interest on 5th " " " " 1 yr. at 6% = 1.80

Total interest, \$207.00

Principal, 500.00

Amount, \$707.00

SOLUTION 2.

Interest of \$500 for 1 yr. at 6% = \$30

Interest of \$500 for 6 yr. at 6 " = \$180

Interest of \$30 for 15 yr. at 6 " = 27 = $\left\{ \begin{array}{l} \text{int. of \$30 for } 5+4+3+2+1 \\ \quad \quad \quad = 15 \text{ yr. at } 6\%. \end{array} \right.$

Annual interest, = \$207

Principal, = 500

Amount, = \$707

NOTE.—The number of years annual interest may also be found by multiplying the given number of years by one less than the number, and dividing by two (2). Thus, $\frac{6 \times 5}{2} = 15$.

RULE.

To find the amount of annual interest due, add to the interest of the given principal to the time of settlement the interest on 1 year's interest for as many years as the sum of all the periods during which successive payments of interest have been due.

2. Find the amount of \$660, at 5% annual interest, for 4 years? *Ans.* \$801.90.

3. What will \$4500 amount to in 12 yr., annual interest at 6%? *Ans.* \$8809.20.

4. What is the amount of \$1800 for 7 yr., interest payable annually, at 7%? *Ans.* \$2867.22.

5. Find the interest on a note of \$1550, due in 6 yr., with interest payable annually, at 6%? *Ans.* \$641.70.

6. Find the amount of \$600 for 10 yr. 6 mo. 24 d., interest payable annually, at 6%? *Ans.* \$1089.84.

SECTION XIV.

NOTES.

233. A Promissory Note is a written promise to pay a sum of money either on demand or at a specified time.

234. The sum named in the note is called the *face* of the note.

235. The party who makes the promise or signs the note is called the **Maker** or **Drawer**. The one to whom it is to be paid is called the **Payee**, and he in whose legal possession it is, is called the **Holder** of the note.

236. An **Indorser** is one who writes his name on the back of the note, and thereby makes himself responsible for its payment.

237. The words "value received" must be placed in every note to make it valid; also, in Pennsylvania and New Jersey, the words "without defalcation."

238. A note draws interest from date if the words "with interest" are placed in it; otherwise, it draws interest from the time it falls due at the legal rate of the State.

239. A **Negotiable Note** is one made payable to bearer or to any person's order. If made payable to bearer, it may be transferred; but if to "order," it must first be indorsed by the person to whose order it is made payable.

The following is a simple form of a negotiable note:

\$160⁰⁰/₁₀₀.

PHILADELPHIA, Pa., Feb. 28, 1876.

Three months after date, I promise to pay Jacob Warfel, or order, One Hundred and Sixty Dollars. Without defalcation, value received.

WILLIAM PARSONS.

NOTE.—If Jacob Warfel writes his name across the back of this note, it is called an *indorsement in blank*, and it may then be collected by the *holder*. But if Jacob Warfel wishes to make it payable to James Burnside only, he will write on the back of the note "Pay to James Burnside," and then sign his name. This is called a *special indorsement*.

240. A **Bankable Note** is one made payable at bank. The following is a simple form:

\$120²⁵/₁₀₀.

HARRISBURG, Pa., May 6, 1876.

Sixty days after date, I promise to pay to the order of Henry Houck, at the Dauphin County Bank, One Hundred and Twenty ²⁵/₁₀₀ Dollars. Without defalcation, value received.

HENRY SMITH.

NOTE.—Should Mr. Houck indorse this note, and Mr. Smith fail to

pay it when due, a *notary public* will serve a notice on Mr. Houck to the effect that the note is not paid and that payment will be expected of him. This is called a notice of *protest*. It must be made out the day the note falls due, and served either that day or the day after; otherwise, the responsibility of the indorser ceases.

241. If a note does not contain the words "or order," or the words "or bearer," it is payable to the person only whose name is mentioned in it, and is *not negotiable*.

242. On the payment of notes at bank, *three days of grace* are allowed; and the note is payable really three days after the expiration of the number of months or days mentioned on the face of it.

243. **Time** notes and **Demand** notes differ only in the fact that in the former the time of payment is specified, while in the latter the note is payable at any time.

SECTION XV.

PARTIAL PAYMENTS.

244. Partial Payments are the payments in part of the amount of a note or other obligation. These payments are usually acknowledged on the back of the note, stating the time and the amount of the payments, and are called **Indorsements**.

245. The following rule for the settlement of interest-bearing notes is the one in use in the Supreme Court of the United States, and it is now used by most of the States. It is based on the two following

PRINCIPLES.—1. The principal cannot be diminished until the interest due is paid.

2. Interest shall not draw interest.

SUPREME COURT RULE.

1. *Compute the interest on the principal from the date of the note to the time of the first payment. If this payment equals or exceeds the interest, add the principal and the interest together,*

and from their sum subtract the payment. Consider the remainder a new principal, and proceed to the next payment as before. Continue this process to the time of settlement.

2. If any payment is less than the accrued interest, add such payment to the next, and treat the sum as one payment at such time as the payment shall equal or exceed the accrued interest.

WRITTEN PROBLEMS.

1. On a note dated Feb. 16, 1860, for \$1500, at 6%, are the following indorsements:

Sept. 16, 1860, received.....	\$80
Jan. 1, 1861, "	\$20
July 25, 1861, "	\$600

What is due Apr. 1, 1862? Ans. \$966.39.

OPERATION.

Principal	\$1500
Interest to Sept. 16, 1860.....	52.50
Amount.....	\$1552.50
First payment.....	80.00
Balance.....	\$1472.50
Int. on balance from Sept. 16, 1860, to Jan. 1, 1861.....	\$25.77
Int. on balance from Sept. 16, 1860, to July 25, 1861...	75.83
Amount.....	\$1548.33
2d payment.....	\$20
3d payment.....	\$600
	} = 620.00
Balance.....	\$928.33
Int. from July 25, 1861, to Apr. 1, 1862.....	38.06
Balance due.....	\$966.39

Explanation.—The interest on \$1500 from Mar. 1, 1860, to Sept. 16, 1860, is \$52.50, which added to the principal equals \$1552.50. Subtracting the first payment, \$80, the remainder is \$1472.50. The interest on this sum to the time of the next payment, Jan. 1, 1861, is \$25.77, which is greater than the payment; no account, therefore, is taken of either for the present. The interest on \$1472.50 from Sept. 16, 1860, to July 25, 1861, is \$75.83, which added to the principal equals \$1548.33; the sum of the 2d and 3d payments is \$620, which subtracted from \$1548.33 leaves a balance of \$928.33. The interest on this sum from July 25, 1861, to the time of settlement, Apr. 1, 1862, is \$38.06, which added to the principal equals \$966.39, the balance due.

2. On a note dated Aug. 16, 1872, for \$650, at 6%, are the following indorsements:

Jan. 1, 1873, received.....	\$100
Apr. 1, 1873 " 	\$20
Aug. 25, 1874 " 	\$300

What is the amount due Jan. 1, 1875? *Ans.* \$305.84.

3. What is due July 16, 1875, on a note for \$540, at 6%, dated Sept. 10, 1872, and bearing the following indorsements?—

Jan. 22, 1873, received.....	\$120
Mar. 1, 1874 " 	\$25
Oct. 22, 1874 " 	\$125

Ans. \$341.63.

4. On a note dated June 15, 1865, for \$540, at 7%, were the following indorsements:

Jan. 1, 1866, received.....	\$25
Sept. 16, 1867, " 	\$120
Mar. 4, 1868, " 	\$10
July 4, 1868, " 	\$300

What was due Aug. 16, 1869? *Ans.* \$211.85.

5. How much is due July 4, 1876, on the following note?—
\$850.

PHILADELPHIA, Aug. 1, 1871.

Three months after date I promise to pay Geo. S. Good, Eight Hundred and Fifty Dollars, with interest.

ISAAC A. WILLIAMS.

Indorsements as follows: Jan. 4, 1872, \$25; May 1, 1873, \$250; July 10, 1874, \$250; Sept. 16, 1875, \$20; Dec. 1, 1875, \$400.

Ans. \$82.84.

246. The following method is usually adopted by merchants in adjusting interest accounts and the settlement of notes running not more than a year. It is called

THE MERCANTILE RULE.

1. Find the amount of the principal to the time of settlement.
2. Find the amount of each payment from the time it is made to the time of settlement.

3. *From the amount of the principal subtract the sum of the amounts of the payments.*

NOTE.—In computing by this method the time is reduced to days, and the interest is calculated by the *method for days*.

WRITTEN PROBLEMS.

1. On a note of \$1200 dated May 10, 1870, at 5%, were the following indorsements: June 1, 1870, \$100; July 5, 1870, \$66; Dec. 1, 1870, \$900; what was due April 11, 1871?

Ans. \$166.69.

OPERATION.

Amt. of \$1200 from May 10, 1870, to Apr. 11, 1871 (336 d.),	\$1256.00
Amt. of \$100 from June 1, 1870, to Apr. 11, 1871 (314 d.),	\$104.36
Amt. of \$66 from July 5, 1870, to " " (280 d.),	68.57
Amt. of \$900 from Dec. 1, 1870, to " " (131 d.),	916.38—1089.31
Balance,	\$166.69

2. On a note of \$600, dated June 10, 1875, at 6%, were the following indorsements:

Aug. 12, 1875.....	\$200
Sept. 15, 1875.....	\$200
Dec. 1, 1875.....	\$200

What is due Jan. 1, 1876? *Ans.* \$11.14.

3. On a note for \$1000, dated July 4, 1876, at 6%, were the following indorsements:

Aug. 12, 1876.....	\$350
Oct. 9, 1876.....	\$350
Jan. 1, 1877.....	\$250

How much remained due Feb. 1, 1877? *Ans.* \$67.24.

4. On a note of \$325, dated Aug. 10, 1873, at 7%, were the following indorsements: Sept. 3, 1873, \$100; Nov. 1, 1873, \$100; Jan. 1, 1874, \$100. What is due Apr. 1, 1874?

Ans. \$31.03.

5. On a note of \$650, dated June 26, 1877, at 8%, are the following indorsements: Oct. 20, 1877, \$220; Jan. 8, 1878, \$200; Mar. 28, 1878, \$100; Apr. 16, 1878, \$110: how much is due May 1, 1878?

Ans. \$49.04.

SECTION XVI.

DISCOUNT.

247. Discount is a deduction from the face of a bill, note or debt.

248. Commercial Discount is a percentage deducted from the face of a bill of merchandise, as agreed upon between the buyer and the seller.

WRITTEN PROBLEMS.

1. Bought a bill of goods for \$640, but paying cash, I was allowed a discount of 5% : what was the discount, and what did I pay? *Ans.* Discount, \$32; paid, \$608.

SOLUTION.

$$5\% \text{ of } \$640 = \$32 \\ \$640 - \$32 = \$608$$

NOTE.—Let the pupil derive the rule.

2. The list or fixed price of a book is \$3.75, but the discount to dealers is 20% : what is the price to dealers?

Ans. \$3.

3. Sold a bill of goods for \$650, but gave a discount of 8% for cash : what was the discount? *Ans.* \$52.

4. If I sell flour on credit at \$8 a barrel, but give a discount of 6% for cash, what do I get a barrel in cash?

Ans. \$7.52.

5. What is the discount on a bill of goods, \$1580, at the rate of $2\frac{1}{2}\%$? *Ans.* \$39.50.

6. What must I pay cash for 6 rolls of carpet, 150 yd. each., at 80 cts. a yard, 5% off for cash? *Ans.* \$684.

SECTION XVII.

PRESENT VALUE AND TRUE DISCOUNT.

249. The Present Value of a debt is a sum which, if placed on interest at the legal rate for the given time, will amount to the face of the debt when it comes due.

250. The True Discount is the difference between the whole debt and the present value.

NOTE.—The present value represents the principal, the discount represents the interest, and the debt represents the amount.

ORAL EXERCISE.

1. What is the present worth of \$163.50, due 1 yr. 6 mo. hence, at 6%?

SOLUTION 1.—The interest of \$1 for 1 yr. 6 mo., at 6%, is 9 cts., and the amount of \$1 for the same time and at the same rate per cent. is \$1.09; hence, the present worth of \$163.50 is as many dollars as \$1.09 is contained times in \$163.50, or \$150.

SOLUTION 2.—Since the amount of \$1 for 1 yr. 6 mo., at 6%, is \$1.09, $\frac{1}{1.09}$ of the present worth or principal equals the amount. If \$163.50 is $\frac{1}{1.09}$ of the present worth, $\frac{1}{1.09}$ of the present worth is $\frac{1}{1.09}$ of \$163.50, or \$1.50, and $\frac{1}{1.09}$ of the present worth is 100 times \$1.50, or \$150.

2. What is the present worth of \$240, due 3 yr. 4 mo. hence, at 6%?

3. What is the present worth of \$150, due 4 yr. 2 mo. hence, at 6%?

4. What is the discount on \$228, due 2 yr. hence, at 7%?

5. What is the discount on \$92, due 3 yr. hence, at 5%?

6. A note for \$300 bears interest at 6%, and is due 5 yr. hence: what is the discount?

7. A bill of goods was bought amounting to \$1575, due 10 mo. hence, interest 6%; the seller offers to take \$1490 cash: will the buyer make or lose by accepting the seller's offer?

WRITTEN PROBLEMS.

1. What is the present value of \$1302, due 1 yr. 5 mo. hence, interest at 6%? *Ans.* \$1200.

SOLUTION.

ANALYSIS.—The interest of

Int. of \$1 for 1 yr. 5 mo., at 6% = $8\frac{1}{2}$ cts. \$1 for the given time and at
Amt. of \$1 = \$1.08 $\frac{1}{2}$ the given rate per cent. is

$$\$1302 \div \$1.08\frac{1}{2} = 1200.$$

$8\frac{1}{2}$ cts.; and the amount of

\$1 for the same time and at the same rate per cent. is \$1.08 $\frac{1}{2}$; hence, the present worth of \$1.08 $\frac{1}{2}$ is \$1, and the present worth of \$1302 is as many dollars as \$1.08 $\frac{1}{2}$ is contained times in \$1302, or \$1200.

NOTE.—Let the pupil derive a rule.

2. What must be paid now to cancel a debt of \$994.50, due 1 yr. 9 mo. hence, at 6%? *Ans.* \$900.

3. What is the true discount on \$800, due 2 yr. 5 mo. hence, at 6%? *Ans.* \$101.31.

4. What is the present value of a debt of \$1000, due 2 yr. 6 mo. 18 d. hence, at 7%? *Ans.* \$848.54.

5. What is the present value of a note for \$750, due 1 yr. 5 mo. 24 d. hence, at 6%? *Ans.* \$688.71.

6. Bought a store for \$15000, and sold it again for \$20000, $\frac{1}{4}$ to be paid cash, and the remainder in 2 yr.: what did I make, money being worth 6%? *Ans.* \$3392.86.

7. Bought two lots for \$2500, on 3 years' time, without interest: what is the cash value, money being worth 7%? *Ans.* \$2066.11.

8. A merchant buys \$900 worth of goods at 25% discount, and sells them at regular retail price on 9 months' credit: what is his cash gain, money being worth 6%? *Ans.* \$186.24.

9. Which is the more profitable, to buy lumber at \$25 a thousand on 9 months' credit, or at \$24.50 on 6 months' credit, money being worth 6%? *Ans.* The latter—13 cts. per M.

10. Mr. Wallace bought a farm for \$12500, paying \$2500 cash, and the remainder in two equal yearly payments: what was the cash value, money being at 6%? *Ans.* \$11681.27.

11. Bought a lot for \$600, of which I paid \$50 cash, \$150 in 9 months, \$200 in 1 yr. 9 mo., and \$200 in 2 yr. 9 mo., interest at 8%: what was the cash value, money being worth 6%? *Ans.* \$617.92.

SECTION XVIII.

BANKING AND BANK DISCOUNT.

251. A Bank is a corporation chartered by law for receiving deposits, loaning money and issuing notes or bills to circulate as money.

252. A Deposit is money or its equivalent placed in the care of a bank.

253. A Check is a written order or request for money, addressed to a bank, by a person having a deposit in that bank.

The following is the usual form of a check :

No.

PHILADELPHIA, PA., July 6, 1876.

First National Bank of Philadelphia, Pa.

Pay to _____ *William Parsons*, _____ or order,
_____ *One Hundred and Thirty* _____ $\frac{12}{100}$ Dollars.

\$130 $\frac{12}{100}$

R. S. BARKER.

254. Bank Discount is the allowance made to a bank for the payment of a note before it matures.

255. Bank Discount is the interest on the face of the note for the time it has to run, including three days of grace.

256. The proceeds of a note is the sum received when the note is discounted, and is the remainder after subtracting the *discount* from the *face*.

NOTE 1.—Bank discount may be computed by any of the methods of computing interest, but the method for days seems to be the shortest and simplest.

2. Bankers in computing discount add *four* days of grace. They claim that the party receiving the proceeds can, by having the discount made early in the day and paying the note late, practically have the use of the money both on the day of discount and also on the day of payment, thus making four extra days.

CASE I.

To find the Bank Discount or the Proceeds.

WRITTEN PROBLEMS.

1. What is the bank discount on a note of \$650 for 60 d., at 6% ?

OPERATION.

$$\begin{array}{r} 65 \quad 21 \\ \$650 \times \frac{6}{100} = \frac{1365}{2} = 682\frac{1}{2} \\ \frac{60}{20} \\ 2 \quad \text{or, } \$6.82\frac{1}{2} \end{array}$$

Or, when 4 days of grace are reckoned,

$$\begin{array}{r} 65 \quad 32 \\ \$650 \times \frac{6}{100} = \frac{2080}{3} = 693\frac{1}{3} \\ \frac{60}{30} \\ 3 \quad \text{or, } \$6.93\frac{1}{3} \end{array}$$

For explanation see Method of Interest by Days, page 226.

2. What is the bank discount on a note of \$1320 for 90 d., at 7%?

OPERATION.

$$\begin{array}{r} 11 \\ 22 \\ 1320 \times 90 \times 7 \\ \hline 80 \times 8 \\ 3 \end{array} = 2387, \text{ or } \$23.87.$$

At 4 days' grace,

$$\begin{array}{r} 11 \\ 22 \\ 1320 \times 94 \times 7 \\ \hline 80 \times 8 \\ 3 \end{array} = \frac{7238}{3} = 2412\frac{2}{3}, \text{ or } \$24.12\frac{2}{3}.$$

RULES.

1. At 6% add the days of grace; then multiply the number of dollars by the number of days, and divide by 60; the result is the bank discount in cents.

2. At any other per cent. than 6 add the days of grace; then multiply the number of dollars, the number of days and the rate together, and divide by 60 times 6, or 360; the result, as before, is the bank discount in cents.

The following problems are solved by reckoning either 3 or 4 days of grace, the first answer being for three days of grace, the second for four. The date of maturity is marked at the bottom; thus, Dec. 13/16.

3. What is the bank discount on a note for \$726 for 30 days, at 6%?

Ans. { \$3.99.
 { \$4.11.

4. Find the proceeds of a note of \$180 due in 60 days, at 7%.

Ans. { \$177.79.
 { \$177.76.

5. What is the discount on a note of \$420.50 for 90 days, at 8%?

Ans. { \$8.69.
 { \$8.78.

6. What are the discount and the proceeds of a note of \$75.66 for 60 days, at 10%?

Ans. Dis., { \$1.32; proceeds, { \$74.34.
 { \$1.34; { \$74.32.

CASE II.

To find the Face of the Note.

1. The proceeds of a note for 30 days, at 6%, are \$556.92: what is the face?

SOLUTION.

Bank discount of \$1 for 33 d., at 6% = \$.0055.

Proceeds of \$1 " " " = \$1 - \$.0055 = \$.9945.

Hence, $\$556.92 \div \$.9945 = 560$, or \$560, the face of the note.

NOTE.—In the following problems 3 days' grace are reckoned.

2. What is the face of a note which, discounted for 60 d. at 7%, yields \$237.06? *Ans.* \$240.

3. What is the face of a note which, discounted for 90 d. at 10%, yields \$650? *Ans.* \$667.24.

4. Bought 10 loads of hay, weighing 2750 lb. each, at \$20 a ton, and gave my note at 60 d. to pay for it: what must be the face of the note to cover the discount at 6%? *Ans.* \$277.91.

5. I owe a bill for flour and feed amounting to \$73.25, and give my note for 90 d.: how must I draw it to cover the discount at 8%? *Ans.* \$74.80.

6. Sold a horse for \$250, a carriage for \$175, and a set of harness for \$120, and took the purchaser's note for 90 d. so as to cover the discount at 6%: what was the face of the note? *Ans.* \$553.58.

SECTION XIX.**BONDS.**

257. Bonds are interest-bearing notes issued by cities, States, nations, railway companies and other corporations as a means of borrowing money.

They bear annual interest, payable annually or semi-annually, and are made negotiable like certificates of stock.

258. Coupons are certificates of interest attached to certain bonds, which, as the interest comes due, are detached and presented for payment.

259. United States Bonds, known as "United States securities" or "government securities," are of two kinds—those made payable at a fixed date, and those which, while payable at a fixed date, may be paid earlier if the government so determine.

260. The principal United States securities are the following:

1. "6's of '81." These bear interest at 6% in gold, payable semi-annually, and are made payable in 1881.

2. "5-20's," which bear interest at 6% in gold, payable semi-annually, redeemable at the option of the government any time after *five* years from date, and are payable in *twenty* years. The "5-20's" were issued in the years 1862, 1864, 1865, 1867 and 1868.

3. "10-40's," issued in 1864, bearing interest at 5% in gold, payable semi-annually, except the \$50 and the \$100 bond, on which the interest is payable annually. They are redeemable in 10 years at the option of the government, and payable in 40 years.

4. "5's of '81," which bear 5% interest in gold, payable quarterly.

Bonds of States or cities and corporations are quoted in the same manner as the foregoing. Thus, "Pennsylvania 6's" means bonds issued by the State of Pennsylvania, and bearing 6% interest.

Bonds are bought and sold like Stocks. The same rules employed in the solution of problems in Stocks will therefore apply in the solution of problems in Bonds.

WRITTEN PROBLEMS.

1. What cost \$4000 in 10-40's when they are quoted at 118½?
Ans. \$4740.

2. What is the cost of \$5000, 6's of '81, when quoted at 123?
Ans. \$6150.

3. A capitalist invests \$5265 in 5-20's when they are quoted at 117: what amount can he buy?
Ans. \$4500.

4. When Philadelphia and Erie 7's sell at 95, if I buy \$5000 worth of bonds, what per cent. do I make on my investment?

Ans. $7\frac{7}{8}\%$.

5. What is the income in gold of a person who invests \$57000 in 5's of '81 when they sell at 95? *Ans.* \$3000.

6. Which will yield the better income—8% bonds at 90, or 7% bonds at 80? *Ans.* The former, $\frac{5}{8}\%$.

7. What must be paid for 5's of '81 that they may yield an income of $6\frac{1}{2}\%$ per annum? *Ans.* $76\frac{1}{2}\%$.

8. What is my annual income in currency if I invest \$4800 in 6's of '81 when selling at 96, and what per cent. do I make on my investment, gold being at a premium of 15?

Ans. \$345; profit, $7\frac{3}{8}\%$.

9. A man pays \$21200 for 5-20's when selling at 106: what is his annual income in currency, and per cent., gold being 112 $\frac{1}{2}$? *Ans.* \$1350; profit, $6\frac{3}{8}\%$.

SECTION XX.

EXCHANGE.

261. Exchange is a method of making payments at a distance, without sending money, by means of *drafts* or *bills of exchange*.

262. A Draft or Bill of Exchange is a written request or order from one party to another to pay to a third party a certain sum of money at a specified time.

263. The person who signs the draft is called the **Drawer** or **Maker**; the person to whom the order is addressed, the **Drawee**; and the person to whom the money is to be paid, the **Payee**.

264. When the order is drawn in one country and made payable in another it is called **Foreign Exchange**.

265. When the order is drawn and made payable in the same country it is called **Inland** or **Domestic Exchange**, and the *bill of exchange* is usually called a **Draft**.

266. A Sight Draft is one made payable on presentation.

267. A Time Draft is one made payable at a specified time after sight or presentation. On a *time draft* three days of grace are allowed, as on a note, and in some States grace is also allowed on sight drafts.

268. When the draft is presented to the drawee, if he agrees to pay it he writes the word "Accepted" across the face of the draft, and signs his name. This is called the *Acceptance*. If he desires to pay it at some other bank than the one presenting it, he writes across the face as follows: "Accepted. Payable at ——— Bank," and signs his name, the blank being filled by writing the name of the bank.

269. The Buyer or Remitter of a bill of exchange is one who purchases it.

270. The comparative value of the money of different countries is called the **Par of Exchange**.

271. When a draft costs more than its face, it is at a *premium*, and when less, at a *discount*. Thus, if Philadelphia be indebted to New York, a draft in Philadelphia on New York will be at a premium, and in New York on Philadelphia at a discount.

272. The following is the usual form of a draft:

\$650⁷⁵/₁₀₀.

BALTIMORE, Md., Mar. 2, 1876.

Five days after Sight, pay to the order of *G. Kintzing*,

Cashier, *Six Hundred and Fifty* ⁷⁵/₁₀₀ *Dollars*. Value received,
and charge to the account of

To MELICK & SCHUYLER, }
573 Market St., Phila. }

A. C. GEARY.

To make the above a *Sight Draft*, it is only necessary to write the words "At sight" instead of "Five days after sight."

273. The Course of Exchange is the variation in the cost of the draft as affected by the relative condition of trade between the places of exchange.

CASE I.

DOMESTIC EXCHANGE.

All problems relating to Domestic or Inland Exchange may be solved by the rules for *Percentage* and *Bank Discount*. In applying these rules the *face* of the bill is equivalent to the *base*, the *premium* or *discount* to the *rate per cent.*, and the *cost* of the bill or draft to the *amount*.

WRITTEN PROBLEMS.

1. What is the cost of a sight draft for \$640 in Philadelphia, at $1\frac{1}{4}\%$ premium? *Ans.* \$648.

SUGGESTION.— $640 \times \$1.0125$.

2. What is the cost of a draft on St. Louis for \$1230, premium $\frac{1}{2}\%$? *Ans.* \$1236.15.

3. What is the cost of a draft on Boston for \$125.60, discount $\frac{3}{8}\%$? *Ans.* \$125.13.

4. What is the cost of a draft of \$850 for 60 d. on Baltimore, exchange being at $101\frac{1}{2}$, interest 6% per annum. *Ans.* \$853.825.

SOLUTION.

$$\begin{array}{rcl} \$850 \times 1.01\frac{1}{4} & = & 862.75 \quad = \text{cost.} \\ 8.92\frac{1}{4} & = & \text{bank discount on } \$850 \text{ for 63 d.} \\ \hline \$853.82\frac{1}{4} & = & \text{cost of draft.} \end{array}$$

Or,

$$\begin{array}{rcl} .015 & = & \text{premium.} \\ .0105 & = & \text{discount.} \\ .0045 & = & \text{difference.} \end{array}$$

1.

$$1.0045 = \text{cost of } \$1 \text{ draft.}$$

$$850 \times 1.0045 = 853.825 = \text{cost of } \$850 \text{ draft.}$$

5. What is the cost of a draft of \$2100 for 30 d. on New York, exchange being 102, interest at 7% per annum? *Ans.* \$2128.52 $\frac{1}{2}$.

6. What is the cost of a draft of \$500 for 60 d., premium at $\frac{3}{4}\%$ and interest at 6% ? *Ans.* \$498.50.

7. What must I pay for a draft of \$350.40 for 90 d., discount being $\frac{1}{2}\%$ and interest 6% ? *Ans.* \$343.22.

8. When exchange on Chicago is $98\frac{1}{2}$, what is the cost of a draft for \$362 for 30 d., interest at 6%? *Ans.* \$354.58.

9. What is the face of a sight draft that can be bought for \$544.05, premium at $\frac{1}{4}\%$? *Ans.* \$540.

SUGGESTION.— $\$544.05 + 1.00\frac{1}{4} = \$540$.

10. What is the face of a draft payable 60 d. after sight that will cost \$652.925, exchange $1\frac{1}{2}\%$ and interest at 6%? *Ans.* \$650.

SUGGESTION. $\frac{652.925}{1.015 - .0105} = 650$.

11. What is the face of a sight draft which costs \$122.46, exchange being $2\frac{1}{2}\%$ discount? *Ans.* \$125.60.

12. What is the face of a draft for 30 d. which costs \$1560, interest at 6% per annum, exchange being $98\frac{1}{4}$? *Ans.* \$1588.595.

CASE II.

FOREIGN EXCHANGE.

274. Foreign Exchange is that in which the exchange takes place between different countries.

Foreign Exchange is computed in the same manner as inland exchange, except that the currency of one country must be reduced to that of the other.

In Foreign Exchange three separate bills of the same tenor and date are made out and sent in a different manner, each of them containing a condition that it shall continue payable only while the others remain unpaid.

The following table will be of service in changing from one currency to another:

TABLE.

Country.	Denomination.	Standard.	Value in U. S. gold.
Austria.....	Florin.....	Gold.....	\$.453
Belgium.....	Franc.....	Gold and silver..	.193
Bolivia.....	Dollar.....	Silver.....	.965
Brazil.....	Milreis of 1000 reis.....	Gold.....	.545
British North America.....	Dollar.....	Gold.....	1.00
Central America.	Dollar.....	Silver.....	.918
Chili	Peso.....	Gold.....	.912

Country.	Denomination.	Standard.	Value in U. S. gold.
China.....	Tael.....	Silver.....	\$1.61
Cuba.....	Peso.....	Gold.....	.9258
Denmark.....	Crown.....	Gold.....	.268
Ecuador.....	Dollar.....	Silver.....	.918
Egypt.....	Dollar of 20 piasters.....	Silver.....	.9948
France.....	Franc.....	Gold and silver..	.193
Great Britain.....	Pound sterling.....	Gold.....	4.8665
Greece.....	Drachma.....	Silver.....	.193
German Empire.....	Mark.....	Gold.....	.238
Hayti.....	Dollar.....	Silver.....	.952
Jamaica.....	Pound sterling.....	Gold.....	4.8665
Japan.....	Yen.....	Gold.....	.997
India.....	Rupee of 16 annas.....	Silver.....	.436
Italy.....	Lira.....	Gold and silver..	.193
Liberia.....	Dollar.....	Gold.....	1.00
Mexico.....	Dollar.....	Silver.....	.998
Netherlands.....	Florin.....	Silver.....	.385
Norway.....	Crown.....	Gold.....	.268
Paraguay.....	Peso.....	Gold.....	1.00
Peru.....	Dollar.....	Silver.....	.918
Porto Rico.....	Peso.....	Gold.....	1.00
Portugal.....	Milreis.....	Gold.....	1.084
Russia.....	Rouble of 100 copecks.....	Silver.....	.734
Sandwich Islands.....	Dollar.....	Gold.....	1.00
Spain.....	Peseta of 100 centimes.....	Gold and silver..	.193
Sweden.....	Crown.....	Gold.....	.268
Switzerland.....	Franc.....	Gold and silver..	.193
Tripoli.....	Mahbub of 20 piasters.....	Silver.....	.829
Tunis.....	Piaster of 16 caroubs.....	Silver.....	.118
Turkey.....	Piaster.....	Gold.....	.043
United States of Colombia.....	Peso.....	Silver.....	.965
Uruguay.....	Patacon.....	Gold.....	.949
Venezuela.....	Peso.....	Silver.....	.7773

WRITTEN PROBLEMS.

1. What must be paid for a draft on Paris for 1200 francs, premium of 7% ?

Ans. \$247.812.

SOLUTION.

$$1200 \times \$1.93 = \$231.60$$

$$\$231.60 \times 1.07 = \$247.812$$

RULE.

Reduce the face of the bill into the currency of the country in which the bill is purchased, and proceed as in Domestic Exchange.

2. What is the cost of a draft on Liverpool for 600 £, 1 £ equalling \$4.8665, premium at $5\frac{1}{2}\%$? *Ans.* \$3080.49.
3. What is the cost of a draft on London for 325 £, when exchange is quoted at \$4.86 to the £? *Ans.* \$1579.50.
4. What cost a draft in Paris on New York for \$1200, when 1 franc = \$.193, premium 4%? *Ans.* 6466.32 francs.
5. How large a bill can be bought in Paris for \$1500, when \$1 = 5.18 francs? *Ans.* 7770 francs.
6. What is the cost of a bill on Berlin for 1600 marks, when a mark is \$.238? *Ans.* \$380.80.
7. What is the cost of a draft in Berlin on Philadelphia for \$523.60, when a mark is \$.238? *Ans.* 2200 marks.
8. Bought a bill on London for 400 £ when 1 £ = \$4.865, and a bill on St. Petersburg for 800 roubles when 1 rouble = .734: what was the cost of both in currency, gold being at a premium of 12%? *Ans.* \$2837.18.

CASE III.

ARBITRATION OF EXCHANGE.

275. Arbitration of Exchange is an exchange between two places by means of one or more intermediate exchanges.

WRITTEN PROBLEMS.

1. A merchant owes a bill in Paris to the amount of 15000 francs, and remits by indirect exchange on London: what is the cost of the draft, 1 £ being equal to \$4.86, and in Paris 24.75 francs equal to a £.

SOLUTION.

$$\frac{15000 \times \$4.86}{24.75} = \$2945.45.$$

Let the pupil give the analysis.

2. A merchant living in Chicago owes a debt of 6000 marks in Berlin: what does it cost him by indirect exchange through Liverpool and Paris, the rates being as follows: in Chicago

on Liverpool 1 £ = \$4.86, in Liverpool on Paris 1 £ = 25.25 francs, and in Paris on Berlin 1 franc = $1.08\frac{1}{2}$ marks?

Ans. \$1067.33.

3. A merchant in Decatur, Illinois, owes a merchant in Reading, Pa., \$3500; he sends a draft by Chicago and New York, exchange between Decatur and Chicago being $1\frac{1}{2}\%$ premium, between Chicago and New York at par, and between New York and Reading at $\frac{1}{2}\%$ discount: what does he pay for the draft?

Ans. \$3534.74.

SECTION XXI.

EQUATION OF PAYMENTS.

276. Equation of Payments is the process of finding the average time of payment of several sums of money due at different times.

277. The Term of Credit is the time a debt has to run.

278. The Average Term of Credit is the average of the several terms of credit.

279. The Equated Time is the time at which the payment of the several debts may be made together without loss to either debtor or creditor.

CASE I.

When the Terms of Credit begin at the Same Time.

ORAL EXERCISE.

1. If a man borrow \$40 for 5 months, how long should he lend \$50 to repay an equal amount of interest?

SOLUTION.—The interest on \$40 for 5 months is the same as the interest on \$1 for 40 times 5 months, or 200 months, and the interest on \$1 for 200 months is the same as the interest on \$50 for $\frac{1}{10}$ of 200 months, or 4 months. Hence, the interest on \$40 for 5 months is the same as the interest on \$50 for 4 months.

2. If a man lend me \$100 for 6 months, how long should I lend him \$50 to reciprocate the favor?

3. If I owe a merchant \$30 for 10 months, how long should he owe me a debt of \$75 to balance the favor?

4. A lends B \$600 for 5 months: how long should B lend A \$150 to reciprocate the favor?

WRITTEN PROBLEMS.

1. Sold a house and lot, payable as follows: \$300 in 3 mo., \$600 in 5 mo., \$500 in a year and \$1000 in 2 years: what was the average term of credit? *Ans.* $14\frac{1}{2}$ mo.

SOLUTION.

$$\begin{array}{rcl} 300 \times 3 \text{ mo.} & = & 900 \text{ mo.} \\ 600 \times 5 \text{ " } & = & 3000 \text{ " } \\ 500 \times 12 \text{ " } & = & 6000 \text{ " } \\ 1000 \times 24 \text{ " } & = & 24000 \text{ " } \\ \hline 2400 & &) 33900 \text{ mo.} \\ & & 14\frac{1}{2} \text{ mo.} \end{array}$$

ANALYSIS.—A credit of \$300 for 3 mo. is equivalent to a credit of \$1 for 900 mo.; a credit of \$600 for 5 mo. is equivalent to a credit of \$1 for 3000 mo.; a credit of \$500 for 12 mo. is equivalent to a credit of \$1 for 6000 mo.; a credit of \$1000 for 24 mo. is equivalent to a credit of \$1 for 24000 mo.;

the sum of the credits therefore is equal to a credit of \$1 for 33900 mo., or a credit of \$2400 for as many months as 2400 is contained times in 33900, or $14\frac{1}{2}$ mo.

2. Bought some land on which I was to make the following payments: \$50 cash, \$150 in 10 mo., \$200 in 22 mo., \$200 in 2 yr. 10 mo.: what is the average time of credit?

Ans. $21\frac{1}{2}$ mo.

3. I owe \$600, due in 5 mo.; \$1000, due in 10 mo.; \$1200, due in $7\frac{1}{2}$ mo.: what is the average term of credit?

Ans. 8 mo.

4. Bought goods June 1st, to be paid as follows: \$300 in 30 d., \$600 in 60 d. and \$900 in 90 d.: what is the equated time of payment?

Ans. Aug. 10.

5. Sold to a merchant goods, Apr. 1, to be paid for as follows: \$300 in 4 mo., \$800 in 5 mo. and \$1000 in 8 mo: what is the equated time of payment?

Ans. Oct. 10.

6. Bought a farm for \$10000, to be paid for as follows: 25% in 6 mo., 20% in 1 yr. 6 mo., and the remainder in $2\frac{1}{2}$ yr.: what is the average term of credit?

Ans. 21 mo. 18 d.

7. Bought a bill of goods amounting to \$10500, payable in 6 mo. I made the following payments: \$2000 in 2 mo. and

\$4500 in 4 mo.: how much time should I have on the remainder?

Ans. $10\frac{1}{4}$ mo.

SOLUTION.

$$\begin{array}{r} 2000 \times 4 = 8000 \\ 4500 \times 2 = 9000 \\ \hline 6500 \quad 17000 \end{array}$$

$$10500 - 6500 = 4000$$

$$17000 \div 4000 = 4\frac{1}{4}$$

$$4\frac{1}{4} \text{ mo.} + 6 \text{ mo.} = 10\frac{1}{4} \text{ mo.}$$

an extra credit of as many months as 4000 is contained times in 17000, or $4\frac{1}{4}$ mo. beyond the given 6 mo.

Explanation.—The credit on the first payment, \$2000, 4 mo. before due is equivalent to a credit of 8000 mo. for \$1; and the second payment, \$4500, 2 mo. before due is equivalent to a credit of 9000 mo. for \$1, or a total credit of 17000 mo. for \$1. Hence, the unpaid remainder, \$4000, is entitled to

8. On a debt of \$3000, due in 8 mo. from Jan. 1, 1875, the following payments were made: Mar. 1, \$1000; Apr. 1, \$200; July 1, \$800: when is the remainder due?

Ans. May 19, 1876.

9. I borrowed \$1600 for 5 mo. At the expiration of 2 mo. I paid back \$500; and at the expiration of 4 mo., \$500 more: how long am I entitled to keep the remainder after the expiration of the 5 mo.?

Ans. $3\frac{1}{2}$ mo.

CASE II.

When the Terms of Credit begin at Different Dates.

WRITTEN PROBLEMS.

1. Bought goods as follows:

June 1, \$500 on 3 mo. credit.

July 5, \$600 " 3 " "

Aug. 6, \$300 " 3 " "

Oct. 2, \$800 " 2 " "

Required the equated time of payment.

Ans. 52 d. after Sept. 1.

SOLUTION.

$$\text{Due Sept. 1, } 500 \times 0 = 0000$$

$$\text{" Oct. 5, } 600 \times 34 \text{ d.} = 20400 \text{ d.}$$

$$\text{" Nov. 6, } 300 \times 66 \text{ d.} = 19800 \text{ d.}$$

$$\text{" Dec. 2, } 800 \times 92 \text{ d.} = 73600 \text{ d.}$$

$$2200$$

$$113800$$

$$113800 \div 2200 = 51\frac{1}{11}$$

Analysis.—The 1st payment, \$500, is due Sept. 1; the 2d payment, \$600, is due Oct. 5, or 34 d. after Sept. 1; the 3d payment, \$300, is due Nov. 6, or 66 d. after Sept. 1; and the 4th payment, \$800, is due Dec. 2, or 92 d. after

Sept. 1; the equated time on these payments is therefore $51\frac{1}{11}$ d, or in full numbers 52 d., after Sept. 1, or Oct. 23.

2. Bought goods as follows :

Mar. 22, \$200 on 2 mo. credit.

Mar. 28, \$300 " 3 " "

June 16, \$500 " 2 " "

July 12, \$200 " 4 " "

What is the equated time of payment?

Ans. 75 d. from May 22, or Aug. 5.

3. What is the average of the following account?—

1875, Oct. 6, Mdse. on 60 d., \$150,

Oct. 31, " " 60 " \$200,

Nov. 26, " " 60 " \$180,

1876, Jan. 16, " " 60 " \$320,

Feb. 14, " " 60 " \$240,

Apr. 3, " " 60 " \$347.

Ans. Equated time, 98 d. from Dec. 5, 1875, or Mar. 12, 1876.

4. Sold the following to a retail merchant :

June 1, 1875, 1200 yd. carpeting, @ \$1.50, on 60 d.

" 16, " 500 " cloth @ \$2.50, on 60 "

Aug. 12, " 600 " cloth @ \$2.25, on 30 "

" 15, " 800 " cassimere @ \$.90, on 60 "

" 20, " 800 " carpet @ \$1.30, on 30 "

What is the equated time of payment?

Ans. 30 d. after July 31, or Aug. 30.

5. I owe four notes, as follows: June 5, for 90 d., \$100; July 12, at 60 d., \$200; Aug. 5, at 60 d., \$300; Sept. 3, at 60 d., \$500: what should be the date of a new note for 90 d. to take the place of the four? *Ans.* July 12 for \$1100; due, Oct. 10/13.

SECTION XXII.

EQUATION OF ACCOUNTS.

280. An **Account** is a written statement of the items of *debit* and *credit* between two parties.

281. **Equation of Accounts** is the process of determining the equated time for the payment of the balance of an account.

WRITTEN PROBLEMS.

1. In the following account what is the balance, and when due?

Dr.		W. N. CHAMBERLIN.		Cr.	
1875.				1875.	
Mar. 10	Mdse.	\$150	00	Apr. 3	Cash \$180 00
Apr. 6	"	50	00	May 12	" 10 00
Aug. 10	"	300	00	July 3	" 250 00
Sept. 7	"	125	00	Aug. 1	" 100 00

SOLUTION.

Due.	Time.	Items.	Product.	Due.	Time.	Items.	Product.
Mar. 10,		\$150		Apr. 3,	24 d.	\$180	4320
Apr. 6,	27 d.	50	1350	May 12,	63 d.	10	630
Aug. 10,	153 d.	300	45900	July 3,	115 d.	250	28750
Sept. 7,	181 d.	125	22625	Aug. 1,	144 d.	100	14400
		625	69875			540	48100
		540	48100				
		85	21775				
			257 d.				

Mar. 10 + 257 d. = Nov. 22.

Explanation.—The item first due is \$150, Mar. 10. Averaging the debits from this date, we find the sum of them equal to a debit of \$1 for 69875 days. Averaging the credits in the same manner from the same date, we find the sum of them equal to a credit of \$1 for 48100 days. The debits exceed the credits \$85, and the difference in time is 21775 days; therefore, the balance, \$85, should have a credit of as many days from Mar. 10 as 85 is contained times in 21775, or 257 days. Hence, the equated time is 257 days after Mar. 10, or Nov. 22.

RULES.

1. Find the date at which each item is due.
2. Multiply each sum by the number of days between its time and the time of the first item due.
3. Add the columns of products and the columns of items.
4. Divide the difference between the columns of products by the difference between the columns of items; this will give the number of days at which the balance is payable from the time the first item is due.
5. If the difference in products and the difference in items are

on different sides of the account, the number of days should be subtracted from the time when the first item is due.

2. What is the balance of the following account, and when is it due? *Ans.* \$62, due Apr. 15.

<i>Dr.</i>		R. M. McNEAL.				<i>Cr.</i>	
1873.				1873.			
Apr. 6	Mdse.	\$ 10	00	May 4	Cash.	\$ 90	00
May 13	"	300	00	May 31	"	200	00
June 5	"	182	00	June 2	"	140	00

3. What is the balance, and when due, on the following account? *Ans.* \$300, int. from Oct. 13, 1874.

<i>Dr.</i>		SILAS WRIGHT.				<i>Cr.</i>	
1875.				1875.			
Jan. 3	Mdse.	\$600	00	Feb. 3	Cash.	\$400	00
Feb. 12	"	450	00	Mar. 28	"	400	00
Apr. 1	"	250	00	Apr. 6	"	200	00

4. What is the balance of the following account, and when due? *Ans.* \$956, due Mar. 6, 1877.

<i>Dr.</i>		JESSE NEWLIN.				<i>Cr.</i>	
1877.				1877.			
Feb. 20	Mdse.	\$140	00	June 15	Cash.	\$300	00
Mar. 15	"	150	00	Aug. 4	"	200	00
Apr. 3	"	650	00	Aug. 18	"	200	00
Apr. 20	"	217	00				
May 12	"	144	00				
Aug. 10	"	355	00				

5. What is the balance, and when due, of the following account? *Ans.* \$65, interest from July 29, 1878.

<i>Dr.</i>		T. F. GAHAN.				<i>Cr.</i>	
1878.				1878.			
Aug. 4	Mdse at 3mo.	\$600	00	Oct. 4	Cash.	\$200	00
Sept. 5	" " 3 "	200	00	Dec. 15	"	600	00
Oct. 4	" " 2 "	300	00	1879.			
Nov. 15	" " 3 "	120	00	Jan. 1	Cash.	\$500	00
Dec. 5	" " 2 "	145	00				

SECTION XXIII.

CASH BALANCE.

282. The **Cash Balance** of an account is the difference between the two sides, with interest on each of the items to the date of settlement.

283. The **Interest Balance** is the difference between the interest of the several debits and the several credits.

WRITTEN PROBLEMS.

1. Find the cash balance of the following account, Sept. 5, 1876, interest at 6%. *Ans.* \$81.79.

<i>Dr.</i>				<i>W. W. WOODRUFF.</i>				<i>Cr.</i>			
1876.				1876.							
Apr. 20	Mdse.	\$50	00	July 2	Cash	\$50	00				
May 3	"	40	00	" 20	"	60	00				
July 3	"	80	00								
Sept. 14	"	20	00								
" 5	Interest	1	79	Sept. 5	Cash Bal.	81	79				
		<u>\$191</u>	<u>79</u>			<u>\$191</u>	<u>79</u>				

SOLUTION.

Dr.

Int. on \$50 from Apr. 20 to Sept. 5, 138 days, \$1.15
 " " \$40 " May 3 " Sept. 5, 125 " .83
 " " \$80 " July 3 " Sept. 5, 64 " .85 — **\$2.83**

Cr.

Int. on \$20 from Sept. 5 to Sept. 14, 9 days, \$.03
 " " \$50 " July 2 " Sept. 5, 65 " .54
 " " \$60 " July 20 " Sept. 5, 47 " .47 — **1.04**
 Balance of interest **\$1.79**

Adding this balance to the debit side, the cash necessary to balance the account we find to be \$81.79.

From this we derive the following

RULE.

Find the interest on each item from the time it is due or paid to the time of settlement.

Add the interest to the item if due before the time of settlement,

and subtract when the item is due after the time of settlement. The difference between the footings of the two sides is the cash balance.

(2.)

Dr.		JOHN A. ROBB,		Cr.	
1873.				1873.	
Jan. 5	Mdse.	\$ 60	00	Mar. 15	Cash
Feb. 3	"	40	00	Apr. 10	"
July 10	"	120	00	May 3	"
Sept. 4	"	35	00		

What is the cash balance due Aug. 1, 1873? *Ans.* \$126.17.

(3.)

Dr.		L. A. MACKEY.		Cr.	
1878.				1878.	
Oct. 1	Rent	\$125	00	Oct. 6	Cash
Nov. 5	Mdse.	40	00	Oct. 15	"
Dec. 1	Rent	125	00	Dec. 10	"
" 16	Hay	40	00	Dec. 16	"
Jan. 1	Rent	62	50		

What is the cash balance due Jan. 1, 1879? *Ans.* \$123.35.

CHAPTER X.

PARTNERSHIP.

284. A **Partnership** is the association of two or more individuals for the transaction of business; the persons so associated are called partners. The association is also sometimes called a company or a firm.

285. A partner who has a voice in the management of the business is called an **Active Partner**, and one who furnishes capital, but has no voice in the management, is called a **Silent Partner**.

286. The debts of a company are called its **Liabilities**, and its property, with the amount due the firm, its **Assets**.

SECTION I.

SIMPLE PARTNERSHIP.

287. A Simple Partnership is one in which each of the partners has his capital invested for the same time.

WRITTEN PROBLEMS.

1. A and B engage in trade; A furnishes \$6000, and B \$4000; they gain \$1200: what is each one's share of the gain?

Ans. A's, \$720; B's, \$480.

SOLUTION.

$$\$6000 + \$4000 = \$10000 = \text{capital.}$$

$$\$1200 \div \$10000 = .12, \text{ or } 12\% = \text{gain.}$$

$$12\% \text{ of } \$6000 = \$720 = \text{A's gain.}$$

$$12\% \text{ of } \$4000 = \$480 = \text{B's gain.}$$

Or,

$$\$6000 + \$4000 = \$10000 = \text{capital.}$$

$$\$6000 = \frac{6}{10} \text{ of capital.}$$

$$\$4000 = \frac{4}{10} \text{ of capital.}$$

$$\text{A's gain} = \frac{6}{10} \text{ of } \$1200 = \$720.$$

$$\text{B's gain} = \frac{4}{10} \text{ of } \$1200 = \$480.$$

RULES.

1. Find what per cent. the gain or loss is of the entire capital, and take the same per cent. of each partner's capital to find his gain or loss.

Or, 2. Find such a proportionate share of the gain or loss as each partner's capital is of the whole capital.

2. Three men purchase a store, paying as follows: the 1st, \$2000; the 2d, \$4000; and the 3d, \$10000; they gain \$6400: how much does each gain?

Ans. 1st, \$800; 2d, \$1600; 3d, \$4000.

3. Four men, A, B, C and D, buy a marble-quarry for \$10000, A paying \$1500; B, \$2000; C, \$2500; and D, \$4000; they sell it for \$45000; what is each one's share of the gain?

Ans. A's, \$5250; B's, \$7000; C's, \$8750; D's, \$14000.

4. A, B and C buy a house for \$2500; A pays \$500; B, \$1200; C, \$800; they rent it for \$300: what is each one's share of the rent? *Ans.* A's, \$60; B's, \$144; C's, \$96.

5. Rankin, Barton and Boggis rent a farm for \$1200; Rankin pastures 16 cows; Barton, 14 cows; and Boggis, 12 cows: how much should each pay?

Ans. Rankin, \$457 $\frac{1}{2}$; Barton, \$400; Boggis, \$342 $\frac{1}{2}$.

6. A, B and C enter into a partnership; A puts in \$18000; B, \$6000; C, \$10000; they lose 30% of their stock by fire, and sell the remainder at a gain of 50%: what is each one's share of the gain? *Ans.* A's, \$900; B's, \$300; C's, \$500.

7. A, B and C gain \$2250; A's gain is \$800; B's, \$1000; and C's capital, \$3000: how much was the stock of each?

Ans. A's, \$5333 $\frac{1}{3}$; B's, \$6666 $\frac{2}{3}$; C's, \$3000.

8. A man dying, willed his son \$6500, his widow \$8000, and his daughter \$5500; but his estate amounted to only \$12000: how much did each get?

Ans. Son, \$3900; widow, \$4800; daughter, \$3300.

9. John, James and George buy a store for \$16000; John pays \$5000; James, \$6500; and George, \$4500; they pay \$2500 clerk hire, \$600 rent, and sell the store for \$23000: how much did each gain?

Ans. John, \$1218.75; James, \$1584.37 $\frac{1}{2}$; George, \$1096.87 $\frac{1}{2}$.

SECTION II.

COMPOUND PARTNERSHIP.

288. Compound Partnership is that in which the capital of the partners is employed for different periods of time.

WRITTEN PROBLEMS.

1. A, B and C engaged in partnership; A put in \$600 for 6 mo.; B, \$800 for 5 mo.; and C, \$1000 for 4 $\frac{1}{2}$ mo.; they gain \$605: what is each one's share of the gain?

Ans. A's, \$180; B's, \$200; C's, \$225.

SOLUTION 1.

$$\begin{aligned} \$600 \times 6 &= \$3600 \\ \$800 \times 5 &= \$4000 \\ \$1000 \times 4\frac{1}{2} &= \$4500 \\ &\$12100 \end{aligned}$$

$$605 \div 12100 = .05, \text{ or } 5\%.$$

$$5\% \text{ of } \$3600 = \$180 = \text{A's gain.}$$

$$5\% \text{ of } \$4000 = \$200 = \text{B's gain.}$$

$$5\% \text{ of } \$4500 = \$225 = \text{C's gain.}$$

Explanation.—\$600 for 6 mo. is equivalent to \$3600 for 1 mo.; \$800 for 5 mo. is equivalent to \$4000 for 1 mo.; \$1000 for 4½ mo. is equivalent to \$4500 for 1 mo., or a total of \$12100 for 1 mo. If the gain on \$12100 for 1 mo. is \$605, 5% of the capital equals the gain. 5% of \$3600, or \$180, equals A's gain, etc.

SOLUTION 2.

$$\begin{aligned} \$600 \times 6 &= \$3600 \\ \$800 \times 5 &= \$4000 \\ \$1000 \times 4\frac{1}{2} &= \$4500 \\ &\$12100 \end{aligned}$$

$$\$3600 = \frac{\$605}{\$12100}, \text{ or } \frac{5}{121}, \text{ of the capital.}$$

$$\$4000 = \frac{\$605}{\$12100}, \text{ or } \frac{5}{121}, \text{ " "}$$

$$\$4500 = \frac{\$605}{\$12100}, \text{ or } \frac{5}{121}, \text{ " "}$$

$$\frac{5}{121} \text{ of } \$605 = \$180, \text{ or A's gain.}$$

$$\frac{5}{121} \text{ of } \$605 = \$200, \text{ or B's gain.}$$

$$\frac{5}{121} \text{ of } \$605 = \$225, \text{ or C's gain.}$$

Let the pupil give the explanation.

RULES.

1. *Multiply each partner's capital by the time for which it is invested, and find what per cent. the gain or the loss is of the sum of these products; then find the same per cent. of each product.*

2. *Multiply each partner's capital by the time for which it is invested, and divide the whole gain or loss among the partners in the ratio of these products.*

2. Three men hire a pasture for \$87.20: A puts in 3 cows for 16 wk.; B, 5 cows for 20 wk.; and C, 7 cows for 10 wk.: how much does each pay, and what is the pasturage of a cow for a week?

Ans. A, \$19.20; B, \$40; C, \$28; pasturage, 40 cts. a week.

3. D, E and F, on closing business, paid \$2706; D had \$4200 capital invested; E, \$3600; and F, \$4500: how much did each pay?

Ans. D, \$924; E, \$792; F, \$990.

4. Salmon, McNerney and Scheid agree to do a piece of work for \$1600; Salmon furnished 7 men 30 d.; McNerney,

5 men 40 d.; and Scheid, 6 men 32 d.: how much should each receive if they paid \$95 clerk hire?

Ans. Salmon, \$525; McNerney, \$500; Scheid, \$480.

5. A, B and C enter into a partnership Jan. 1; A furnishes \$6000; B, \$5000; C, \$4500. A adds \$2000 July 1; B adds \$3000 Aug. 1; and C adds \$3500 Sept. 1; they gain during the year \$4540: what is each one's share of the gain?

Ans. A, \$1680; B, \$1500; C, \$1360.

6. Bowman, Johnson and Bitner enter into partnership; they put in money as follows: Jan. 1, Bowman put in \$5000; Johnson, \$6000; Bitner, \$7500; July 1, Bowman withdraws \$1000; Aug. 1, Johnson withdraws \$2500; and Sept. 1, Bitner withdraws \$5000; they make \$7340: after paying \$1835 for clerk hire, what is each one's gain?

Ans. Bowman, \$1620; Johnson, \$1785; Bitner, \$2100.

7. Leshner, Clark and Bickford enter into partnership with Schuyler as a special partner, contributing as follows: Leshner, \$3000; Clark, \$2500; Bickford, \$4000; Leshner puts in \$3000 more Apr. 1; Clark, \$4000, May 1; and Bickford, \$2000, June 1. Leshner withdraws \$2000 Aug. 1; Clark, \$3000, Sept. 1; and Bickford, \$2500, Oct. 1. Schuyler furnished \$10000 Jan. 1, and was to have one-fourth of the net profits; they gain \$7000, and pay out \$700 of this amount as expenses: how much capital has each at the end of the year, and what is each one's share of the gain?

Ans. Leshner, $\left\{ \begin{array}{l} \text{Cap., } \$4000 \\ \text{Gain, } \$1590 \end{array} \right.$; Clark, $\left\{ \begin{array}{l} \text{Cap., } \$3500 \\ \text{Gain, } \$1500 \end{array} \right.$;
Bickford, $\left\{ \begin{array}{l} \text{Cap., } \$3500 \\ \text{Gain, } \$1635 \end{array} \right.$; Schuyler, $\left\{ \begin{array}{l} \text{Cap., } \$10000 \\ \text{Gain, } \$1575 \end{array} \right.$

8. Noyes, Barrows and Hipple enter into partnership; they furnish money as follows: Jan. 1, Noyes, \$10,000; Barrows, \$6000; and Hipple, \$8000; Sept. 1, Noyes withdraws \$3000; Oct. 1, Barrows puts in \$2000; and Nov. 1, Hipple withdraws \$600. They gain \$3842, of which Barrows receives \$800 for managing the business: what is each one's share of the net gain? *Ans.* Noyes, \$1170; Barrows, \$845; Hipple, \$1027.

CHAPTER XI.

ALLIGATION.

289. Alligation treats of the mixing or combining of two or more articles of different values.

290. Alligation Medial is the process of finding the average value or quality of the several articles.

291. Alligation Alternate is the process of determining the proportion of the several articles used in forming a mixture.

The word *alligation* is derived from the Latin *alligo*, "I bind," and is so called because in solving some of the problems the numbers may be joined or linked together.

CASE I.

To find the Average.

ORAL EXERCISE.

1. If a grocer mix 3 lb. of sugar at 10 cts., 5 lb. at 8 cts. and 2 lb. at 15 cts., what is the average price per pound?

SOLUTION.—3 lb. at 10 cts. are worth 30 cts.; 5 lb. at 8 cts. are worth 40 cts., and 2 lb. at 15 cts. are worth 30 cts.: 30 cts. plus 40 cts. plus 30 cts. are 100 cts.; 3 lb. + 5 lb. + 2 lb. equal 10 lb. If 10 lb. are worth 100 cts., 1 lb. is worth $\frac{1}{10}$ of 100 cts., or 10 cts.

2. If a wine-dealer mix 6 gal. of wine worth \$2 a gallon with 4 gal. worth \$3 a gallon, what must he sell it at per gallon so as not to lose?

3. A farmer bought 20 sheep at 5 dollars a head, 10 at 4 $\frac{1}{2}$ dollars a head, and 20 at 3 dollars a head: what was the average price?

WRITTEN PROBLEMS.

4. A miller has 4 loads of wheat, as follows: 60 bu. at \$1.25, 47 bu. at \$1.30, 55 bu. at \$1.32, and 38 bu. at \$1.20: how must he sell it per bushel so as not to lose? *Ans.* \$1.27 $\frac{3}{4}$.

5. A merchant bought 150 yd. muslin at 10 cts., 120 yd. at $7\frac{1}{2}$ cts., 144 yd. at $14\frac{1}{2}$ cts., and 160 yd. at 15 cts. : what must he sell it at so as to gain 25% ? *Ans.* 15 cts. per yard.

6. A grocer bought 600 lb. coffee at 23 cts., 800 lb. at 26 cts., and 960 lb. at 30 cts. : he sold it at an average price of 31 cts. a pound : what was the entire gain ? *Ans.* \$97.60.

CASE II.

To find the Proportion of Different Articles of Given Value or Quality when the Average Value or Quality is given.

WRITTEN PROBLEMS.

1. How much muslin at 10, 13 and 15 cts. a yard must a merchant sell that the average price may be 12 cts. a yard ?

SOLUTION.

$$12 \left\{ \begin{array}{c|c|c|c|c} 10 & \frac{1}{2} & 1 & \frac{1}{2} & 3 \\ 13 & & & & \\ 15 & & \frac{1}{2} & \frac{1}{2} & 2 \end{array} \right. \begin{array}{l} 1 - 4 \\ 2 - 2 \\ - 2 \end{array}$$

Analysis.—On the 10-ct. muslin the gain is 2 cts. a yard; hence, to gain 1 ct. $\frac{1}{2}$ yd. must be sold. On the 13-ct. muslin the loss is 1 ct. on 1 yd.; and on the 15-ct. muslin the loss is 3 cts. a yard, or a cent

on $\frac{1}{3}$ of a yard. Since the gain on $\frac{1}{2}$ yd. of 10-ct. muslin is 1 ct., and the loss on $\frac{1}{2}$ yd. of 15-ct. muslin is 1 ct., the loss and the gain are equal, and we take $\frac{1}{2}$ yd. at 10 cts. as often as $\frac{1}{2}$ yd. at 15 cts., or 3 yd. at 10 cts. as often as 2 yd. at 15 cts. In a similar manner, since the gain on $\frac{1}{2}$ yd. at 10 cts. balances the loss on 1 yd. at 13 cts., we take $\frac{1}{2}$ yd. at 10 cts. as often as 1 yd. at 13 cts., or 1 yd. at 10 cts. as often as 2 yd. at 13 cts. The requisite numbers of yards are, therefore, 3+1, or 4 at 10 cts., 2 at 13 cts., and 2 at 15 cts.

2. In what proportion must sugar worth 9 cts. a pound be mixed with that worth 14 cts., so that the mixture may be worth 12 cts. a pound ? *Ans.* 2 lb. at 9, and 3 lb. at 14.

3. How many dozens of eggs at 12, 15, 18 and 20 cts. a dozen must be sold that the average price per dozen may be 16 cts. ? *Ans.* 1 at 12, 2 at 15, 1 at 18, and 1 at 20.

4. If I have wine worth \$1.50, \$1.75, \$2.20 and \$2.40 a gallon, in what proportions must it be mixed that I may sell it at \$2 a gallon ?

Ans. 4 at \$1.50, 4 at \$1.75, 5 at \$2.20, and 5 at \$2.40.

5. A wine-merchant has wine worth respectively \$3 and \$3.50 a gallon. He wishes to mix these two and sell a grade at \$2.50 a gallon: what proportion of each and what proportion of water must he take?

Ans. 5 gal. at \$3, 5 gal. at \$3.50, and 3 gal. of water.

6. A farmer bought sheep at \$5, \$6, \$7 and \$10 a head: how many of each did he buy if the average price was \$7.50?

Ans. 1 at \$5, 5 at \$6, 5 at \$7, and 5 at \$10.

7. If I have cloth worth \$2½, \$2¼, \$2⅓ and \$4 a yard, how many yards of each must I sell that the average price may be \$3½ a yard? *Ans.* 3 at \$2½, 1 at \$2¼, 3 at \$2⅓, and 7 at \$4.

CASE III.

To find the Proportions of the Different Parts of a Given Value or Quality when the Average Value or Quality is given, and the Proportion of One or more of the Parts.

1. A merchant sold 25 yd. of muslin at 12 cts. a yard: how much must he sell at 8 and 15 cts. respectively that the average price may be 13 cts.?

SOLUTION.

Analysis.—By Case II. we determine

$13 \left\{ \begin{array}{l|l} 8\frac{1}{2} & 2 \\ \hline 12\frac{1}{2} & 2 \\ 15 & 5+1 \end{array} \right\} \times 12\frac{1}{2} = \begin{cases} 25 & \text{that 2 yd. of the 8-ct. muslin are taken} \\ 25 & \text{as often as 2 yd. at 12 cts. and 6 yd. at} \\ 75 & \text{15 cts. The required number of yards} \end{cases}$
at 12 cts. is 25, or 12½ times 2; hence we take 12½ times 2 yd., or 25 yd., at 8 cts., and 12½ times 6 yd., or 75 yd., at 15 cts.

2. A dry-goods merchant has 30 yd. of damaged merino which he wishes to sell at 40 cts. a yard: how many yards must he sell worth \$1.20 that he may get cost for his goods, which was 90 cts. a yard? *Ans.* 50 yd.

3. A miller buys corn at 60, 62 and 65 cts. a bushel: how much did he buy of each if he bought 70 bu. at 70 cts., and the average price is 67 cts.? *Ans.* 15 bu. of each.

4. A merchant bought cloth at \$4, \$3.50, \$2.20 and \$2 a yard, and 30 yd. at \$2.50: how many yards of each did he buy if the average price is \$3.20 a yard?

Ans. 50 at \$4, 110 at \$3.50, 40 at \$2.20, 10 at \$2

5. If a druggist have 6 gal. of alcohol 95% strong, and some 90% strong, how much water and how much of the 90% alcohol must be mixed with the 6 gal., that the strength may be reduced to 80%?

Ans. 3 gal. at 90%, and $1\frac{1}{2}$ gal. water.

CASE IV.

To find the Proportion of the Different Parts of a Given Value or Quality when the Entire Quantity is given.

1. How much sugar at 8, 9, 12 and 15 cts. a pound must be mixed so as to fill a barrel of 300 lb. at 10 cts.?

SOLUTION.

$$10 \left\{ \begin{array}{c|c|c} 8 & \frac{1}{2} & 1 \\ 9 & 1 & 1 \\ 12 & \frac{1}{3} & 1 \\ 15 & \frac{1}{5} & 1 \end{array} \right\} \times 30 = \begin{array}{r} 150 \\ 60 \\ 30 \\ 60 \\ \hline 300 \\ 30 \end{array}$$

Analysis.—By the analysis, as in the previous cases, we determine that 5 lb. of the 8-ct. sugar shall be taken as often as 2 lb. of the 15-ct.; and 2 lb. of the 9-ct. as often as 1 lb. of the 12-ct. The sum of these quantities is 10 lb.

To make a mixture of 300 lb. will therefore require 30 times as many pounds of each, or 150 lb. of the 8-ct., 60 lb. of the 9-ct., 30 lb. of the 12-ct., and 60 lb. of the 15-ct.

Another form of solution may be given as follows:

SOLUTION.

$$10 \left\{ \begin{array}{c|c|c|c|c} 8 & \frac{1}{2} & 1 & \frac{a}{1} & \frac{b}{2} \\ 9 & 1 & 1 & 1 & 1 \\ 12 & \frac{1}{3} & 1 & 1 & 1 \\ 15 & \frac{1}{5} & 1 & 1 & 1 \end{array} \right\} \times 30 = \begin{array}{r} 60 \\ 150 \\ 60 \\ 30 \\ \hline 300 \\ 30 \end{array}$$

Analysis.—By the analysis, as in previous cases, we determine that 1 lb. of the 8-ct. shall be taken as often as 1 lb. of the 12-ct., and 5 lb. of the 9-ct. as often as 1 lb. of the 15-ct. The sum of these numbers is 8, which is not an exact

divisor of 300. Either column *a* or column *b*, or both, may be multiplied by any number so that the sum of the two columns may form an exact divisor of 300. Multiplying column *a* by 2, we have 2 lb. at 8 cts., 5 at 9, 2 at 12, and 1 at 15, or a total of 10 lb., which is $\frac{1}{30}$ of 300 lb. Multiplying the resulting columns by 30, we have 60 lb. at 8 cts., 150 at 9 cts., 60 at 12 cts., and 30 at 15 cts. a pound.

2. A man hired men at \$15 a week, and boys at \$5 a week,

to do some work; they worked 5 weeks, and he paid \$450: how many were there of each if the number of laborers was 10?

Ans. 4 men, 6 boys.

3. Sold 100 lb. of butter for \$33; some was sold at 30 cts., some at 28 cts., and some at 35 cts. a pound: how many pounds were there of each?

Ans. 12 lb. at 30 cts., 20 lb. at 28 cts., and 68 lb. at 35 cts.

4. Sold 50 doz. of eggs for \$8: how many dozens of each were there at 13, 14, 18 and 21 cts. respectively?

	6	25	10
<i>Ans.</i>	25	5	17
	9, or	5, or	17
	10	15	6

5. Bought turkeys at 85 cts., ducks at 30 cts., and chickens at 40 cts. each; in all, 80 fowls for 48 dollars: how many of each were there? *Ans.* 40 turkeys, 20 ducks, 20 chickens.

CHAPTER XII.

POWERS AND ROOTS.

SECTION I.

POWERS.

292. A **Power** of a number is the number itself, or that number taken two or more times as a factor. Thus, 3 is the *first* power of 3; $3 \times 3 = 9$, or 3 taken twice as a factor is the *second* power, or square, of 3.

$3 \times 3 \times 3 = 27$, or 3 taken three times as a factor, is the *third* power, or cube, of 3.

293. The power of a number is usually indicated by a small figure, called the exponent, written to the right and above it; thus, the fourth power of 5 is written 5^4 .

From the preceding definitions it is evident that any power of a number may be found by the following

RULE.

To find the power of a number, take the number as a factor as many times as is indicated by the exponent.

WRITTEN PROBLEMS.

1. What is the 4th power of 21?

OPERATION.— $21^4 = 21 \times 21 \times 21 \times 21 = 194481$.

Find the indicated powers of the following—

- | | | | |
|------------------------------------|----------------|---|----------------------------------|
| 2. 1^2 . | Ans. 1. | 9. 8.14^2 . | Ans. 66.2596. |
| 3. 23^2 . | Ans. 529. | 10. 7.03^2 . | Ans. 347.428927. |
| 4. 314^2 . | Ans. 98596. | 11. $(\frac{1}{2})^2$. | Ans. $\frac{1}{4}$. |
| 5. 216^2 . | Ans. 10077696. | 12. $(\frac{1}{2})^6$. | Ans. $\frac{1}{64}$. |
| 6. 874^2 . | Ans. 763876. | 13. $(2\frac{1}{2})^4$. | Ans. $24\frac{1}{16}$. |
| 7. 19^2 . | Ans. 2476099. | 14. $(3\frac{1}{2} \times \frac{2}{3})^2$. | Ans. $4\frac{1}{3}$. |
| 8. 6.1^2 . | Ans. 37.21. | 15. $(3\frac{1}{2} + 2\frac{1}{2})^2$. | Ans. $31\frac{1}{4}$. |
| 16. $(\frac{7}{3\frac{1}{4}})^3$. | | | Ans. $1\frac{1}{8}\frac{1}{8}$. |

SECTION II.

ROOTS.

294. A **Root** of a number is one of its equal factors.

295. The **Square Root** of a number is one of its two equal factors; thus, 4 is the square root of 16.

296. A number is said to be a **perfect square** when its square root can be taken; that is, when it consists of two equal factors.

297. The **Cube Root** of a number is one of its three equal factors.

298. A number is said to be a **perfect cube** when it is composed of three equal factors.

299. The radical sign, $\sqrt{}$, when placed before a number indicates that some root is to be found.

300. The index of the root is a small figure written above the radical sign to indicate what root is to be found; thus, $\sqrt[3]{64}$ indicates that the third or cube root of 64 is to be found. When no index is written, the figure 2, indicating the square root, is understood.

Sometimes the root is indicated by a fractional exponent; thus, $625^{\frac{1}{3}}$ means the same as $\sqrt[3]{625}$.

To find the Root of any Number by Factoring.

Find the cube root of 91125.

OPERATION.

$$\begin{array}{r}
 5 \overline{)91125} \qquad 5 \times 3 \times 3 = 45 = \sqrt[3]{91125} \\
 5 \overline{)18225} \\
 5 \overline{)3645} \\
 3 \overline{)729} \\
 3 \overline{)243} \\
 3 \overline{)81} \\
 3 \overline{)27} \\
 3 \overline{)9} \\
 3
 \end{array}$$

Explanation.—Every perfect cube consists of three equal factors. The prime factors of 91125 are $(5 \times 3 \times 3) \times (5 \times 3 \times 3) \times (5 \times 3 \times 3)$. Therefore, the cube root of 91125 is one of the three equal factors $(5 \times 3 \times 3)$, or 45.

In a similar manner find—

1. The square root of 625; 1728; 6561.
2. The cube root of 729; 3375; 970299; 13824.

1. SQUARE ROOT.

GENERAL METHOD.

301. The General Method of finding the square root of numbers depends upon the following

PRINCIPLES.—1. The square of any number less than 10 consists of one or two places.

2. The square of any number between 9 and 100 consists of three or four places.

3. In general, the square of any number contains either twice as many places as the number, or twice as many less one. Hence,

4. If any number be separated into periods of two figures

each, beginning with the units' place, the number of periods will represent the number of places in the square root of the number.

WRITTEN PROBLEMS.

1. Find the square root of 58564.

OPERATION.

$$\begin{array}{r}
 58564 \overline{)242} \\
 \underline{4} \\
 44 \overline{)185} \\
 \underline{176} \\
 482 \overline{)964} \\
 \underline{964}
 \end{array}$$

Analysis.—Since the number consists of 3 periods of figures, the square root will consist of 3 figures—namely, units, tens and hundreds. The greatest number whose square is contained in 5 ten-thousands is 2 hundreds, the square of 2 hundreds is 4 ten-thousands; subtracting, we have 185 hundreds; doubling the root already found, placing a cipher after it, and using it as a trial divisor into 185, we find it contained 4 tens times; substituting 4 for the cipher, and multiplying the entire divisor, 44 tens, by 4 tens, we have 176 hundreds; subtracting this from 185 hundreds, we have 9 hundreds and 64 units, or 964. Doubling the root, 24 tens, and writing a cipher after it, it becomes a trial divisor into 964; 480 is contained in 964 about 2 times; substituting 2 for the cipher, and multiplying the entire divisor by 2, we have 964, which being subtracted from 964 leaves no remainder; 242 is therefore the square root of 58564.

The foregoing process gives us the following rules for extracting the square root:

RULES.

1. *Separate the number into periods of two figures each, beginning with the units' figure.*
2. *Find the greatest square in the first period on the left, and place its square root on the right as the first figure of the root.*
3. *Subtract the square of this root from the first period, and to the difference annex the next period of the dividend.*
4. *Double the root found, and place it on the left with a cipher annexed for a trial divisor. The quotient will be the next figure of the root.*
5. *Put this figure of the root in place of the cipher, and then multiply the entire divisor by this last figure of the root; subtract the product from the dividend, and to the difference annex the next period for a new dividend.*
6. *Double the root already found, with a cipher annexed, for a*

new trial divisor, and proceed as before until all the periods have been brought down and used.

NOTES.—1. The explanation of the principles of the foregoing rule belongs properly to Algebra, and will not be given here.

2. In extracting the square root of a fraction, we extract the root of both numerator and denominator separately, and of decimals as of whole numbers, first separating into periods both right and left from the decimal point.

3. When the number is not a perfect square, annex ciphers and extract the root on to decimals.

4. When the terms of a common fraction are not both perfect squares, the square root may be found by reducing it to a decimal fraction, or by multiplying both numerator and denominator by the denominator, and then extract the square root. Thus, to find the $\sqrt{\frac{7}{16}}$: $\sqrt{\frac{7}{16}} = \sqrt{\frac{7}{16} \times \frac{16}{16}} = \sqrt{\frac{7 \times 16}{16 \times 16}} = \sqrt{\frac{112}{256}} = \frac{1}{16} \sqrt{112} = \frac{1}{16} \sqrt{16 \times 7} = \frac{1}{16} \times 4 \sqrt{7} = \frac{1}{4} \sqrt{7}$ plus, = .935.

Extract the square root of the following:

1. 225.	Ans. 15.	11. $\frac{81}{100}$.	Ans. $\frac{9}{10}$.
2. 529.	Ans. 23.	12. $\frac{144}{169}$.	Ans. $\frac{12}{13}$.
3. 5929.	Ans. 77.	13. $\frac{441}{1089}$.	Ans. $\frac{21}{33}$.
4. 7396.	Ans. 86.	14. .0289.	Ans. .17.
5. 15376.	Ans. 124.	15. .003969.	Ans. .063.
6. 104976.	Ans. 324.	16. 64.112049.	Ans. 8.007.
7. 334084.	Ans. 578.	17. 49.7025.	Ans. 7.05.
8. 2047761.	Ans. 1431.	18. 10.	Ans. 3.1622+.
9. 6375625.	Ans. 2525.	19. $\frac{9}{16}$.	Ans. .7746-.
10. 9247681.	Ans. 3041.	20. $\frac{1}{9}$.	Ans. $\frac{1}{3} \sqrt{3}$.

PROBLEMS.

1. What is the length of the side of a square field containing one acre?
Ans. 208 ft. 8.52 in.

2. What is the length of a square piece of land containing 4 A.?
Ans. 417 ft. 5.04 in.

3. What will it cost to fence a square field containing 10 A. with post and rail fence 5 rails high, allowing 15 ft. to the panel, rails being worth \$6 a hundred and posts \$40 a hundred, the builder working 10 d., at \$1.50 a day?

Ans. \$138.20.

4. I have a farm of 122 A. 80 P. in the shape of a square which I wish to divide into 4 equal square fields: what will the fencing cost per acre if my neighbors pay for half the boundary fences, the panels being 15 ft. long, 5 rails to the panel, and the setting of my part requiring the work of 2 men 24 d., each at \$1.75 a day, posts being worth 42 cts. each, and rails \$6.25 per hundred?

Ans. \$4.37.

2. CUBE ROOT.

The *general* method of finding the cube root of a number depends upon the following

PRINCIPLES.—1. The cube of any number less than 10 never contains any figure higher than hundreds.

2. The cube of any number of tens never exceeds hundred thousands, and is never less than one thousand.

3. In general, the cube of a number contains either three times as many places as the number itself, or three times as many less one or two.

4. Any number of which the cube root is to be taken may therefore be separated into periods of three figures each, beginning with the place of units, and the number of periods will represent the number of figures in the root.

Ex. 1. Find the cube root of 10077696.

OPERATION.		<i>Explanation.</i> —By separating the	
	10077696	200	given number into periods of three
$2^3 = 8$	8	10	figures each, we find the root consists
$20^3 \times 3 = 1200$	2077	6	of three places. The greatest cube
$20 \times 1 \times 3 = 60$	216		contained in the first period is 2.
$1 \times 1 = 1$			Cubing this number and subtracting,
$1 \times 1261 = 1261$			we have a difference of 2, to which we
$210^3 \times 3 = 132300$	816696		annex the next period. Annexing a
$210 \times 6 \times 3 = 3780$			cipher to the root already found,
$6 \times 6 = 36$			squaring it, and multiplying by 3,
$6 \times 136116 = 816696$			we have 1200 for a trial divisor,

which is contained in 2077 about 1 time; multiplying 20, the root before found, by the root last found, and this product by 3, we have 60; squaring 1, the root last found, and adding the partial divisors together, we have

for a complete divisor 1261, which, being multiplied by the quotient 1 and subtracted from the dividend, leaves a difference of 816.

Annexing the next period to this difference, we have for a dividend 816696.

Annexing a cipher to the root 21, before found, squaring the result, and multiplying it by 3, we have for a trial divisor 132300, which is contained in the dividend about 6 times. Multiplying the root before found, with a cipher annexed, 210, by the root 6 just found, and this product by 3, we have 3780. Squaring the root just found, we have 36; adding these partial divisors, we have for a complete divisor 136116, which being multiplied by the quotient 6 produces 816696, and this subtracted from the dividend leaves no remainder. The cube root of 10077696 is therefore 216.

From the foregoing principles and the explanation we have the following rules for Cube Root:

RULES.

1. *Separate the number into periods of three figures each, beginning with the units' place.*
2. *Find the greatest cube contained in the first period on the left; this is the first figure of the root. Subtract the cube of this root from the first period on the left, and to the remainder annex the next period to form a dividend.*
3. *Consider the root just found as tens, square it, and multiply by 3; this forms a trial divisor. Divide the dividend by this divisor, and the quotient forms the second figure of the root.*
4. *To complete the divisor, multiply the first figure of the root, with a cipher annexed, by the second figure, and this product by 3; also take the square of the second figure, add these two results to the trial divisor, and the divisor becomes complete.*
5. *Multiply the complete divisor by the second figure of the root, and subtract the product from the dividend.*
6. *If there be a remainder and any more periods, to this remainder annex the next period, and proceed as before, by annexing a cipher to the root already found, squaring it, and multiplying by 3 to find a trial divisor; this, divided into the dividend, will give the next figure of the root.*
7. *Complete the divisor as before, multiply by the last figure*

of the root found, and subtract the product from the dividend. If there be no remainder, the number found is the cube root of the given number.

NOTE 1.—If after using up all the periods there be a remainder, the number is not a perfect cube, and its root may be further found by annexing ciphers and finding its cube root to one or more decimal places.

2. In extracting the cube root of decimals, point off from left to right, beginning at the decimal point.

3. Should the dividend at any time not contain the trial divisor, place the cipher in the root and annex another cipher to the previous root before squaring and multiplying by 3 to find a trial divisor.

Find the cube root of—

1. 15625.	<i>Ans.</i> 25.	9. 941192.	<i>Ans.</i> 98.
2. 74088.	<i>Ans.</i> 42.	10. 8615125.	<i>Ans.</i> 205.
3. 13824.	<i>Ans.</i> 24.	11. 347428927.	<i>Ans.</i> 703.
4. 300763.	<i>Ans.</i> 67.	12. $\frac{1728}{15625}$.	<i>Ans.</i> $\frac{12}{5}$.
5. 11390625.	<i>Ans.</i> 225.	13. 274.625.	<i>Ans.</i> 6.5.
6. 36926037.	<i>Ans.</i> 333.	14. 41.063625.	<i>Ans.</i> 3.45.
7. 150568768.	<i>Ans.</i> 532.	15. 9.	<i>Ans.</i> 2.08008+.
8. 432081216.	<i>Ans.</i> 756.	16. 12895213625.	<i>Ans.</i> 2345.

PROBLEMS.

1. What is the inside measure of a cubic box which shall hold just a bushel? *Ans.* 12.89+in.

2. What is the height of a cubical grain-bin that holds just 200 bu.? *Ans.* 75.49—in.

3. What is the difference between 6 cu. in. and 6 in. cube? *Ans.* 210 cu. in.

4. A cubic pile of wood contains 80 cd.: what are its dimensions? *Ans.* 21.71+ft. cube.

5. What is the size of a cubic cistern which shall hold 1500 gal. of water? *Ans.* 70.24—in. cube.

6. A man has a hall which he wishes to paint, including the floor; its measure is 15 ft. high, 36 ft. wide and 50 ft. long: how large must a cubic hall be to contain the same number of cubic feet of air; and which is the more expensive to paint, and how much at 5 cts. per square foot?

Ans. Size of hall, 30 ft. cube; 1st cost \$39 more than the 2d.

SECTION III.

SIMILAR SURFACES AND VOLUMES.

302. Similar Volumes are such as have the same shape, though they may differ in size.

According to the principles of geometry,

1. Similar surfaces are to each other as the squares of their like dimensions.

2. Similar volumes are to each other as the cubes of their like dimensions.

3. The dimensions of similar volumes are to each other as the cube roots of their contents or the square roots of their surfaces.

PROBLEMS.

1. If a half-inch pipe discharge a gallon of water in a certain time, how much will a 2-in. pipe discharge in the same time?

STATEMENT.— $(\frac{1}{2})^2 : (2)^2 :: 1 \text{ gal.} : (?)$.

2. If a building-lot 40 ft. square cost \$200, what will a lot 100 ft. square cost at the same rate? *Ans.* \$1250.

3. If the area of a circle whose diameter is 10 ft. is 78.54 sq. ft., what is the area of a circle whose diameter is 50 ft.?

Ans. 1963.50 sq. ft.

4. If a water-pipe 1 in. in diameter discharge 1 gal. in 40 sec., how much will a 2-ft. pipe discharge in a minute?

Ans. 864 gal.

5. A piece of ground 200 ft. square was sold for \$1920 : how much at the same rate is a piece 25 ft. square worth?

Ans. \$30.

6. If a 5-in. cube of marble weighs 12 lb., what will a cubic foot weigh?

Ans. 165 lb. $14\frac{2}{3}$ oz.

STATEMENT.— $5 \text{ in.}^3 : 12 \text{ lb.} :: 12 \text{ lb.} : (?)$.

7. If a ball of metal 2 in. in diameter weigh 5 lb., what will a ball of the same metal 6 in. in diameter weigh?

Ans. 135 lb.

8. If a man 6 ft. high weigh 180 lb., what will one 5 ft. high of the same proportions weigh? *Ans.* $104\frac{1}{2}$ lb.

9. If $\frac{1}{4}$ -inch round iron weigh 1 lb. to the foot, how much ought $2\frac{1}{4}$ -in. round iron to weigh? *Ans.* 25 lb. to the foot.

10. Which weighs the most—an apple 3 in. in diameter, or 3 apples 2 in. in diameter? *Ans.* 1 apple 3 in. in diameter.

CHAPTER XIII.

PROGRESSIONS.

303. A **Progression** is a series of numbers which increase or decrease according to a fixed law.

304. Progressions are of two kinds—**Arithmetical** and **Geometrical**.

305. The **Terms** of a progression are the numbers of which the series consists.

306. The first and the last term are called the *Extremes*, and all the others the *Means*.

SECTION I.

ARITHMETICAL PROGRESSION.

307. In an Arithmetical Progression the terms increase or decrease by a common difference. Thus, 1, 4, 7, 10, 13, 16, etc. is an increasing arithmetical progression whose common difference is 3; and 10, 8, 6, 4, 2 is a decreasing arithmetical progression whose common difference is 2.

308. The quantities considered are—

- | | |
|----------------------------------|-----------------------------------|
| 1. The first term (a). | 3. The common difference (d). |
| 2. The last term (l). | 4. The number of terms (n). |
| 5. The sum of the terms (S). | |

CASE I.

To find either Extreme when the other Extreme, the Common Difference and the Number of Terms are given.

WRITTEN PROBLEMS.

1. The first term of a series is 5, the common difference 3, and the number of terms 15: what is the last term?

SOLUTION.

$$\begin{aligned} 15 - 1 &= 14 \\ 5 + (14 \times 3) &= 47 \end{aligned}$$

Explanation.

$$\begin{aligned} \text{The 1st term} &= 5 \\ \text{The 2d} \quad &= 5 + 3 \\ \text{The 3d} \quad &= 5 + (2 \times 3) \\ \text{The 4th} \quad &= 5 + (3 \times 3) \\ \text{The 5th} \quad &= 5 + (4 \times 3) \end{aligned}$$

And in the same manner to the 15th term, which equals the 1st + (14×3) , or $5 + (14 \times 3)$.

2. The last term of an arithmetical series is 40, the common difference 4, and the number of terms 8: what is the first term?

$$\begin{aligned} 8 - 1 &= 7 & \text{Explanation.} & \text{—The 8th term} = a + 7d; \text{ hence, } a = 40 \\ 40 - (7 \times 4) &= 12 & -7d &= 40 - (7 \times 4) = 12. \end{aligned}$$

From the foregoing solutions we derive the following

RULES.

1. If the smaller extreme be given, add to it the product of the common difference and the number of terms, less one, to find the larger.
2. If the larger extreme be given, subtract from it the product of the common difference and the number of terms, less one, to find the smaller.

These rules may be expressed by the following formulas:

$$\begin{aligned} (1.) \quad l &= a \pm (n - 1) d. \\ (2.) \quad a &= l \mp (n - 1) d. \end{aligned}$$

3. The first term of an increasing series is 5, the common difference 4, and the number of terms 30: what is the last term?

Ans. 121.

4. The last term of an increasing series is 147, the common difference 3, and the number of terms 36: what is the first term? *Ans.* 42.

5. Find the 64th term of a descending series whose first term is 200, and the common difference 3. *Ans.* 11.

6. Find the 1st term of a descending series whose 100th term is $6\frac{1}{2}$, and the common difference $\frac{1}{2}$. *Ans.* 56.

7. How much can a man lift at the end of 300 d. if he can lift 200 lb. the first day, and gain $\frac{1}{2}$ a pound each day?

Ans. 349 $\frac{1}{2}$ lb.

8*. If a town is located 420 ft. above the level of the sea, and a town 118 mi. west of the former is reached by a railway whose average up-grade is 12 ft. to the mile, how high is the second town? *Ans.* 1836 ft.

CASE II.

From the preceding formulas and rules we derive the following

RULES.

1. *To find the common difference when the extremes and the number of terms are given, divide the difference between the extremes by the number of terms, less one.*

2. *To find the number of terms when the extremes and the common difference are given, divide the difference between the extremes by the common difference, and add 1 to the quotient.*

WRITTEN PROBLEMS.

1. The extremes of a series are 7 and 28, and the number of terms 8: what is the common difference? *Ans.* 3.

2. The extremes of a series are 1 and 30, and the number of terms 117: what is the common difference? *Ans.* $\frac{1}{2}$.

3. The amount of \$500 for 25 yr. at simple interest is \$1375: what is the rate? *Ans.* 7%.

4. If I start in business with \$5000, and at the end of 5 yr. have \$8000, what is my annual gain? *Ans.* \$600.

* The number of terms here is 119.

5. I have at the beginning of the year \$500, and work on a monthly salary; at the end of the year I have \$1400: what was my salary? *Ans.* \$75 monthly.

6. The extremes of a series are 3 and 45, and the common difference 3: what is the number of terms? *Ans.* 15.

7. The extremes of a series are 17 and 125, and the common difference 6: how many terms are there? *Ans.* 19.

8. If I am worth now \$6000, and loan my money at 6%, in how many years will I be worth \$13560? *Ans.* 21 yr.

9. If a young man clerk on a salary of \$300 the first year, and his salary be increased at the rate of \$75 a year, how long will it require to have his salary reach \$1500 a year? *Ans.* 16 yr.

The formulas for Case II. are—

$$(1.) d = \frac{l-a}{n-1}, \text{ or } d = \frac{a-l}{n-1}, \text{ if descending.}$$

$$(2.) n = \frac{l-a}{d} + 1, \text{ or } n = \frac{a-l}{d} + 1, \text{ “}$$

CASE III.

To find the Sum of all the Terms when the Extremes and the Number of Terms are given.

1. The extremes of a series are 3 and 59, and the number of terms 5: what is the sum?

FORMULA.

$$S. (\text{sum}) = (a+l) \frac{n}{2}, \text{ or } S. = \frac{3+59}{2} \times 5 = 155$$

NOTE.—The foregoing formula is derived as follows: The common difference in the series given is 14.

$$\begin{array}{rcl} S. & = & 3+17+31+45+59 = 155 \\ \text{or, } S. & = & 59+45+31+17+ 3 = 155 \\ 2 S. & = & 62+62+62+62+62 = 2 \times 155 \\ 2 S. & = & 5 \times 62, \text{ or } 5 \times 3+59 \\ S. & = & 5 \times \frac{3+59}{2} = 155 \end{array}$$

Since 3 and 59 are the extremes, we have the following

RULE.

Multiply the sum of the extremes by half the number of terms.

2. The extremes of an arithmetical series are 5 and 149, and the number of terms 25: what is the sum? *Ans.* 1925.

3. How many strokes does a clock strike in half a day?

Ans. 78.

4. How many strokes does a clock strike in a week of 7 d.?

Ans. 1092.

5. A man starts to walk, going 10 mi. the first day, 13 the second, 16 the third, and so on: how far does he walk in 10 d.?

Ans. 235 mi.

6. If a clerk earn \$600 the first year, and his salary is increased \$150 a year, how much does he earn in 10 yr.?

Ans. \$12750.

INTEREST ON DEFERRED PAYMENTS.

CASE I.

To find the Interest on Monthly Payments.

309. From the rule for the sum of any arithmetical progression we derive the following rule for calculating interest on payments due in equal instalments.

The same rule applies in calculating the value of *Building Association Shares*.

Ex. 1. A man rents an office at \$10 a month, payable monthly in advance: what is the interest due if his instalments remain unpaid for 12 mo.?

Ans. \$3.90.

OPERATION.

$$\frac{12 \times 13}{4} \times 10 = 390 = \$3.90$$

Explanation.—Regarding the 1st term of the series as 1, and the common difference as 1, the last, or 12th, term is 12. By the rule for the sum of the series, we have the sum as

follows: $S. = \frac{12+1}{2} \times 12$, or $\frac{13 \times 12}{2}$; but since the interest on \$1 for 1 mo. at 6% is $\frac{1}{2}$ ct., we take $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$ of the number of terms, as a multiplier

instead of $\frac{1}{4}$; hence, the sum of the interest on \$12, payable \$1 monthly in advance, is $\frac{12 \times 13}{4}$, and the interest on \$10 monthly for 12 months is 10 times as much, or $\frac{12 \times 13}{4} \times 10$, or 390, equivalent to \$3.90.

From this we have the formula $\frac{n \times (n+1)}{4}$, or the

RULE.

To find the interest at 6% on monthly instalments, multiply the number of months by the number plus 1, and divide by 4; multiply this result by the number of dollars payable monthly, and the answer will be the interest in cents.

2. A man rents a house at \$300 a year, payable monthly in advance; the rent remains unpaid for 3 years: what is the interest due, at 6%? *Ans. \$83.25.*

3. If I rent an office for \$240 a year, payable monthly in advance, and the tenant does not make payment till the end of 5 yr., what does he owe me, interest at 7%? *Ans. \$1413.50.*

4. The monthly instalment on a building association share is \$1: what is a share worth at 6% interest at the expiration of 4 years? *Ans. \$53.88.*

5. What is the value of a building association share, at 6% interest, at the expiration of 9 years? *Ans. \$137.43.*

6. What is the value of 15 building association shares, at 6% interest, at the expiration of 6 yr. 3 mo.? *Ans. \$1338.75.*

7. A building association pays 10% interest: what is the value of a share if the association has been in operation 6 yr.? *Ans. \$93.90.*

NOTE.—This case and the following may be employed also in finding the final value of annuities.

CASE II.

To find the Interest on Instalments when Payable otherwise than Monthly.

Ex. 1.—A man buys a lot for \$360, payable in 2 yr. in quarterly instalments, with interest at 6%: what amount is due at the end of the time if he fail to pay quarterly?

OPERATION.
 $\frac{8 \times 9 \times 3}{4} \times 45 = 2430$, or \$24.30.

Explanation.—The number of quarterly payments is 8. By the last case, multiplying 8 by 9, and dividing by 4, would give the interest at 6% if the instalments were \$1, payable monthly. Since there are 3 mo. in a quarter, however, the quarterly interest is 3 times as great; hence, $\frac{8 \times 9 \times 3}{4}$ is the interest in cents for 2 yr. on quarterly instalments of \$1, and on quarterly instalments of \$45 the interest in cents is $\frac{8 \times 9 \times 3}{4} \times 45 = 2430$.

From the foregoing we derive the following

RULE.

To find the interest on deferred payments other than monthly, multiply the number of payments by the number of payments plus 1, and this by the number of months in a payment. Divide this result by 4 to find the interest in cents on \$1 of the regular instalment.

To find the total interest, multiply the foregoing result by the number of dollars in a regular payment.

2. A house was rented for \$300 a year, payable in advance quarterly, but at the expiration of 3 years none of the rent was paid: how much was due, interest at 6%? *Ans.* \$987.75.

3. I buy a farm for \$12000, payable in 10 annual instalments. Suppose nothing is paid until the expiration of the 10 yr., how much is due, interest at 6%? *Ans.* \$15960.

4. What is the difference between a debt of \$2000 which should have been paid in 5 annual instalments, and one of the same amount which should have been paid in 20 quarterly instalments, both remaining unpaid at the expiration of 5 yr., and drawing interest at 6%?

Ans. Int. on annual, \$45 the greater.

5. A gentleman rented a room to a merchant at \$350 per annum, payable quarterly in advance, but failed to secure any settlement till the expiration of 4 yr.: what amount should be paid to him at that time, interest at 6% per annum?

Ans. \$1578.50.

SECTION II.

GEOMETRICAL PROGRESSION.

310. A Geometrical Progression is a series of numbers varying by a fixed multiplier or factor. Thus, 3, 9, 27, or 27, 9, 3, is a geometrical series. The former, increasing from left to right, is called an *ascending series*, and the latter a *descending series*.

311. The *rate* or *ratio* of the series is the common multiplier.

312. An Infinite Series is a decreasing series, in which the number of terms is unlimited.

313. The following are the quantities considered in a geometrical progression :

- | | |
|--------------------------------------|---------------------------------|
| 1. The first term (a). | 3. The ratio (r). |
| 2. The last term (l). | 4. The number of terms (n). |
| 5. The sum of all the terms (S). | |

CASE I.

To find either the First or the Last Term when the other Extreme, the Ratio and the Number of Terms are given.

Ex. 1. The first term of a geometrical series is 2, the ratio 3, and the number of terms 8 : what is the last term ?

SOLUTION. *Explanation.*—The 2d term equals $1st \times 3$; the 3d term $3^1 = 2187$ equals the $1st \times 3 \times 3$, or $1st \times 3^2$; the 4th term equals the $2 = a$ $1st \times 3 \times 3 \times 3$, or $1st \times 3^3$; and so on. Hence, the 8th term equals $1st \times 3^7$, or $2 \times 3^7 = 4374$.

2. The last term of a geometrical series is 4374, the ratio 3, and the number of terms 8 : what is the first term ?

SOLUTION. *Explanation.*—The first term is a number which $\frac{4374}{3^7} = \frac{4374}{2187} = 2 = a$ if multiplied by 3^7 will produce the 8th term ; hence, if we divide the 8th term by 3^7 it will produce the 1st term. $4374 \div 3^7$, or $4374 \div 2187 = 2$, the 1st term.

From the foregoing we derive the following

RULES.

1. *If the given extreme is the first term, multiply it by the ratio raised to a power one less than is indicated by the number of terms.*

2. *If the given extreme is the last term, divide it by the ratio raised to a power one less than is indicated by the number of terms.*

From the foregoing rules we have the following formulas:

$$l = ar^{n-1},$$

$$a = \frac{l}{r^{n-1}}.$$

3. The first term of a geometrical series is 5, the ratio 2, and the number of terms 6: what is the last term? *Ans.* 160.

4. The last term of a series is 2187, the ratio 3, and the number of terms 5: what is the 1st term? *Ans.* 27.

5. The last term of a series is 384, the number of terms 7, and the ratio 2: what is the 1st term? *Ans.* 6.

6. What is the 6th term of the series 2187, 729, 243, etc.
Ans. 9.

7*. What is the compound amount of \$50 for 5 yr., at 6%?
Ans. \$66.91+.

8. If a farmer plant a quart of corn, and it produce 9 bu., how much corn will he have at the expiration of 4 yr. if he plant the entire harvest each succeeding spring?
Ans. 214990848 bu.

CASE II.

To find the Sum of a Geometrical Series when the Extremes and the Ratio are given.

Ex. 1. The first term of a series is 6, the last term 162, and the ratio 3: what is the sum of the series? *Ans.* 240.

$$\text{SOLUTION. } \frac{(3 \times 162) - 6}{3 - 1} = \frac{486 - 6}{2} = 240.$$

* Rate = 1.06.

NOTE.—The foregoing statement is derived as follows:

$$\text{Sum (S.)} = 6 + 18 + 54 + 162$$

$$3 \text{ S.} = 18 + 54 + 162 + 486$$

$$3 \text{ S.} - \text{S.} = 2 \text{ S.} = 486 - 6$$

$$\text{S.} = \frac{486 - 6}{2}, \text{ or } \frac{r \times l - a}{r - 1}.$$

From the above we derive the rule for finding the sum of a geometrical series, as follows:

RULE.

Multiply the last term by the ratio, and from this take the first term, and divide the remainder by the ratio, less one.

The following is the formula: $\text{S.} = \frac{rl - a}{r - 1}$.

2. A clerk whose first year's salary is \$75 has his salary doubled every year: how much does he earn in 5 years?

Ans. \$2325.

3. A merchant engaging in business makes \$500 the first year, and continues in business 6 yr., making each year 3 times as much as the preceding year: how much does he make?

Ans. \$182000.

4. What is the compound amount of \$50 for 8 yr. at 6%? *

Ans. \$79.6924.

CASE III.

INFINITE SERIES.

In a decreasing infinite series the last term becomes so small that it may be considered zero; and the formula $\frac{r \times l - a}{r - 1}$

becomes $\frac{r \times 0 - a}{r - 1}$, or $\frac{-a}{r - 1}$, which is the same as $\frac{a}{1 - r}$; hence we have the following

RULE.

The sum of an infinite series equals the 1st term, divided by 1 minus the rate.

1. What is the sum of the series $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, etc.?

$$\text{S.} = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = \frac{2}{1} = 2.$$

$$* r = 1.06.$$

2. What is the sum of the series $3, \frac{3}{2}, \frac{9}{4}, \text{etc.}$? *Ans.* $3\frac{1}{2}$.
 3. What is the sum of the series $2, \frac{1}{2}, \frac{1}{4}, \text{etc.}$? *Ans.* $2\frac{2}{3}$.
 4. A dog and a rabbit are 100 yd. apart; if the rabbit run 60 yd. while the dog runs 100, how far will the dog run to catch the rabbit? *Ans.* 250 yd.
-

CHAPTER XIV.

ANNUITIES.

314. An **Annuity** is a sum of money payable at certain periods, usually annually.

315. A **Certain Annuity** is one which is payable for a certain or definite time.

316. A **Perpetual Annuity**, or **Perpetuity**, continues for ever.

317. A **Contingent Annuity** is one payable for an uncertain period, usually for a lifetime. It is dependent on some contingent event, such as a death.

318. An **Immediate Annuity** begins at once; a **Deferred Annuity** begins at some future time.

319. The **Final Value** of an annuity is the sum of all the payments, with interest on each from the time it becomes due.

320. The **Present Value** of an annuity is such a sum of money as will, if put on interest for the given time and at the given rate per cent., amount to the final value; or, in other words, it is the present worth of the final value.

SECTION I.

ANNUITIES AT SIMPLE INTEREST.

All annuities at simple interest may be solved by applying the rules and principles pertaining to *Arithmetical Progression* and *Interest*.

CASE I.

To find the Final Value of an Annuity at Simple Interest.

Ex. 1. What is the final value of an annuity of \$500 for 10 yr., at 6% ? Ans. \$6350.

SOLUTION. *Explanation.*—The 10th payment = \$500; the 9th, \$500, with interest for 1 yr., at 6%, or \$530; the 8th, \$500, with interest for 2 yr., at 6%, or \$560; and so on to the first, which is (\$770 + \$500) $\frac{1}{2}$ = \$6350 \$500, with interest for 9 yr., at 6%, or \$770; the whole forming an arithmetical series of 10 terms, with \$500 for the first term, and \$30, the interest for 1 yr., at 6%, for the common difference. By the rule for the sum of an arithmetical series, we have the sum = the first term (\$500) + the last term (\$770), multiplied by 5, half the number of terms, equal to \$6350, the final value of the annuity.

The final value may also be found by the rule for deferred payments.

Explanation.—The number of interest-bearing payments or terms is 9; hence the total interest is \$1350; and 10 payments of \$500 each are \$5000; both payment and interest, therefore, equal \$1350 + \$5000, or \$6350.

$$\frac{9 \times 10 \times 12}{4} \times 500 = \$1350$$

$$10 \times 500 = \frac{5000}{\$6350}$$

2. What is the final value of an annuity of \$400 for 20 yr., at 5% ? Ans. \$11800.

3. If a man buy a farm on which he is to pay \$300 annually for 15 yr., what is due at the expiration of the time if he fail to make the annual payments, interest at 6% ?

Ans. \$6390.

CASE II.

To find the Present Value of an Annuity at Simple Interest.

NOTE.—Since the present worth of an annuity is the present worth of the final value, it is only necessary to find the final value, as in the preceding case, and then find the present worth of this amount.

1. What is the present worth of an annuity of \$1200 for 15 yr., at 6% ?

$$\frac{14 \times 15 \times 12}{4} \times 1200 = \$7560 = \text{int.}$$

$$15 \times 1200 = \frac{18000}{\$25560} = \text{annuity.}$$

$$\text{final value.}$$

\$25560 + 1.90 = 13452.63 = present worth of \$25560 due 15 yr. hence, at 6%.

2. What is the present value of an annuity of \$500 for 10 yr., at 6%? *Ans.* \$3968.75.

3. An author has an annual income of \$1200 as copyright. Suppose his copyright is still good for 25 yr., what is the present value, interest at 6%? *Ans.* \$20640.

4. A widow has a dower of \$300 to continue 16 yr., at 6%: what is the present value? *Ans.* \$3551.02.

5. I buy a house for \$18000, to be paid in instalments of \$1500 annually, without interest: what cash payment will cancel the debt, money being worth 6% interest? *Ans.* \$13918.60.

6. I rent a house for \$300 a year, the rent to be paid monthly in advance: what amount of cash at the beginning of the year will pay one year's rent? *Ans.* \$292.22.

SECTION II.

ANNUITIES AT COMPOUND INTEREST.

All problems in annuities at compound interest may be solved by applying the rules and principles pertaining to *Geometrical Progression* and *Compound Interest*.

CASE I.

To find the Amount of an Annuity at Compound Interest.

1. What is the amount of an annuity of \$200 for 5 yr., at 5%, compound interest? *Ans.* \$1105.128.

SOLUTION.

$$\begin{aligned}
 & \$1 \times (1.05)^4 = \$1.215506 \\
 & \underline{\$1.215506 \times 1.05 - \$1} \quad \underline{\$1.276282 - \$1} \\
 & \quad .05 \qquad \qquad .05 \\
 & \quad \underline{\$.276282} = \$5.52564 \\
 & \quad \quad .05 \\
 & 200 \times \$5.52564 = \$1105.128
 \end{aligned}$$

Explanation.—Taking 1 as the first term of the geometrical series, and 1.05, the amount of \$1 for 1 yr., as the rate, the 5th term is 1×1.05^4 ; and since the sum of the series $= \frac{l \times r - a}{r - 1}$, we have $\frac{1 \times 1.05 - \$1}{.05} = \$5.52564$, the final value of \$1 for the given time and rate per cent., and for \$200, $200 \times \$5.52564$, or \$1105.128.

NOTE.—The rate to any power may be found by consulting the Compound Interest Table, page 235, in which the required power of the rate is given as the amount of \$1 for the given time at the given rate per cent.

The following is the rule for finding the amount of an annuity at compound interest:

RULE.

Find the compound interest of \$1 for the given rate and time; divide this by the rate, and multiply this result by the annuity.

2. What is the final value of an annuity of \$300 for 6 yr., at 4%? *Ans.* \$1989.75.

3. If I deposit \$600 a year in a bank for 10 yr. at 5%, compound interest, what is the value at the expiration of the period? *Ans.* \$7546.80.

4. If it cost me \$123.90 yearly to have my life insured, what ought my policy to be worth at the end of 10 yr., money being worth 7%? *Ans.* \$1711.94.

5. A gentleman deposited \$50 every six months in a savings bank, the amount to draw interest at 6% semi-annually: what is it worth at the end of 10 yr.? *Ans.* \$1343.50.

CASE II.

To find the Present Value of an Annuity at Compound Interest, first find the Final Value as above, and Divide by the Amount of \$1 for the given Rate and Time, at Compound Interest.

1. What is the present value of an annuity of \$200 at compound interest for 7 yr., at 6%? *Ans.* \$1116.43.

2. What amount of cash deposited in a savings bank at 5% compound interest would amount to as much as \$500 annually for 15 yr.? *Ans.* \$5189.76.

3. What amount of cash deposited in a savings bank at 4% compound interest would amount to as much as \$100 deposited annually for 12 yr.? *Ans.* \$938.48.

4. What is the present value of a semi-annual deposit of \$600 at compound interest for 7 yr. at 8%, payable semi-annually? *Ans.* \$6337.99.

Ex. Reduce the decimal $.754\dot{3}$ to a common fraction.

OPERATION.

$$10000 \text{ R.} = 7543.\dot{4}\dot{3}, \text{ etc.}$$

$$\underline{100 \text{ R.} = 75.4\dot{3}, \text{ etc.}}$$

$$9900 \text{ R.} = 7468$$

$$\text{R.} = \frac{7468}{99} = \frac{1867}{24\frac{1}{2}}.$$

Reduce the following circulating decimals to common fractions:

1. $\dot{3}$.	Ans. $\frac{1}{3}$.	11. $.25\dot{6}\dot{4}$.	Ans. $\frac{2564}{99}$.
2. $\dot{6}$.	Ans. $\frac{2}{3}$.	12. $.351\dot{3}\dot{5}$.	Ans. $\frac{13}{3}$.
3. $\dot{2}\dot{1}$.	Ans. $\frac{7}{9}$.	13. $.2\dot{2}9\dot{7}$.	Ans. $\frac{71}{4}$.
4. $.75\dot{3}$.	Ans. $\frac{253}{33}$.	14. $.4\dot{3}0\dot{6}$.	Ans. $\frac{717}{1885}$.
5. $\dot{5}3\dot{1}$.	Ans. $\frac{531}{111}$.	15. $.842\dot{3}\dot{4}$.	Ans. $\frac{127}{22}$.
6. $.823\dot{2}$.	Ans. $\frac{2744}{3333}$.	16. $\dot{5}9\dot{4}$.	Ans. $\frac{27}{4}$.
7. $\dot{8}5\dot{2}$.	Ans. $\frac{284}{333}$.	17. $.072\dot{0}\dot{9}$.	Ans. $\frac{801}{49950}$.
8. $\dot{5}2\dot{7}$.	Ans. $\frac{527}{333}$.	18. $\dot{0}000\dot{3}$.	Ans. $\frac{1}{33333}$.
9. $\dot{0}4\dot{8}$.	Ans. $\frac{16}{333}$.	19. $9.32\dot{0}\dot{9}$.	Ans. $9\frac{853}{1100}$.
10. $\dot{0}00\dot{3}$.	Ans. $\frac{1}{3333}$.	20. $18.637\dot{5}$.	Ans. $18\frac{25}{88}$.

CHAPTER XVI.

DUODECIMALS.

327. Duodecimals are fractions of which 12 of any order equal 1 of the next higher order.

328. The unit is the *foot*, which is divided into 12 equal parts called *primes* ($'$), each prime ($'$) being divided into 12 equal parts called *seconds* ($''$), each second ($''$) in the same manner into 12 thirds ($'''$), and each $'''$ into twelve fourths ($''''$).

TABLE.

12 fourths ($''''$)	= 1 third, $'''$.
12 thirds ($'''$)	= 1 second, $''$.
12 seconds ($''$)	= 1 prime, $'$.
12 primes ($'$)	= 1 foot, ft.

1. ADDITION AND SUBTRACTION OF DUODECIMALS.

Problems in Addition and Subtraction of Duodecimals are solved like those in Denominate Numbers.

1. Add 6 ft. 4' 2"; 15 ft. 4" 3''' ; 8' 9" 4''' 3''', and 7'''.

PROCESS.

$$\begin{array}{r}
 6 \text{ ft. } 4' 2'' \\
 15 \text{ ft. } 0' 4'' 3''' \quad 3'''' \\
 \quad 8' 9'' 4''' \quad 7'''' \\
 \hline
 22 \text{ ft. } 1' 3'' 7''' 10''''
 \end{array}$$

2. From 9 ft. 1' 3" take 2 ft. 6' 1" 3'''.

PROCESS.

$$\begin{array}{r}
 9 \text{ ft. } 1' 3'' \\
 2 \text{ ft. } 6' 1'' 3''' \\
 \hline
 6 \text{ ft. } 7' 1'' 9'''
 \end{array}$$

3. Add 7 ft. 1' 3" 6''' ; 1 ft. 3" 6''' 1'''' ; 7 ft. 8' 7" 9''' ; 8 ft. 10' 6''.

Ans. 24 ft. 8' 8" 10'''.

4. From 18 ft. 3" 9''' take 10 ft. 2' 2" 6'''.

Ans. 7 ft. 10' 6" 9'''.

2. MULTIPLICATION OF DUODECIMALS.

1. What is the product of 9 ft. 6' 5" and 6 ft. 4' ?

SOLUTION.

$$\begin{array}{r}
 9 \text{ ft. } 6' 5'' \\
 6 \text{ ft. } 4' \\
 3 \text{ ft. } 2' 1'' 8''' \\
 57 \text{ ft. } 2' 6'' \\
 \hline
 60 \text{ ft. } 4' 7'' 8'''
 \end{array}$$

Explanation.—5'', or $1\frac{1}{2}$ ", multiplied by 4', or $1\frac{1}{2}$ ', equals $1\frac{1}{2}$ " $\times$ $1\frac{1}{2}$ " = 20''' = 1'' 8''' ; in the same manner, 6' multiplied by 4' = 24'', which added to 1'' is 25'', or 2' 1'' ; 9 ft. multiplied by 4' = 36', which added to 2' is 38', or 3 ft. 2'. Multiply by 6 ft. in the same manner, and we have for the second product 57 ft. 2' 8'', which being added to the first product, the result is 60 ft. 4' 7'' 8'''.

2. What is the entire surface of a floor 15 ft. 8' long and 16 ft. 3' wide ?

Ans. 254 sq. ft. 7'.

3. What is the amount of surface to be plastered in a room 18 ft. 3' long, 15 ft. 2' wide, and 10 ft. 3' high, allowing 8' for the width of the base-board ?

Ans. 917 sq. ft. 3' 4''.

4. What are the solid contents of a block of stone 3 ft. 2' long, 2 ft. 3' 6" wide, and 4 ft. 2' high?

Ans. 30 cu. ft. 2' 10" 2".

5. What is the surface measure of the foregoing block?

Ans. 60 sq ft.

3. DIVISION OF DUODECIMALS.

1. Divide 22 ft. 3' 9" by 5 ft. 3'.

SOLUTION. $\begin{array}{r} 5 \text{ ft. } 3' \overline{) 22 \text{ ft. } 3' 9''} \\ \underline{21 \text{ ft. } 0'} \\ 1 \text{ ft. } 3' 9'' \\ \underline{1 \text{ ft. } 3' 9''} \\ 0 \end{array}$	Explanation.—5 ft. are contained in 22 ft. 4 times. Multiplying the entire divisor by 4, and subtracting, we have 1 ft. 3' 9" remaining. Reducing 1 ft. to primes, we have 1 ft. 3' 9" = 15' 9"; the divisor is contained in this number 3' times; multiplying as before and subtracting, we have no remainder. Hence, 4 ft. 3' is the quotient.
--	---

2. If a floor contain 216 sq. ft. 5' 10" 6", and is 10 ft. 6' wide, how long is it?

Ans. 20 ft. 7' 5".

3. A block of marble contains 112 cu. ft. 5' 4", and is 5 ft. 4' wide and 3 ft. 10' thick: how long is it?

Ans. 5 ft. 6'.

4. A board contains 26 sq. ft. 6", and is 12 ft. 6' long: how wide is it?

Ans. 2 ft. 1'.

5. A plank is 5' thick, 20 ft. 2' long, and contains 14 cu. ft. 8": how wide is it?

Ans. 1 ft. 8'.

CHAPTER XVII.

MEASUREMENTS.

329. The process of finding the measure of *lines, surfaces* and *volumes* is called **Mensuration**. The rules of Mensuration depend on the principles of Geometry. Many of them are of much importance, and are therefore given here without an attempt to explain them by means of geometrical demonstration.

DEFINITIONS.—1. LINES.

330. A **Line** is that which has length, but no breadth or thickness.

331. A Straight Line is one whose direction does not change.



332. A Curved Line is one whose direction changes at every point.



333. Parallel Lines are those which extend in the same direction, and are equally distant from each other at all points.



334. A Horizontal Line is a line parallel with the horizon or water-level.

2. ANGLES.

335. An Angle is the difference in the direction of two lines meeting at a common point, called the *vertex*.



336. A Right Angle is one in which the two lines are perpendicular to each other.



337. An Obtuse Angle is one greater than a right angle.



338. An Acute Angle is one less than a right angle.



339. A Polygon is a figure bounded by straight lines. The distance around a polygon is called its *perimeter*.

340. A polygon of three sides is called a *triangle*; of four sides, a *quadrilateral*; of five, a *pentagon*, etc.

341. The *Area* of a polygon is the surface included within the lines which bound it.

3. TRIANGLES.

342. A Triangle is a polygon of three sides and three angles; as, A B C. The side A B in the figure is called the *base*; B C, the *altitude*; and A C the *hypotenuse*.



343. A Right-angled Triangle is one which has one right angle.



344. An Equilateral Triangle is one whose sides are equal.



345. An Isosceles Triangle has two sides equal.



346. A Scalene Triangle has all its sides unequal.



The following are the rules for the measurements of triangles :

It was proved in Denominate Numbers (Art. 107) that the area of a parallelogram is equal to the product of its base and altitude, and since a triangle is half a parallelogram, we derive the following

RULE.

1. *To find the area of a triangle when the base and the altitude are given, take half the product of the base and altitude.*

Ex. 1. What is the area of a triangle whose base is 16 in. and altitude 25 in.?

$$\text{SOLUTION. } \frac{16 \text{ in.} \times 25 \text{ in.}}{2} = 200 \text{ sq. in.}$$

2. What is the surface of the gable end of a house 30 ft. wide and 12 ft. 6 in. high? *Ans.* $187\frac{1}{2}$ sq. ft.

3. How many acres in a triangular field whose base is 920 ft. and perpendicular 816 ft. *Ans.* 8 A. 98 P. $199\frac{1}{2}$ sq. ft.

4. What will it cost to paint the two gables of a house 25 ft. wide and 8 ft. 3 in. high, at 6 cts. a square foot. *Ans.* \$12.37 $\frac{1}{2}$.

2. *When the three sides of a triangle are given, to find the area, from half the sum of the sides subtract each side separately; multiply the three remainders together, and this product by the half sum of the sides; the square root of the product is the area.*

1. What is the area of a triangle whose sides are 20, 40 and 36 ft. respectively? *Ans.* 359.2 - sq. ft.

PROCESS.

$$\begin{aligned} \frac{20+40+36}{2} &= 48 = \text{half-sum.} \\ 48-20 &= 28 \\ 48-40 &= 8 \\ 48-36 &= 12 \\ 28 \times 8 \times 12 \times 48 &= 129024 \\ \sqrt{129024} &= 359.2 \end{aligned}$$

2. What is the area of an equilateral triangle whose sides are each 60 ft.? *Ans.* 1559 - sq. ft.

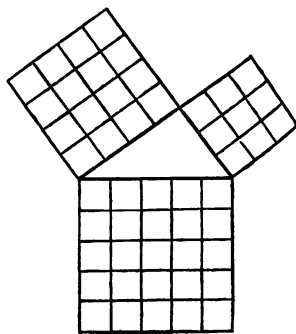
3. A man owns a triangular field at the junction of two roads; its sides are 40 rd., 60 rd. and 50 rd. respectively: how many acres does it contain? *Ans.* 6 A. 32.16 P.

The following principles have been established by Geometry:

1. The square described on the hypotenuse of a right-angled triangle is equal to the sum of the squares described on the two other sides.

2. The square of the base, or of the perpendicular, of a right-angled triangle is equal to the square of the hypotenuse, less the square of the other side.

Hence, the following rule to find the hypotenuse:



RULE.

To the square of the base add the square of the perpendicular, and extract the square root of this sum.

Ex. 1. What is the hypotenuse of a right-angled triangle whose base is 20 and the perpendicular 25? *Ans.* 32.015 +.

SOLUTION.

$$\begin{aligned} 20^2 + 25^2 &= 400 + 625 = 1025 \\ \sqrt{1025} &= 32.015 + \end{aligned}$$

2. If the two sides of a rectangular field are 500 ft. and 700 ft. respectively, how far from one corner to the opposite one?

Ans. 860+ft.

3. If the sides of a house are 40 ft. and 30 ft. respectively, what must be the length from one corner to the opposite that the corners may be perfect right angles?

Ans. 50 ft.

4. The height of a gable is 8 ft., and the width of the house 30 ft.: how long must the rafters be that they may project over the edge 2 ft.?

Ans. 19 ft.

5. Two vessels start at the same port, one sailing south at the rate of 12 mi. an hour, the other east at the rate of 10 mi. an hour: how far apart will they be at the end of 72 hr.?

Ans. 1124.67+mi.

From the *second principle* the following rule, to find the base or the perpendicular when the hypotenuse and one of the other sides are given, is derived:

RULE.

From the square of the hypotenuse subtract the square of the given side; the square root of the remainder is the other side.

Ex. 1. A ladder 40 ft. long, whose foot is 10 ft. from a building, reaches to a window: how high is the window from the ground?

Ans. 38.73 - ft.

$$40^2 - 10^2 = 1600 - 100 = 1500$$

$$\sqrt{1500} = 38.73 - \text{ft.}$$

2. A man standing 200 ft. from the foot of a tree shoots a bird from the top; the bullet travels 250 ft.: how high is the tree if the man's eye was 5 ft. higher than the foot of the tree?

Ans. 155 ft.

3. A boat in crossing a stream 620 ft. wide goes 800 ft.: how far does it drift down stream?

Ans. 505.57 ft.

4. The rafters on a house are 20 ft. long, the width of the gable is 30 ft., the rafters project 2 ft.: what is the height of the gable?

Ans. 9.95 ft.

5. A room is 18 ft. long, 15 ft. wide and 10 ft. high: what is the distance from an upper corner to the opposite lower one?

Ans. 25.475 ft.

4. QUADRILATERALS.

347. A Quadrilateral is a plane figure bounded by four straight lines.

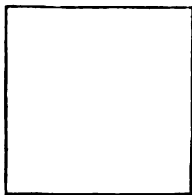
348. A Parallelogram is a quadrilateral whose opposite sides are equal and parallel.

349. A Rectangle is a parallelogram whose angles are right angles.

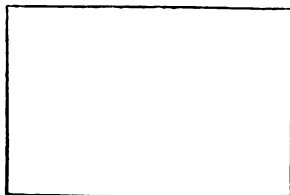
350. A Square is a rectangle whose sides are equal.

351. A Rhombus is a parallelogram whose sides are all equal, but its angles are not right angles.

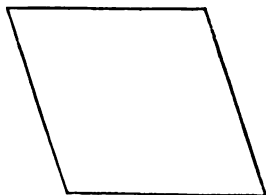
352. A Rhomboid is a parallelogram whose opposite sides only are equal, but whose angles are not right angles.



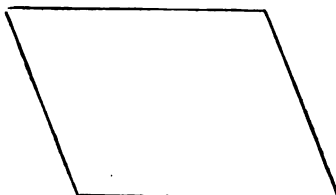
SQUARE.



RECTANGLE.

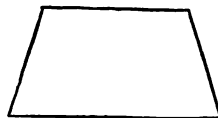


RHOMBUS.



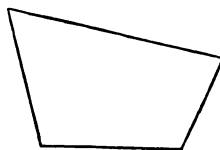
RHOMBOID.

353. A Trapezoid is a quadrilateral only two of whose sides are parallel.



354. A Trapezium is a quadrilateral none of whose sides are parallel.

355. The altitude of a parallelogram, or of a trapezoid, is the perpendicular distance between the parallel sides.



The following principles have been established by Geometry:

1. The area of any parallelogram is equal to the product of its base and altitude.

2. The area of a trapezoid is equal to one-half the sum of its parallel sides, multiplied by its altitude.

1. What is the area of a room 15 ft. long, 9 ft. 6 in. wide?

Ans. $142\frac{1}{2}$ sq. ft.

2. A board 16 ft. long is 12 in. wide at one end, and 10 in. wide at the other: how many square feet does it contain?

Ans. $14\frac{2}{3}$ sq. ft.

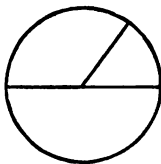
3. A field 900 ft. long is 720 ft. wide at one end, and 600 ft. at the other: how many acres does it contain? *Ans.* $13\frac{1}{11}$ A.

NOTE.—A *trapezium* may be divided by a diagonal into two triangles, and the area of these be found by the rule for finding the area of triangles.

4. The sides of a field are 30, 40, 60 and 70 rods, respectively, and the diagonal joining the two opposite angles 50 rods: what is the area of the field? *Ans.* 12 A. 149.7 P.

5. CIRCLES.

356. A **Circle** is a plane figure bounded by a curved line called the *circumference*, every point of which is equally distant from a point within, called the *centre*.



357. The *diameter* of a circle is a straight line passing through the centre and ending on both sides in the circumference.

358. The *radius* is a line drawn from the centre to the circumference.

The following are the most important rules relating to the circle:

RULES.

1. To find the circumference of a circle, multiply the diameter by 3.1416.

2. To find the diameter of a circle, divide the circumference by 3.1416, or multiply the circumference by .3183.

PROBLEMS.

1. What is the circumference of a circular plot of ground 25 ft. in diameter? *Ans.* 78.54 ft.

2. The circumference of a tree is 12.5664 ft.: what is the diameter? *Ans.* 4 ft.

3. A circular race-course is 1 mi. in extent: what is the diameter of the enclosure? *Ans.* 1680.67 ft.

3. *The area of a circle = the circumference $\times \frac{1}{2}$ diameter; or, radius squared $\times 3.1416$; or, diameter squared $\times .785398$.*

4. What is the area of a circle whose circumference is 1 mi.? *Ans.* 50 A. 148 P. 194.4—sq. ft.

5. What is the area of a circle $\frac{1}{4}$ mi. in circumference? *Ans.* 12 A. 117 P. 48.6—sq. ft.

6. What is the area of a circle whose radius is 100 ft.? *Ans.* 31416 sq. ft.

4. *To find the side of an inscribed square when the diameter of a circle is given—1. Multiply the diameter by .7071; or, 2. Find the square root of half the square of the diameter.*

7. How large a square piece of lumber can be cut from a log 3 ft. in diameter? *Ans.* 25.4556 in.

8. How large a square piece of lumber can be cut from an 18-in. log? *Ans.* 12.7278 in.

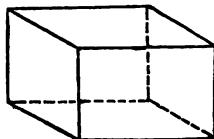
6. VOLUMES.

359. A Solid or Volume is a body which has three dimensions—length, breadth and thickness.

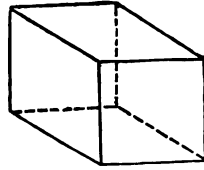
360. A Prism is a volume whose ends are equal and parallel polygons, and its sides are parallelograms. These sides are called *faces*, and the ends *bases*.

361. A prism is triangular, rectangular, square, etc., according to the name of its bases.

362. A prism bounded by six parallelograms, the opposite ones being equal and parallel, is called a **Parallelepipedon**.



363. A parallelopipedon whose faces are all squares is called a **Cube**.



364. A **Cylinder** is a prism whose ends are circles, and whose body is bounded by a uniformly curved surface.



RULES.

1. To find the convex surface of a prism or a cylinder, multiply the perimeter of its base by the altitude.
2. To find the entire surface of a prism or a cylinder, to its convex surface add the area of its bases.

PROBLEMS.

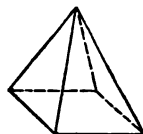
1. What is the convex surface of a log 30 in. in diameter and 10 ft. long? *Ans.* 78.54 sq. ft.
2. What is the entire surface of a stick of timber 20 ft. long, 2 ft. wide and $1\frac{1}{2}$ ft. thick? *Ans.* 146 sq. ft.
3. What is the convex surface of a triangular prism whose altitude is 12 ft. and its sides 12, 15 and 18 in. respectively? *Ans.* 45 sq. ft.
4. What is the entire surface of a cylinder 30 ft. long and 40 in. in diameter? *Ans.* 331.6133 sq. ft.
5. What will it cost to paint a rectangular prism 6 ft. high whose sides are 30 in. each, one base and the sides only to be painted, at 5 cts. a square foot? *Ans.* \$3.31 $\frac{1}{2}$.
3. To find the contents of a prism or cylinder, multiply the area of the base by the altitude.
6. What are the contents of a stick of timber 60 ft. long, 18 in. wide and 15 in. thick? *Ans.* $112\frac{1}{2}$ cu. ft.
7. What are the contents of a cylindrical column $2\frac{1}{2}$ ft. in diameter and 30 ft. high? *Ans.* $147.2621 +$ cu. ft.

8. What is the value of a log 50 ft. long whose thickness is 24 in., at 20 cts. a cubic foot? *Ans.* \$31.416.

9. What are the contents of a cylindrical water-tank whose height is 20 ft. and diameter 30 ft.? *Ans.* 105753.6 gal.

7. PYRAMIDS AND CONES.

365. A **Pyramid** is a volume whose base is a polygon, and whose sides are triangles terminating in a common point called the vertex.



366. A **Cone** is a volume having a circular base, and whose surface slopes uniformly to the vertex.



367. The *altitude* of a pyramid or a cone is the perpendicular distance from the vertex to the base.

368. The *slant height* of a pyramid or a cone is the perpendicular distance from the vertex to the perimeter of the base.

NOTE.—In a right pyramid the slant height is the altitude of each of the equal triangles which compose the convex surface.

369. The **Frustum** of a cone or a pyramid is the part which remains after cutting off the top by a plane parallel with the base.



The following are the principles applying to pyramids and cones:

PRINCIPLES.—1. The convex surface of a cone or a pyramid equals the perimeter multiplied by one-half the slant height.

2. The volume of a cone or a pyramid equals the area of the base multiplied by one-third the altitude.

3. The convex surface of the frustum of a cone or a pyramid equals the sum of its perimeters multiplied by one-half the slant height.

4. The volume of the frustum of a cone or a pyramid equals the sum of the areas of the two bases, plus the square root of

the product of the two areas, all multiplied by one-third of the altitude.

NOTES.—1. To find the entire surface of a cone or a pyramid, add to the convex surface the area of the base.

2. To find the entire surface of the frustum of a pyramid or a cone, add to the convex surface the areas of the two bases.

PROBLEMS.

1. What is the convex surface of a cone whose slant height is 60 ft. and the circumference of its base 40 ft.?

Ans. 1200 sq. ft.

2. What will it cost to paint an octagonal church-spire whose slant height is 80 ft. and the sides of whose base are each 8 ft., at $5\frac{1}{2}$ cts. a square foot?

Ans. \$140.80.

3. What is the entire surface of a square pyramid whose slant height is 8 ft. and the sides of whose base are each 5 ft.?

Ans. 105 sq. ft.

4. What was the volume of the pyramid of Ghizeh, which was 480 ft. high, with a square base 764 ft. on each side?

Ans. 93,391,360 cu. ft.

5. What is the value of a block of marble, in the shape of a square pyramid, whose height is 20 ft., and the sides of whose base are each 20 in., at \$2.25 per cubic foot?

Ans. \$41.67.

6. What is the convex surface of the frustum of a cone whose slant height is 6 ft., the diameter of its lower base 5 ft., and of the upper 4 ft.?

Ans. 84.8232 sq. ft.

7. What is the convex surface of the frustum of a square pyramid whose slant height is 10 ft., the side of the upper base being 20 in. and the lower 30 in.?

Ans. $83\frac{1}{2}$ sq. ft.

8. What are the contents of a square pyramid of marble 20 ft. high, the side of whose upper base is 8 in. and of the lower base 20 in.?

Ans. $28\frac{2}{3}$ cu. ft.

9. What are the contents of a water-tank, in the shape of the frustum of a cone, whose depth is 15 ft., the diameter of the lower base being 16 ft. and of the upper 12 ft.?

Ans. 2324.784 cu. ft.

10. What amount of timber in a log 40 ft. long, 3 ft. thick at one end and 2 ft. at the other? *Ans.* 198.968 cu. ft.

11. What amount of lumber in 20 posts sawed 6 in. square at one end and 4 by 4 at the other, the posts being 12 ft. long? *Ans.* 42 $\frac{2}{3}$ cu. ft.

8. SPHERES.

370. A **Sphere** is a volume bounded by a uniformly-curved surface, every point of which is equally distant from a point within called the centre.



371. The *diameter* of a sphere is a straight line passing through the centre and ending in the surface.

372. The *radius* of a sphere is a straight line drawn from the centre to any point in the surface.

The following are the principles applying to the sphere:

PRINCIPLES.—1. The surface of a sphere equals the diameter multiplied by the circumference of a great circle of a sphere.

2. The surface of a sphere equals the square of the diameter multiplied by 3.1416.

3. The volume of a sphere equals the surface multiplied by $\frac{1}{3}$ of the diameter or by $\frac{1}{3}$ of the radius, or the cube of the diameter multiplied by $\frac{1}{6}$ of 3.1416.

4. The largest cube that may be cut from a given sphere equals the square root of $\frac{1}{6}$ of the square of the diameter.

PROBLEMS.

1. What is the surface of a sphere 10 in. in diameter?

Ans. 314.16.

2. What will it cost to gild a spherical ball 60 in. in diameter, at 2 cts. a square inch?

Ans. \$226.19 $\frac{1}{2}$.

3. What is the volume contained in a globe 30 in. in diameter?

Ans. 14137.2 cu. in.

4. How much water will a globe 40 in. in diameter hold if the shell of the globe is 1 in. thick?

Ans. 124.376 + gal.

5. What is the largest cube that could be cut from a sphere whose diameter is 30 in.?
Ans. 17.32+in.

9. BOARD MEASURE.

373. Boards and Lumber are usually measured by the square foot, one inch thick. This is called *Board Measure*. *Hewn and Round Timber* are measured by the cubic foot.

RULES.

1. To find the contents of a board, multiply the length in feet by the breadth in inches, and divide by 12; this will give the number of square feet.
2. To find the contents of plank, etc., multiply the product of the width and thickness in inches by the length in feet, and divide by 12.

PROBLEMS.

1. What is the number of feet in a board 16 ft. long and 9 in. wide?
Ans. 12.
2. What is the number of feet in 12 boards 16 ft. long and 15 in. wide?
Ans. 240.
3. Find the number of feet in a joist 20 ft. long, 8 in. wide and 4 in. thick.
Ans. 53½.
4. What is the value of the following pieces of lumber, at \$15 a thousand?—

6 pieces	16 ft.,	3 × 4,
12	"	18 ft., 3 × 8,
12	"	16 ft., 10 × 2,
40	"	16 ft., 2 × 4.

Ans. \$19.12.

5. Sold the following bill of lumber, at \$18 a thousand:

150 boards	16 ft. by 10 in.,
160	" 12 ft. " 12 in.,
60 plank	14 ft. " 10 × 3,
60 scantling	16 ft. " 4 × 2,
40 rafters	20 ft. " 5 × 3.

What was the value of it?

Ans. \$137.88.

6. What is the value of the following bill of lumber?—

120 pieces	20 ft., 6×3,	at \$15 per M.
32 "	20 ft., 8×6,	at \$16 "
120 "	18 ft., 10×3,	at \$16 "
8 "	24 ft., 8×8,	at \$15 "
200 boards	16 ft. by 12 in.,	at \$22 "
150 "	12 ft. " 9 in.,	at \$20 "
60 plank	14 ft., 2×8,	at \$18 "
130 scantling	16 ft., 2×4,	at \$15 "

Ans. \$335.08.

3. To measure square timber if tapering, add the areas of the two ends in inches, and multiply by half the length in feet, and divide by 12 for board feet, or by 144 for cubic feet.

4. To measure round timber, multiply the square of one-fourth the medium girt in inches by the length in feet, and divide by 144 to find the cubic feet.

NOTE.—The medium girt is the circumference of the log $\frac{1}{2}$ the distance from the larger to the smaller end.

The last rule gives about $\frac{1}{4}$ less than the actual contents, $\frac{1}{4}$ being allowed for waste in working.

PROBLEMS.

1. What are the contents of a stick of timber 40 ft. long, measuring 18 by 16 in. at one end and 16 by 12 at the other?

Ans. 800 sq. ft.

2. What is the value of a log 20 ft. long whose medium girt is 60 in., at $2\frac{1}{2}$ cts. a cubic foot?

Ans. 78 $\frac{1}{2}$ cts.

10. MEASUREMENT OF STONE-WORK.

374. Stone Masonry is usually estimated by the perch.

A *perch* of stone is $16\frac{1}{2}$ ft. long, 1 ft. high and $1\frac{1}{2}$ ft. thick, and is equivalent to $24\frac{1}{4}$ cu. ft.

The following is the common rule to find the contents of a wall or other stone-work.

RULE.

Find the contents in cubic feet, and divide by 24.75.

PROBLEMS.

1. What are the contents of a stone wall 80 ft. long, $2\frac{1}{2}$ ft. thick and 10 ft. high? *Ans.* $80\frac{1}{2}$ pch.

2. What will be the cost of the cellar-wall of a house 60 ft. by 45 ft., the wall to be 8 ft. high and $1\frac{1}{2}$ ft. thick, at \$2.50 a perch, no allowance being made for corners or openings?

Ans. \$254.54.

11. MEASUREMENTS OF GRAIN AND HAY.

375. Grain is usually measured in the United States by the bushel.

The standard bushel is 2150.42 cu. in.

Since a cubic foot is 1728 cu. in., a cubic foot is nearly .8 of a bushel; hence the

RULE.

To find the contents of any quantity of grain in bushels, find the contents in cubic feet, and multiply by .8.

For the same reason any number of bushels increased by $\frac{1}{4}$ the number will equal the number of cubic feet.

Heaped measure is regarded as containing $\frac{1}{4}$ more than *stricken* or *even* measure.

NOTE.—The results under this rule are only approximately correct.

PROBLEMS.

1. How many bushels of grain may be put in a bin 10 ft. long, 8 ft. deep and 8 ft. wide? *Ans.* 512 bu.

2. How many bushels may be put in a cubical bin whose sides are each 9 ft.? *Ans.* 583.2 bu.

3. How many bushels of potatoes, heaped measure, may be put in a wagon-box 18 ft. long, 3 ft. wide and 2 ft. deep?

Ans. 69.12 bu.

4. How many cubic feet in 600 bu. of wheat?

Ans. 750 cu. ft.

5. How many cubic feet in 400 bu. of potatoes, heaped measure?

Ans. 625 cu. ft.

6. How much wheat will 12 bins hold, each 10 ft. long, 5 ft. wide and 8 ft. high? *Ans.* 3840 bu.

In making measurements of hay, about 550 cubic feet of clover or 450 cubic feet of meadow-hay, well settled, equal a ton; hence the

RULE.

To find the quantity of hay in tons in a mow well settled, divide the cubic contents in feet by 550 for clover, or by 450 for meadow-hay.

PROBLEMS.

1. How much clover hay in a mow 30 ft. by 20 ft., and 20 ft. deep? *Ans.* $21\frac{2}{11}$ T.

2. How much meadow-hay in a mow 20 ft. square and 15 ft. deep? *Ans.* $13\frac{1}{4}$ T.

3. What is the value of a cylindrical stack of clover-hay 12 ft. in diameter and 10 ft. high, at \$15 a ton? *Ans.* \$30.84.

12. GAUGING.

376. Gauging is the process of finding the capacity or volume of casks and similar vessels.

377. Ullage is the quantity a cask lacks of being full.

378. The *mean diameter* of a cask is the diameter of a cylinder whose capacity is that of a cask of the same length.

The *mean diameter* is nearly equal to the *head diameter* of the cask, plus $\frac{2}{3}$ of the difference between the *head* and *bung* diameters; or plus $\frac{1}{10}$ the difference of the *head* and *bung* diameters if the cask is but slightly curved.

RULES FOR GAUGING.

1. *Take the square of the mean diameter in inches, and multiply by the length of the cask in inches, and the product by .0034, to find the capacity in gallons.*

2. *When the cask is not full, multiply the square of $\frac{1}{2}$ of the sum of the head, mean and bung diameters, in inches, by the depth of the liquid in inches, and this product by .0034, to find the amount of liquid in gallons.*

PROBLEMS.

1. What are the contents of a cask whose length is 36 inches and the mean diameter 25 inches?

SUGGESTION.— $25^3 \times 36 \times .0034 = 76.5$, or 76.5 gal.

2. What are the contents of a cask 40 inches long whose mean diameter is 30 inches? *Ans.* 122.4 gal.

3. What are the contents of a cask whose head diameter is 18 inches, its bung diameter 24 inches, and the depth of the liquid 15 inches?

SUGGESTION.

$$\text{Mean diameter} = 18 + 4 = 22; \quad \frac{18 + 22 + 24}{3} = \frac{64}{3};$$

$$\left(\frac{64}{3}\right)^3 \times 15 \times .0034 = 23.21 + \text{gal.}$$

4. How much wine in a cask having a head diameter of 20 in. and a bung diameter of 29 in., the depth of the wine being 20 in.?

Ans. 42.5 gal.

CHAPTER XVIII.

METRIC SYSTEM.

379. The **Metric System** is a decimal system of weights and measures, having the *meter* for the base or unit.

380. The **Meter** is one ten-millionth part of the distance from the equator to the pole, measured on the earth's surface at the level of the sea, and is equal to 39.37079 in.

381. The names of the denominations lower than the unit are indicated by prefixing the Latin words *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$) and *milli* ($\frac{1}{1000}$) to the unit.

382. The names of the denominations greater or higher than the unit are formed by prefixing the Greek terms *deca* (10), *hecto* (100), *kilo* (1000) and *myria* (10000) to the unit.

NOTE.—All these terms are accented on the first syllable. The *c* in *deca* is sounded like *k*.

The Metric System was first adopted by France in 1795, and has since been legalized by most other nations, although the system is not compulsory. It is largely used in all countries by scientific men, as being the simplest system of weights and measures in existence.

1. LINEAR MEASURE.

383. The **Meter** (meter) is the unit of linear measure.

TABLE.

10 millimeters (mm.)	= 1 centimeter (cm.)	= .3937 in.
10 centimeters	= 1 decimeter (dm.)	= 3.937 in.
10 decimeters	= 1 meter (m.)	= 39.37 in.
10 meters	= 1 decameter (dm.)	= 393.7 in. = 32.8 + ft.
10 decameters	= 1 hectometer (hm.)	= 328 + ft.
10 hectometers	= 1 kilometer (km.)	= .62138 + mi.
10 kilometers	= 1 myriameter (Mm.)	= 6.2138 + mi.

The meter is used in measuring short distances. It is equal to a little more than 1 yd. $3\frac{1}{8}$ in.

The nickel 5-cent piece is $\frac{1}{10}$ of a meter in diameter.

The kilometer is used in measuring long distances. It is a little less than $\frac{1}{2}$ of a mile.

2. SURFACE MEASURE.

The *square meter* is the unit of measurement when applied to ordinary surfaces, as floors, ceilings, etc.

384. The **Are** (air), equal to ten meters square, or the square decameter, is the unit of land measurement.

TABLE.

100 sq. millimeters (sq. mm.)	= 1 sq. centimeter (sq. cm.)	= .155 + sq. in.
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)	= 15.5 + sq. in.
100 sq. decimeters	= 1 sq. meter (sq. m.)	= 1.196 + sq. yd.

Also,

100 centiares	= 1 are (a.)	= 119.6 + sq. yd.
100 ares	= 1 hectare (ha.)	= 2.471 + A.

A square meter, or 1 centiare, is about $10\frac{1}{4}$ sq. ft., and a hectare a little less than $2\frac{1}{2}$ acres.

3. CUBIC MEASURE.

385. The Cubic Meter, or Stere (stare), is the unit of volume.

TABLE.

1000 cu. millimeters	(cu. mm.) = 1 cu. centimeter	= .061 + cu. in.
1000 cu. centimeters	(cu. cm.) = 1 cu. decimeter	= 61.026 cu. in.
1000 cu. decimeters	(cu. dm.) = 1 cu. meter	= 35.316 + cu. ft.

The cubic meter, or *stere*, is about $1\frac{1}{8}$ cu. yd., and is used for measuring embankments, etc.

The *stere* is the unit of wood or solid measure. It is equal to .2759 of a cord.

TABLE.

10 decisteres (ds.)	= 1 stere	= 35.316 + cu. ft.
10 steres (st.)	= 1 decastere (ds.)	= 13.079 + cu. yd.

4. LIQUID AND DRY MEASURE.

386. The Liter (leeter) is the unit of measure of capacity. It is equal to a cube each of whose sides is $\frac{1}{10}$ of a meter, and is equivalent to about $1\frac{1}{8}$ liq. qt., or $\frac{2}{10}$ of a quart, dry measure.

TABLE.

10 milliliters (ml.)	= 1 centiliter (cl.)	= .338 fl. oz.
10 centiliters	= 1 deciliter (dl.)	= .854 gi.
10 deciliters	= 1 liter (l.)	= 1.0567 liq. qt.
10 liters	= 1 decaliter (dl.)	= 2.6417 gal.
10 decalitters	= 1 hectoliter (hl.)	= 2 bu. 3.35 pk.
10 hectoliters	= 1 kiloliter (kl.)	= 28 bu. $1\frac{1}{2}$ pk.
10 kiloliters	= 1 myrialiter (ml.)	

The **Hectoliter** is used in measuring liquids, grain, fruits and roots in large quantities. It is equal to about $2\frac{1}{2}$ bu.

387. A Kiloliter is a cubic meter or stere.

5. WEIGHT.

388. The Gram is the unit of weight. It is equal to the weight of a cube of distilled water the edge of which is $\frac{1}{100}$ of a meter, equal to 15.432 Troy gr.

TABLE.

10 milligrams (mg.)	- 1 centigram (cg.)	= .15432 + gr. Troy.
10 centigrams	- 1 decigram (dg.)	= 1.54324 + gr.
10 decigrams	- 1 gram (g.)	= 15.4324 + gr.
10 grams	- 1 decagram (dg.)	= .352739 + oz. av.
10 decagrams	- 1 hectogram (hg.)	= 3.52739 + "
10 hectograms	- 1 kilogram (kg.)	= 2.20462 + lb. av.
10 kilograms	- 1 myriagram (mg.)	= 22.0462 + "
10 myriagrams	- 1 quintal	= 220.462 + "
10 quintals	- 1 tonneau, or ton	= 2204.62 + "

389. The **Kilogram** or **Kilo** is the unit of weight in common trade. It is equal to a little less than $2\frac{1}{2}$ lb. av.

390. The **Tonneau** or **Ton** is used for weighing heavy articles. It is equal to a little less than 2205 lb.

391. Metric numbers are written as decimals, with the decimal point after the unit. Thus, 166 meters and 15 millimeters is written 166.015 m.

392. The numbers are read by applying the name of the unit to all on the left of the decimal point, and the name of the lowest denomination indicated to all on the right of the point. Thus, 127.163 m. is read 127 meters and 163 millimeters.

The principles of reduction in this system and the solution of problems are similar to those of integers and decimals, as given in the preceding pages of this work.

CHAPTER XIX.

GENERAL ANALYSIS.

ORAL EXERCISE.

1. A bought hats at \$3 each; had he paid \$5 each they would have cost \$20 more: how many did he buy?
2. John bought oranges at 3 cts. each, and had 40 cts. re-

maining; if he had paid 6 cts. each, he would have but 1 ct. remaining: how many were there?

3. A farmer desiring to purchase some sheep, found if he bought those at $\$4\frac{1}{2}$ each he would have \$75 remaining; but if he bought those at \$6 each, he would lack \$15 of having enough money: how many did he wish to buy?

4. If a man distribute 90 cts. to an equal number of boys and girls, giving to the boys 3 cts. each and to the girls 6 cts. each, how many were there of each?

5. I bought an equal number of oranges, apples and lemons for 72 cts.: if I paid 2 cts. apiece for the apples, 3 cts. apiece for the lemons and 4 cts. apiece for the oranges, how many were there of each?

6. A drover sold a number of pigs and sheep for \$120; for the pigs he got \$8 each, and for the sheep \$4 each: how many of each did he sell if he sold three times as many sheep as pigs?

7. If a man can do a piece of work in 3 d., how much can he do in 1 d.?

8. If a man can do $\frac{7}{8}$ of a piece of work in a day, how long will it take to do $\frac{7}{8}$ of the work?

9. A can do a piece of work in 6 d., and B in 5 d.: how much can each do in a day, and how much both?

10. C can do a piece of work in 3 d., E in 4 d., and F in 6 d.: how much can they together do in a day?

11. Two men can do a piece of work in 5 hr.; the first can do it in 9 hr.: how long will it take the second to do it?

12. Mary can knit a stocking in $\frac{3}{4}$ of a day, and Ida in $\frac{4}{5}$ of a day: how much can they do together in a day, and how long will it take them to knit 6 pairs?

13. A, B and C can do a piece of work in 3 d., C alone in 9 d., and B alone in 10 d.: how long will it take A alone to do it?

14. A, B and C can do a piece of work in 6 d., A and B in 10 d., and A and C in 8 d.: how long will it take each alone to do it?

15. E, F and G can plough a field in 3 wk., E can plough it in 10 wk., and E and F in 6 wk.: how long will it take each to plough it?

16. George and James can cut a quantity of wood in 3 wk.; George alone can cut it in 8 wk.: how long will it take James to cut what remains after George has worked 3 wk.?

17. A barrel of flour will last 3 persons 10 wk.; it would last the first person 40 wk., and the second 25 wk.: how long would it last the third?

18. A man agreed to work 30 d. for \$45 and board; he was to pay \$1 a day board for every day he was idle: how many days did he work if he received \$30?

19. If a man engage to work 30 d. for \$60, and forfeit \$1 for every day he is idle, and at the end of the time gets \$45, how many days was he idle?

20. A man receives \$2 a day for his labor, and pays \$5 a week for his board; at the end of 20 wk. he has saved \$120: how many days was he idle?

21. 5 times a number equals 4 times $\frac{2}{3}$ of the same number, + 30: what is the number?

22. A watch and a chain cost \$120; $\frac{3}{4}$ of the cost of the watch is \$20 more than the cost of the chain: what is the cost of each?

23. The cost of a farm and a house is \$18800: what is the cost of each if $\frac{2}{3}$ of the cost of the farm, + \$1200 equals, $\frac{2}{3}$ the cost of the house?

24. The head of a fish is 3 in. long; the tail is as long as the head + $\frac{1}{4}$ of the body; and the body is as long as the head and the tail: how long is the fish?

25. The foundation-wall of a house is 4 ft. high; the gable is as high as the foundation-wall + $\frac{1}{8}$ of the height of the body of the house; and the body is twice as high as the gable and the foundation-wall: how high is the house?

26. The distance from A to B is 50 mi.; the distance from B to C is equal to the distance from A to B + $\frac{1}{4}$ the distance from C to D; and the distance from C to D is 3 times the distance

from A to C: how far is it from A to D if the four places are in a straight line, with A at one end and D at the other?

27. A boy gave $\frac{3}{4}$ of his money to his sister, $\frac{2}{3}$ of the remainder for a knife, and had 5 cents remaining: how much had he at first?

28. A pole 51 feet in length was broken so that $\frac{3}{4}$ of the longer piece equalled $\frac{2}{3}$ of the shorter: how long was the pole?

29. Two men earn \$100; $\frac{3}{4}$ of what the first earned, + \$8, equals $\frac{2}{3}$ of what the second earned: how much did each earn?

30. In a school of 60 pupils there are 3 girls to every 2 boys: how many girls must leave that the number of girls and boys may be equal?

31. In a school of 50 pupils there are 2 girls to every 3 boys: how many girls should be admitted that there may be 3 girls to 2 boys?

32. An alloy of 60 lb. contains 50 lb. of silver and 10 of copper: how much silver must be added that there may be 9 lb. of silver to 1 lb. of copper?

33. If for every cow a farmer keeps he allow $\frac{1}{4}$ of an acre pasturage and $\frac{2}{3}$ of an acre of corn, how many cows can he keep on 39 acres?

34. If a man can pasture 3 cows on an acre, and 5 cows will eat the produce of 1 acre of roots, how many cows can he keep on 40 acres?

35. If a farmer allow 1 acre of grass to every 2 cows, and 1 acre of roots to every 5 cows, how many cows can he keep on 35 acres, and how much land does he have in grass, and how much in roots?

36. A boy having 60 cents lost $\frac{3}{4}$ of it, and then found $\frac{2}{3}$ as much as he had remaining: how much had he then?

37. James found 20 cents, then lost $\frac{1}{2}$ of what he had, and then had $\frac{2}{3}$ as much as at first: how much had he at first?

38. A man has 197 fowls; $\frac{3}{4}$ of the number of ducks equals $\frac{2}{3}$ of the number of turkeys, and $\frac{1}{2}$ of the number of turkeys equals $\frac{1}{3}$ the number of chickens: how many were there of each?

39. A has 40 dollars, which is $\frac{2}{3}$ as much as B has : if each gives to the other $\frac{1}{2}$ of his money, how much has each then ?

40. How far may a person ride in a buggy at the rate of 6 miles an hour, so that, walking back at the rate of 3 miles an hour, he may reach home in 10 hours ?

41. A boat whose rate of sailing is 8 miles an hour goes down a river whose current is 3 miles an hour, and returns in 8 hours : how far does it go ?

42. A man travels by rail at the rate of 20 miles an hour, and returns by boat at the rate of 12 miles an hour ; he is gone 16 hours : how far does he travel ?

43. What is the time of day if $\frac{1}{4}$ of the time to midnight equals the time past noon ?

44. What is the time of day if $\frac{1}{2}$ of the time past noon equals the time to midnight ?

45. What is the time of day if $\frac{1}{3}$ of the time past midnight equals the time past noon ?

46. What is the time of day if $\frac{1}{2}$ of the time past midnight equals the time to midnight ?

47. What is the time of day if $\frac{1}{2}$ of the time past midnight equals $\frac{1}{3}$ of the time to noon ?

48. What is the time of day if $\frac{1}{6}$ of the time past noon equals $\frac{1}{3}$ of the time to noon again ?

49. At what time after 2 o'clock are the hour-hand and the minute-hand of a watch together ?

50. At what time between 8 and 9 o'clock are the hour-hand and the minute-hand of a watch together ?

51. Two men buy a farm, but by taking in a third as partner the amount to be paid by each is reduced \$2000 : how much is the farm worth ?

52. If 40 men order a supper, but 10 remain away, the expense of the others being increased thereby $\frac{1}{2}$ a dollar each, how much did the supper cost ?

53. A coach is ordered for a party of 6, but 2 fail to pay their share ; the expense of the others is increased \$1 each : what is the total cost ?

54. A man bought some fowls for \$20, but 30 of them having died, he sold $\frac{2}{3}$ of the remainder at cost, and received \$6: how many did he buy?

55. A drover bought a number of sheep for \$500, but having lost 40 of them, he sold $\frac{3}{4}$ of the remainder at cost, and received \$200: how many did he buy?

56. A hare is 40 rd. before a hound, and runs 3 rd. while the hound runs 5: how far will the hound run to catch the hare?

57. A thief is 60 steps before an officer, and runs 5 steps while the officer runs 4, but 4 of the officer's steps equal 6 of the thief's: how far will each run before the thief is caught?

58. Nora is 14 yr. old, and Laura is 4: in what time will Nora be just twice as old as Laura?

59. Edgar is 12 yr. old, and Frank is 2 yr. old: in what time will Frank be $\frac{1}{2}$ as old as Edgar?

60. Bertie is 6 years old, and Mary 9: how long since Bertie was $\frac{1}{3}$ as old as Mary?

61. Three times Jennie's age equals $\frac{3}{4}$ of Gertie's age: in how many years will Gertie be just twice as old as Jennie?

62. Edgar is $\frac{1}{3}$ as old as his father, but in 12 yr. he will be $\frac{1}{2}$ as old: how old is each?

63. Ten years ago, when Mrs. C. was married, she was $\frac{1}{3}$ as old as her husband, but now she is $\frac{2}{3}$ as old: how old was each when they were married?

64. William is 6 times as old as Mary, but in 12 yr. he will be twice as old: how old is each now?

65. A, B and C camping out together, A furnished 6 loaves of bread, and B 9; C paid 40 cts.: how should this be divided between A and B?

66. If A, B, C and D lunch together, A furnishing 7 eggs, B 5, C 4, and D paying 16 cts., how much should each receive of the money?

67. A man being asked the time of day, replied, "It is between 4 and 5 o'clock, and the hands on the dial are exactly opposite:" what was the time of day?

68. A dog ran 100 rd. to catch a deer; $\frac{2}{3}$ of the distance the deer ran equals the distance between them at the start: how much was the deer ahead at starting?

69. A farmer bought some cattle for \$300; had he bought 4 additional at \$5 less each, the whole number would have cost him \$400: how many did he buy, and at what price?

70. A drover sells some sheep for \$200; had he sold 3 times as many at 50 cts. less each, he would have received \$540: how many did he sell, and at what price?

GENERAL REVIEW.

1. A mechanic pays wages as follows: to 6 men, each \$2.25 a day; to 3 men, each \$1.75 a day; and to 5 boys, each 90 cts. a day: how much did he pay to all for 6 days' work?

Ans. \$139.50.

2. A farmer traded his farm of 113 A., worth \$82.50 an acre, for another, valued at \$75 an acre: how many acres did he get?

Ans. 124 A. 48 P.

3. Divide the L. C. M. of 16, 96, 240 by their G. C. D.

Ans. 30.

4. Bought 6 lb. of coffee at 33 cts., 15 lb. of sugar at 12 cts., 12 lb. of rice at 10 cts., 5 lb. of prunes at 17 cts., 12 lb. of cheese at 22 cts., and $1\frac{1}{2}$ doz. dishes at 90 cts. a dozen. Make out a bill.

Ans. \$9.82.

5. What is the G. C. D. of $17\frac{1}{2}$, $18\frac{1}{4}$ and $\frac{14\frac{1}{2}}{3\frac{1}{2}}$? *Ans.* $\frac{1}{2}$.

6. What is the L. C. M. of $16\frac{1}{2}$, $18\frac{1}{3}$ and $\frac{7\frac{1}{2}}{9\frac{1}{3}}$? *Ans.* 4095.

7. Multiply 60.325 by 7.0034, and divide the product by 3.035.

Ans. 139.20267+.

8. What cost 75 A. 10 P. of land at \$165.875 per acre?

Ans. \$12450.99.

9. How many pounds Troy equal 144 lb. avoirdupois?

Ans. 175.

10. How many quarts, wine measure, does 1 bu. contain?

Ans. 37.237 - qt.

11. Two men start at Cincinnati—one travelling east by rail 25 geographical miles an hour, and the other west at the rate of 22 geographical miles an hour: at the end of 36 hours, how much do their watches differ if both have correct time?

Ans. 1 hr. 52 min. 48 sec.

12. A traveller leaves Chicago on Monday, and travels by rail until Saturday night, when he finds his watch 2 hr. 15 min. fast: what direction, and how far east or west from Chicago, has he travelled if his watch has kept correct time?

Ans. 33° 45' W.

13. A farmer bought 100½ A. in one farm, and 87½ in another, and sold 62½ A.: how much had he remaining?

Ans. 125 A. 156 P.

14. Bought 10½ bu. of chestnuts, dry measure, at \$4 a bushel, and sold them at 10 cts. a pint, liquid measure: how much did I make?

Ans. \$36.20.

15. If 15 men can do a piece of work in 40 d., how many will be required to do it in 6 d.?

Ans. 100.

16. If 5 boys or 3 men can do a piece of work in 6 d., how long will it take 5 men and 3 boys to do it?

Ans. 2½ d.

17. If a quantity of feed keep 60 fowls 24 wk., how many must be sold that it may keep the remainder 40 wk.?

Ans. 24.

18. If 25 men earn \$350 in building a wall 650 ft. long, 10 ft. high and 3 ft. thick, how much can they earn in building a wall 800 ft. long, 2 ft. thick and 12 ft. high?

Ans. \$344.62.

19. What cost 10 bu. 3 pk. 1 qt. of beans at \$1.60 per bushel?

Ans. \$17.25.

20. The sum of two numbers is 1750, their difference is 562: what are the numbers?

Ans. $\begin{cases} 594. \\ 1156 \end{cases}$

21. The sum of three numbers is 1540; $\frac{2}{3}$ of the first is $\frac{3}{4}$ of the second, and $\frac{1}{3}$ of the second is $\frac{1}{4}$ of the third: what are the numbers?

Ans. 1st, 540; 2d, 600; 3d, 400.

22. Divide 1200 into four parts which shall be to one another as 2, 3, 4 and 6.

Ans. 1st, 160; 2d, 240; 3d, 320; 4th, 480.

23. John and Samuel together have \$390; 3 times John's, + \$60, equals 4 times Samuel's, - \$30: how much has each?

Ans. John, \$210; Samuel, \$180.

24. Bought a farm of 140 A. 20 P. at \$120 an acre, and sold 72% of it at \$140 an acre: how much had I remaining, and what did it cost me per acre?

Ans. 39 A. 37.6 P., at \$68.57 per acre.

25. A merchant bought \$590 worth of goods; he paid \$35 freight, and sold the goods for \$820: what per cent. did he gain if his store expenses average 12% of his profits?

Ans. 27 $\frac{456}{1000}$ %.

26. I buy 3 doz. shawls at \$13 per dozen, and pay 50 cts. cartage and \$1 freight: how must I sell them apiece to gain 20%?

Ans. \$1.35.

27. If a capitalist invest \$8000 in business, and gain each year 12 $\frac{1}{2}$ % of what he had at the beginning of the year, what will he have at the end of the fifth year? *Ans.* \$14416.26.

28. A merchant failing in business settles with his creditors at 32 cts. on the dollar: how much does A receive whose claim is \$3650; and B, whose claim is \$1650.50?

Ans. A, \$1168; B, \$528.16.

29. If I gain 20% by selling goods at 72 cts. a yard, how much will I gain by selling the same goods at 65 cts. a yard?

Ans. 8 $\frac{1}{3}$ %.

30. Sold 2 horses at \$125 each; on the first I lost 16 $\frac{2}{3}$ %, and on the second I gained 25%: what was the gain or loss?

Ans. Nothing.

31. What will 230 shares of railroad stock cost at 7 $\frac{1}{2}$ premium, par value \$100, and brokerage $\frac{1}{8}$ %?

Ans. \$24696.25.

32. A capitalist bought railroad stocks at 73, par value \$100; the company declared a dividend of 4 $\frac{1}{2}$ %: what per cent. did he realize on his stock?

Ans. 6 $\frac{1}{8}$ %.

33. What is a five-dollar note worth when gold is at a premium of $14\frac{1}{2}$? *Ans.* \$4.37.

34. Which is the cheaper on a currency basis, and how much—to buy a cow at \$60 gold, or at \$68 in currency when gold is at a premium of 14? *Ans.* Currency 40 cts. cheaper.

35. If my watch and chain cost \$147 currency when gold was at a premium of 90, what is the gold value?

Ans. \$77.37.

36. What is the simple interest of \$620.30 in Pennsylvania for 3 yr. 2 mo. 3 d.? *Ans.* \$118.17.

37. What is the simple interest of \$173.40 in New York for 5 yr. 6 mo. 5 d.? *Ans.* \$66.93.

38. A gentleman desires to invest \$2500: which is the better investment—bonds that bear 7% interest or a house which will rent for \$240 a year, his taxes on the real estate being \$30.50 and the annual repairs \$40? *Ans.* Bonds, $2\frac{2}{3}\%$ better.

39. A man dying left an estate of \$23480 to his three sons, aged 15, 13 and 11, to be so divided that, put on interest at 5%, the amounts should be equal as the sons came of age: what amount did he will to each?

Ans. 1st, \$8400; 2d, \$7800; 3d, \$7280.

40. A father left an estate of \$39000 to be so divided between his son and daughter that, being placed on interest at 6%, the son should have twice as much as the daughter when they each reached the age of 21: how much was willed to each if the son was 18 and the daughter 15 years of age?

Ans. Son, \$27200; daughter, \$11800.

41. The amount of C's money for 6 yr. at 6% is \$300 more than the amount of it for 3 yr. at 10%: how much money has he? *Ans.* \$5000.

42. What must be a merchant's marking-price of goods costing 60 cts. a yard, so that falling 20% he may still make 20%? *Ans.* 90 cts.

43. A miller had 800 bbl. of flour, worth \$6 a barrel; a freshet destroyed 30% of it; he sold the remainder at \$8.50: how much did he lose? *Ans.* \$40.

44. A bookseller sold two books for \$3; on one he gained $12\frac{1}{2}\%$, and on the other he lost 20% : did he gain or lose if for the second book he received $\frac{2}{3}$ as much as for the first?

Ans. Loss, 10 cts.

45. The list-price of a certain book is \$18 a dozen; a merchant buys at $33\frac{1}{3}\%$ discount, and sells the books at \$1.25 apiece: what per cent. does he make?

Ans. 25.

46. What is the discount on a note of \$700 for 90 d. at 8% , bankers' rule?

Ans. \$14.62.

47. A merchant desires to borrow \$600 for 60 d. from bank: what is the face of the note, discount being at 6% , bankers' rule?

Ans. \$606.47.

48. What per cent. does a capitalist make on his money if he invest \$3680 in 5's of '81 when they sell at 92?

Ans. $5\frac{1}{4}\%$.

49. How much can I afford to give for 6's of '81, so that I may realize 8% per annum, gold being at a premium of 15?

Ans. 86 $\frac{1}{2}$.

50. A box of glass contains 50 sq. ft.: how many panes in each of four boxes marked respectively 8×10 , 10×12 , 9×16 and 10×18 .

Ans. 1st, 90; 2d, 60; 3d, 50; 4th, 40.

51. Rankin, Parsons and Lander are in partnership; Rankin puts in \$3000 for 5 yr.; Parsons, \$2000 for 8 yr.; and Lander, \$5000 for $3\frac{1}{2}$ yr.; they gain \$14550: what is each one's share of the gain?

Ans. R., \$4500; P., \$4800; L., \$5250.

52. A lot is 60 ft. front and 160 ft. deep: what is the length of its diagonal?

Ans. 170.88 ft.

53. What is the value of a building association share, monthly payments \$1, drawing interest at 6% , 72 instalments having been paid?

Ans. \$85.14.

54. What is the value of a building association share, paying 10% interest to withdrawing members, the shares being 7 yr. old, payable \$1 monthly?

Ans. \$113.75.

55. A man rents an office for \$350 a year, payable monthly in advance; no settlement is made till the expiration of 5 yr.: what is the amount due, interest at 6% ?

Ans. \$2016.87 $\frac{1}{2}$.

56. I rent a house for \$120 per annum, payable quarterly in advance: what amount is due at the expiration of 2 yr., no payments having been made, interest at 6%? *Ans.* \$256.20.

57. What will it cost to carpet a room 18 ft. long, 15 ft. wide, with carpet 27 in. wide, worth \$2.50 a yard? *Ans.* \$100.

58. How many rolls of paper 8 yd. long and $\frac{1}{4}$ yd. wide will it take to paper a room 24 ft. long and 18 ft. wide, the height of the ceiling being 10 ft? *Ans.* 23 $\frac{1}{4}$.

59. How many cubic feet of air in a sleeping room 15 ft. by 13 ft., the ceiling being 9 ft. high? *Ans.* 1755.

60. What will it cost to have a pile of wood 60 ft. long, 4 ft. wide and 6 ft. high sawed at 75 cts. a cord? *Ans.* \$8.44.

61. What will it cost to have a cellar dug, one part of which is 31 ft. long and 16 ft. wide, and the other 23 ft. long and 16 ft. wide, the average depth being 5 ft., at 25 cts. a cubic yard? *Ans.* \$40.

62. What is the value of the oil in a cylindrical tank 30 ft. in diameter and 15 ft. deep, at 2 $\frac{1}{2}$ cts. a gallon?

Ans. \$1982.88.

63. A garden is 160 ft. long and 60 ft. wide; a path 3 ft. from the fence and 3 ft. wide runs around the whole garden: how many square feet does the path contain? *Ans.* 1212.

64. At what time between 9 and 10 o'clock are the hour-hand and the minute-hand of a clock together? *Ans.* 9 hr. 49 $\frac{1}{11}$ min.

65. $\frac{1}{3}$ of the corn a farmer raised he fed to his pigs, $\frac{1}{4}$ to his horses, $\frac{1}{5}$ he sold, and the remainder, 390 bu., he has in his granary: how many bushels did he raise? *Ans.* 1800.

66. If a peddler buy 5 doz. apples at 2 for a cent, and 5 doz. pears at 3 for a cent, and sell them at 5 for 2 cents, what per cent. does he gain or lose? *Ans.* 4% loss.

67. Which is the better investment—5's of '81 at 80%, or 6's of '81 at 90%? *Ans.* 6's, by $\frac{5}{12}$ %.

68. When a five-dollar bill is worth \$4.44 $\frac{4}{5}$ in gold, what is the premium on gold? *Ans.* 12 $\frac{1}{2}$ %.

69. $\frac{1}{3}$ is what per cent. of 2 $\frac{1}{3}$? *Ans.* 13 $\frac{1}{3}$.

70. A dealer buys 40 M lumber at \$15 a thousand, cash,

and sells it for \$16 a thousand, taking a 6 months' note in payment: what does he make if the note is discounted at 6%, no days of grace being allowed? *Ans.* \$20.80.

71. A and B can do a piece of work in $12\frac{1}{2}$ days, B and C in 15 days, and A and C in 14 days: how many days will it take each alone to do it? *Ans.* A, $23\frac{1}{3}$; B, $26\frac{1}{3}$; C, $34\frac{1}{3}$.

72. Boggis, Wilson and Bitner engaged in trade; Boggis furnished \$6000, and Wilson \$5000; they gained \$2950; Bitner's share of the gain was \$750: how much did Boggis and Wilson each gain, and what part of the capital did Bitner furnish?

Ans. Boggis's gain, \$1200; Wilson's, \$1000; Bitner's capital, \$3750.

73. $\frac{2}{3}$ of A's money equals $\frac{1}{4}$ of B's, and the difference of the money is \$126: how much has each?

Ans. A, \$210; B, \$336.

74. Clark and Welliver made a joint stock of \$5240, on which they gained \$786, of which Welliver's gain was \$45 more than Clark's: how much capital did each furnish?

Ans. W., \$2770; C., \$2470.

75. How many brick 8 in. square, and how many 10 in. square, will it take to pave a walk 120 ft. long and 10 ft. wide?

Ans. 2700 eight-inch, or 1728 ten-inch.

76. A merchant buys cloth at \$3.22 a yard, and after keeping it 1 yr. sells it at \$3.50 a yard, and does not collect the account for 6 mo.: does he gain or lose, and how much, interest being at 6%? *Ans.* Loses 9.8 mills per yard.

77. If I take a policy of insurance for \$1000 on my life for 20 yr. when 30 yr. of age, to be paid in 20 annual instalments of \$48.53 each, what do I gain or lose, counting simple interest on my instalments as worth 6% per annum?

Ans. Lose \$21.56.

78. Wishing to find the height of a tree, I erected a stick 4 ft. high which cast a shadow 2 ft. 6 in.; the shadow of the tree was 90 ft. in length: what was the height of the tree?

Ans. 144 ft.

79. A laborer agrees to work for \$1.25 a day and his board; for every day he is idle he pays 35 cts. for his board; he saves \$80.40 in 12 wk.: how many days was he idle? *Ans.* 6.

80. If a manufacturer sell goods to a jobber at 20% above cost of manufacturing, and the jobber sell to the retail merchant at 20% advance, who in turn makes 25% by selling at \$1.08, what is the cost of manufacture? *Ans.* 60 cts.

81. A pile of wheat was heaped in the corner of a room in the form of a portion of a cone: if the room was rectangular, and the height of the pile 6 ft., the radius being 5 ft., how many bushels of wheat were in the pile? *Ans.* 31.556.

82. A pile of wheat is heaped against the side of a wall in the shape of a portion of a cone; the perpendicular height is 4 ft., and the slant height 6 ft. 8 in.: what is the wheat worth at \$1.25 a bushel? *Ans.* \$59.84.

83. How large must a cubical bin be to hold 250 bu.?

Ans. 6 ft. 9 in. +

84. A merchant failing offers to compromise with his creditors at 75 cts. on the dollar, or pay them in full at the end of 6 yr. if permitted to continue his business: which is the more profitable to them, and how much, money being worth 6 per cent.?

Ans. The former, by 2%.

85. A man learning to smoke at the age of 16, spends 10 cts. a day for cigars: had the money been saved and invested at 6 per cent., simple interest, annually, what could he have saved for his family were he to die at the age of 66, not counting extra days in leap year?

Ans. \$4617.25.

86. Suppose a smoker who spends \$30 a year were to abstain from the use of tobacco for 20 yr., and invest the money at compound interest: how much would he save in that time, interest at 6%?

Ans. \$1103.57.

87. The width of the gable of a house is 30 ft., and the height 8 ft.: how long are the rafters if they project 2 ft.?

Ans. 19 ft.

88. Three men, A, B and C, hire a cab for \$15 to drive 30 miles; having gone 12 miles, they take in D, and after having

driven 6 miles more, they take in E: how much should each pay? *Ans.* A, B and C, each \$3.75; D, \$2.25; E, \$1.50.

89. A man dying left an estate worth \$20000 to be divided as follows: to his daughter $\frac{1}{4}$ of his money, to his son $\frac{5}{12}$, and to his widow $\frac{1}{4}$: if the widow die, what is the proportionate share of the son and the daughter each?

Ans. Son, \$12500; daughter, \$7500.

90. A pole whose length is 134 feet stands on a bluff; 16 times the length in the ground, + 6 feet, equals $\frac{1}{3}$ of the length in the air increased by 80 feet: how much of the pole is in the air, and how much in the ground?

Ans. 4 ft. in ground, 130 ft. in air.

91. A farmer sold 100 lb. of butter and cheese for \$37.80; for the butter he received 45 cents a pound, and for the cheese 27 cents: how many pounds of each were there?

Ans. Butter, 60; cheese, 40.

92. The head of a fish weighs 16 oz., the tail weighs as much as the head, + $\frac{1}{4}$ of the body, and the body weighs twice as much as the head and the tail both: what is the weight of the fish?

Ans. 12 lb.

93. A boy has a number of apples at 3 for a dime, and an equal number at 5 for a dime: suppose he sells them at 2 for 5 cents, will he make or lose, and how much?

Ans. Loses $\frac{1}{6}$ of a dime on each.

94. Two men engaged to build a house for \$540; the first worked $\frac{2}{3}$ as many days as the second, plus 40, and received \$300: how many days did each work?

Ans. 1st, 100; 2d, 80.

95. I gave $\frac{1}{4}$ of my money, plus \$100, to James; $\frac{1}{4}$ of the remainder, plus \$120, to John; $\frac{1}{4}$ of the remainder, plus \$150, to George; $\frac{1}{4}$ of the remainder, plus \$400, to Silas, and still have $\frac{1}{5}$ of my money remaining: how much had I at first?

Ans. \$3000.

96. Ten years ago Edwin was $\frac{1}{3}$ as old as his father, but two years hence he will be $\frac{1}{2}$ as old: what is the age of each?

Ans. Edwin, 22; father, 46.

97. If a merchant's gain at retail is 25%, and he sells at wholesale for 10% below his retail price, what is his gain at wholesale? *Ans.* $12\frac{1}{2}\%$.

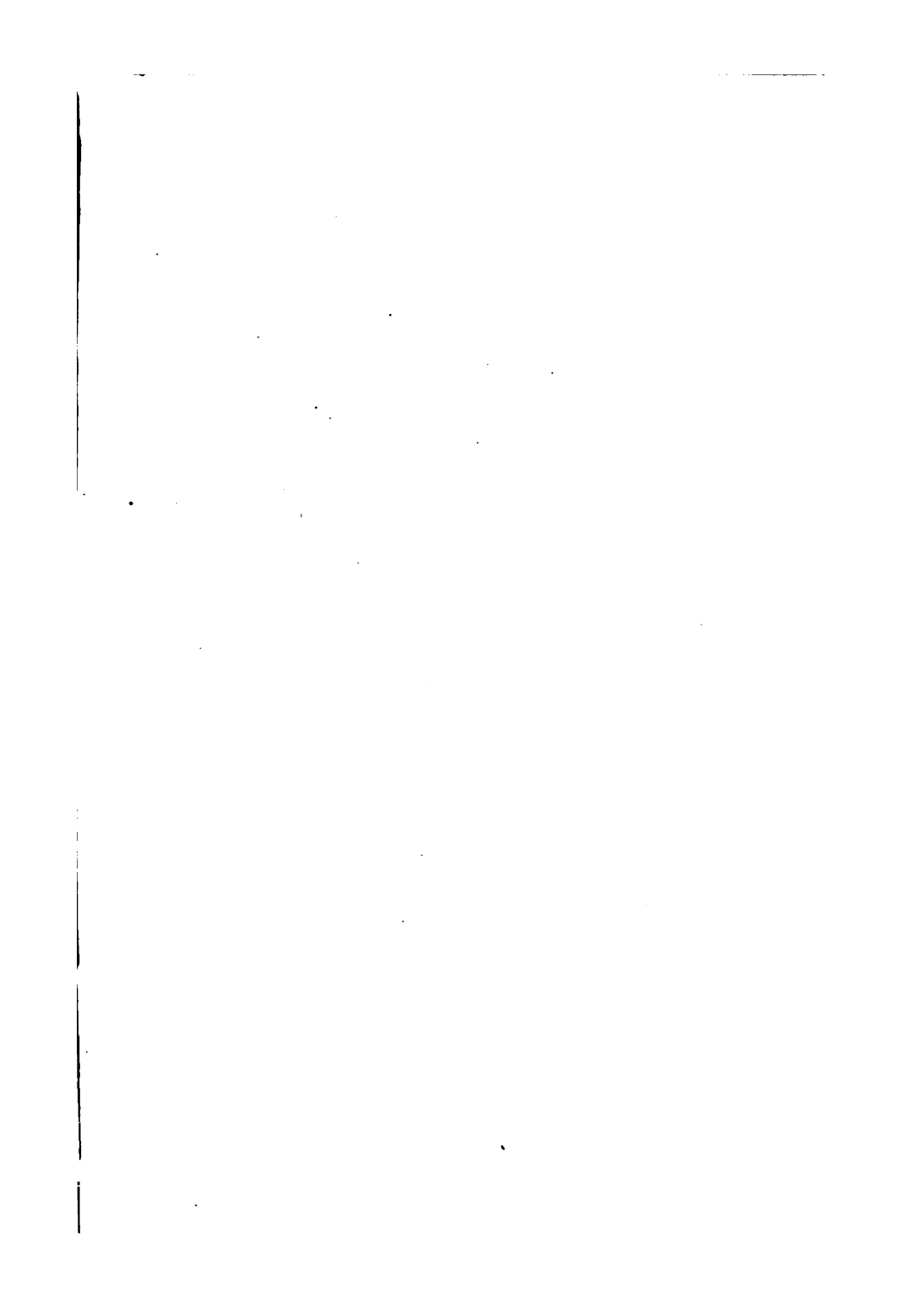
98. A man has two watches and a chain; the first watch is worth \$60; the first watch and the chain together are worth $\frac{5}{4}$ as much as the second watch; and the second watch and the chain are worth twice as much as the first watch: what is the value of each? *Ans.* Chain, \$40; 2d watch, \$80.

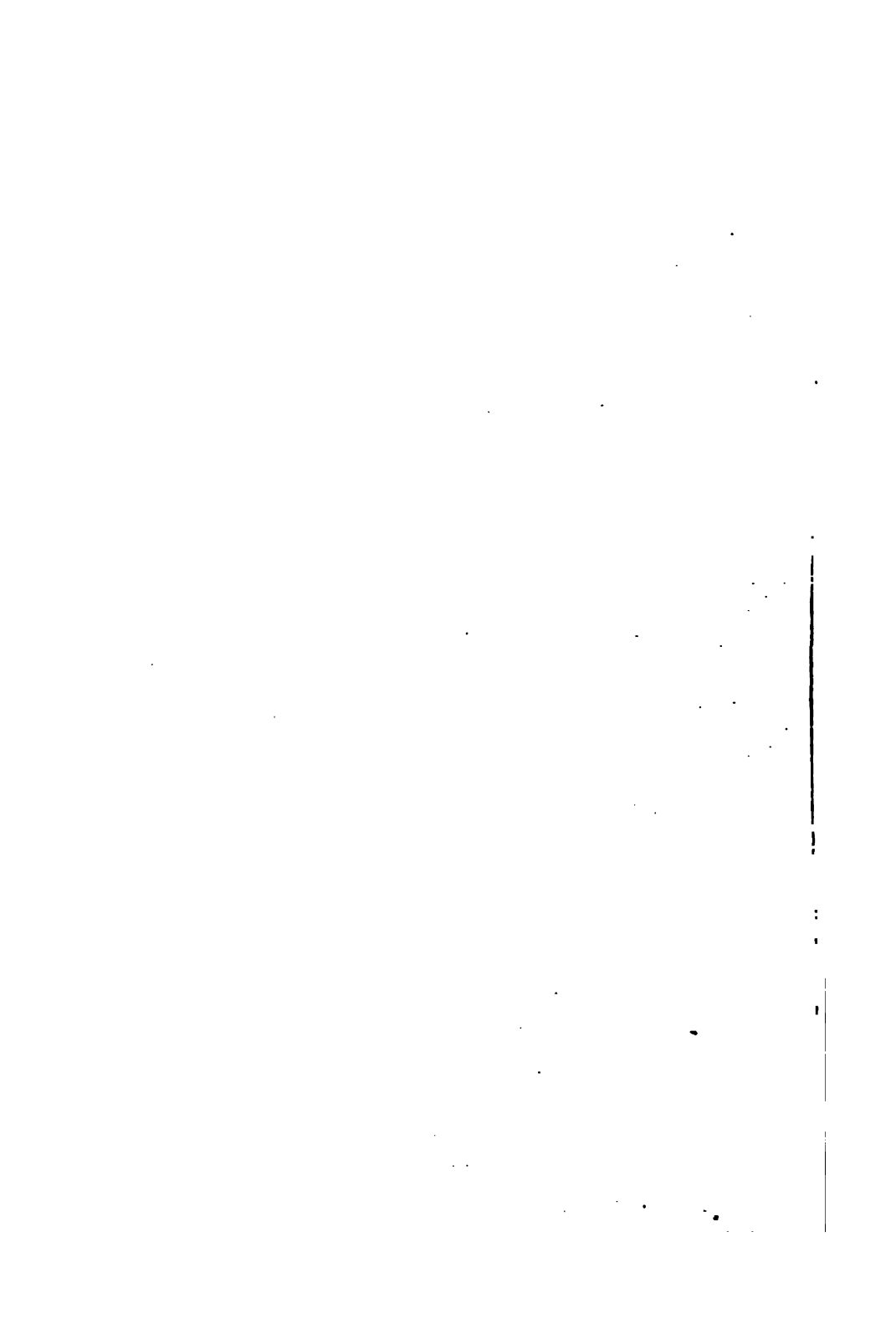
99. If 60 lb. of salt water contains 5 lb. of salt, how much fresh water must be added that 100 lb. of water may contain but 2 lb. of salt? *Ans.* 190 lb.

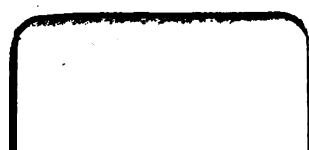
100. How much water will a bucket hold whose upper diameter is 13 in., its lower diameter $8\frac{1}{2}$ in., and its height $8\frac{1}{2}$ in.? *Ans.* 13 qt., 1.1 pt.

THE END.









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